

CHAPTER

1

Properties of Matter

1.1 Introduction

The advancement in technology & Engineering is possible through our detailed understanding of properties of matter. The mechanical properties of solids generally deals with deformations due to the external forces impressed on the materials. The property, elasticity is important for many applications in Engineering & Technology. [e.g a bridge used for traffic is subjected to loads or forces of varying amounts. Before a steel bridge is constructed, the steel samples are sent for testing to find whether the steel can withstand the loads likely to be put on them or not].

The development of dynamics begins with the concept of an infinitely small but massive particles for their behaviour under the influence of applied forces. Later the idea of rigid body came into force. The behaviour of ordinary material under the action of forces constitutes the study of elasticity.

1.2 Rigid Body

A *rigid body* is one in which the relative position of its constituent particles does not change under the influence of impressed forces or in other words an ideal rigid body does not change in shape or size under the influence of applied force of any magnitude. When a body is subjected to external forces, it may get deformed. Such a force is called *Deforming Force*. As a result of deforming forces applied to a body reactionary forces come into play internally due to relative displacement of its molecules, tending to balance the load and restore the body to its original condition. The body undergoes a change in its shape or size or both. These reactionary forces tend to restore the body to its original conditions. When the deforming forces are removed, the body tends to recover its original form.

The property of a material body by virtue of which the bodies are restored to their original shape or size or both after the removal of external deforming force is called elasticity. Extent to which the original form of a body is restored, when

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the deforming forces are removed varies from material to material. Based on their behaviour perfect elastic and perfect plastic bodies are available.

Bodies which can recover completely their original conditions on removal of deforming forces are known as perfectly elastic bodies.	Bodies which do not show any tendency to recover their original condition & retain completely are known as perfectly plastic bodies.
They develop a definite amount of deformation which does not increase when the force is prolonged. (eg) quartz fibre	Partially regain their original form. Putty

Generally no body is perfectly elastic or plastic. Actual bodies behave between these two limit. Concept of perfectly elastic and perfectly plastic bodies is an idealization.

1.3 Stress

When a body is deformed by external force, internal reactionary forces are developed between the molecules of the body to oppose the action of the deforming force, which tend to restore the body to its original condition.

The internal restoring force developed / unit area of the body when subjected to external deforming force is called stress.

Being a disturbed force, it is measured in the same manner as fluid pressure (i.e.,) in terms of load on deforming force applied / unit area of the body, being equal in magnitude but opposite in direction to it, until a permanent change has been brought about in the body.

These forces are self adjusting forces. As deforming force increases restoring force also increases. When deforming force is equal to restoring force, body attains equilibrium. Due to this, at equilibrium, stress can be measured by the deforming force applied on a unit area of the body.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{area}} = \frac{\text{Deforming force}}{\text{area}} = \frac{F}{A}$$

Unit of stress is Newton per metre² (N/m²) and its dimensional formula is [ML⁻¹T⁻²]. The stress developed in a body depends upon how the external forces are applied over it. Depending on this three types of stress, are as follows.

Types of stress

(a) Longitudinal stress (or) Tensile stress

If the deforming force acting on a body is along its longitudinal axis and produces a change in its length, then the deforming force / unit area acting normal to the surface is called longitudinal or normal or tensile stress.

If the applied force is a thrust or produces a compression, then the stress is called normal compressive stress.

Thus, Tensile stress = force / unit area = F/A

$$\text{Tensile stress} = F/A$$

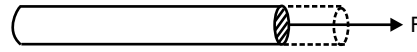


Fig. 1

(b) Volume or Bulk stress

If the equal deforming forces can be applied uniformly on each 6 faces of a cube in outward direction, then the cube suffers an increase in its volume.

Under equilibrium, the applied force/unit area is called volume or bulk stress.

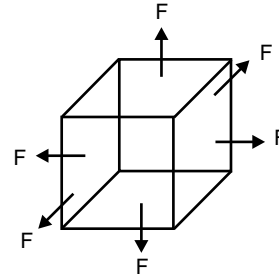


Fig. 2

(c) Shearing stress (or) Tangential stress

If the deforming forces are applied tangentially over the top surface of a cube and bottom surface being kept fixed then, the top face gets displaced towards the direction of applied force.

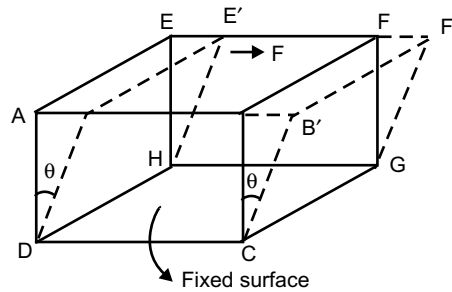


Fig. 3

The tangential force/unit area or the stress which tends to make one part of the body slide across the other part is termed as shearing stress or tangential stress.

1.4 Strain

A body under deforming forces undergoes a change in length, volume or shape. Then the body is said to be under strain. The strain produced in the body is measured in terms of the fractional change produced in the dimensions of a body. Under a system of forces in equilibrium strain is also measured as the ratio of change in dimension of the body to its original dimension. As strain is just a ratio, it is a dimensionless quantity, having no units.

Types of strain

(a) Longitudinal strain or Tensile strain

If the deforming force is of the nature of pull or a tension and acting along the longitudinal axis of a wire of length L , and produces a change in length ΔL without any change in shape, then this fractional change $\Delta L/L$, is called longitudinal strain.

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

If length increases from its natural length, then it is tensile strain. If in case there is decrease in length then it is compressive strain.

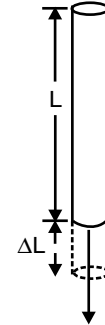


Fig.4

(b) Volume strain

When the forces or pressure are applied uniformly and normally inwards (or outwards) over the whole surface of a body of volume V , then its volume gets decreased (or increased) by an amount ΔV without any change in shape. The ratio of this change in volume to its original volume is called volume strain.

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

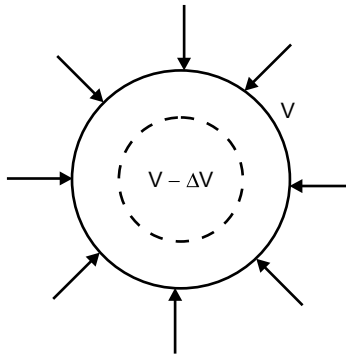


Fig.5

(c) Shearing strain or Shear

When the deforming forces are applied tangentially over the top surface of the body, it suffers a change in shape without any change in volume or length and is said to be sheared. Shear is numerically equal to the ratio of the displacement of any layer in the direction of applied tangential force to its distance from the fixed surface.

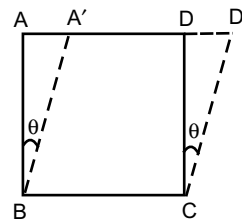


Fig.6

The strain produced is measured by an angle which a tilted surface makes with original vertical surface.

$$\frac{AA'}{AB} = \theta$$

Strain produced by stretching or compressing force, both volume & shape of the body may alter, while the strain produced by shearing forces only the shape of the body is altered though the volume remains constant.

Elastic limit

The maximum stress which produces maximum amount of recoverable deformation is called Elastic limit. If the stress applied exceeds the elastic limit, then the substance does not return to its original state when the stress is removed. The substance is then said to have acquired permanent set.

1.5 Hooke's Law

If a substance is subjected to a stress below the elastic limit, it recovers completely when the stress is removed or within elastic limit, the stress is directly proportional to strain produced.

The linear relationship between the stresses and deformations produced below elastic limit is called Hooke's law. (i.e.,)

$$\begin{aligned} \text{Stress} &\propto \text{Strain} \\ \frac{\text{Stress}}{\text{Strain}} &= \text{Constant} \\ &= E \rightarrow \text{Modulus of elasticity (coefficient of elasticity)} \end{aligned}$$

The value of modulus of elasticity depends upon the type of stress and strain produced.

Young's modulus

If the strain is longitudinal then the modulus of elasticity is called "Young's Modulus" (Y).

$$\begin{aligned} Y &= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} \\ &= \frac{FL}{A\Delta L} \end{aligned}$$

Within elastic limit, the ratio of longitudinal stress to the corresponding longitudinal strain is called Young's modulus of elasticity.

Bulk modulus

When a uniform pressure (normal force) is applied normally over the whole surface of a body of an isotropic material, it suffers a change in its volume though its shape

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remains unchanged within elastic limit. The ratio of the volume stress to the volume strain is called bulk modulus of elasticity of the material.

$$K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{F/A}{\Delta V/V} = \frac{PV}{\Delta V}$$

Reciprocal of K is called compressibility. Unit N/m^2 and its dimensional formula N/m^2 $[\text{ML}^{-1}\text{T}^{-2}]$

Modulus of rigidity

When a body is subjected to tangential deforming force, it suffers a change in shape but volume remains unchanged. Then body is said to be sheared. The stress developed in this case is called shearing stress, due to which a shearing strain is developed.

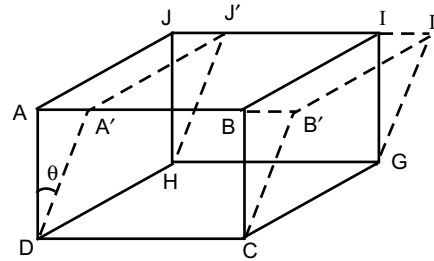


Fig. 7

Within the elastic limit, the ratio of shearing stress or tangential stress to shearing strain is called modulus of rigidity of the material.

If θ is angle of shear, then

$$\begin{aligned} \theta &= \tan \theta = \frac{BB'}{BC} \\ &= \frac{\text{displacement of top surface}}{\text{distance of top surface from fixed surface}} \\ n &= \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\theta} \end{aligned}$$

Unit of n is N/m^2 and dimensional formula is $[\text{ML}^{-1} \text{T}^{-2}]$

1.6 Poisson's Ratio

When a wire is pulled, it not only becomes longer but also thinner. If a force produces elongation or extension in its own direction, a contraction also occurs in a direction perpendicular to it, that is in lateral direction or vice versa. The fractional change in the direction of applied force is longitudinal strain, fractional change in perpendicular direction is lateral strain.

Within elastic limit, ratio of lateral strain to longitudinal strain is constant for a given material and is called poisson's ratio (σ).

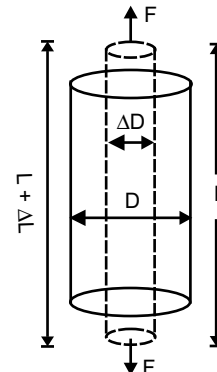


Fig. 8

In the Fig, a wire of original length L and diameter D is acted upon by two equal and opposite force F along the length. Its length increases by ΔL , while its diameter decreases by ΔD .

$$\begin{aligned}\alpha &= \frac{\Delta L}{L}; \quad \beta = \frac{\Delta D}{D} \\ \sigma &= \frac{\beta}{\alpha} = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-(\Delta D/D)}{\Delta L/L} \\ &= -\frac{L}{D} \frac{\Delta D}{\Delta L}\end{aligned}$$

σ as a differential coefficient,

$$\sigma = \frac{L}{D} \frac{dD}{dL}$$

Minus sign indicates that increase in the direction of force would be accompanied by decrease in the direction perpendicular to the force. σ is dimensionless and has no units.

1.7 Relation between the Three Moduli of Elasticity (Y, K and n)

To arrive at the relation between Young's modulus, bulk modulus and rigidity modulus, the three moduli need to be expressed in terms of longitudinal strain (α) and lateral strain (β) and hence in terms of σ (Poisson's ratio) must be known.

(i) Relation between Young's modulus and α

$$Y = \frac{1}{\alpha}$$

(ii) Bulk modulus in terms of α and β

$$K = \frac{1}{3(\alpha - 2\beta)}$$

(iii) Rigidity modulus in terms of α and β

$$n = \frac{1}{2(\alpha + \beta)}$$

(iv) Relation between Y , K and σ

We know

$$\sigma = \frac{\beta}{\alpha}; \quad Y = \frac{1}{\alpha}$$

$$\text{and} \quad K = \frac{1}{3(\alpha - 2\beta)}$$

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$$\text{now } K = \frac{1}{3\alpha(1-2(\beta/\alpha))} = \frac{1/\alpha}{3(1-2\sigma)} = \frac{Y}{3(1-2\sigma)} \quad \left(\because Y = \frac{1}{\alpha} \right)$$

$$\text{(or) } \boxed{Y = 3K(1-2\sigma)} \quad (1)$$

(v) Relation between n , Y and σ

$$\begin{aligned} n &= \frac{1}{2(\alpha + \beta)} \\ &= \frac{1}{2\alpha(1 + \beta/\alpha)} = \frac{1/\alpha}{2(1 + \sigma)} = \frac{Y}{2(1 + \sigma)} \end{aligned}$$

$$\text{(or) } \boxed{Y = 2n(1 + \sigma)} \quad (2)$$

(vi) Relation between Y , K , n and σ and Relation between the three moduli of Elasticity

Now from (1) and (2)

$$Y = 3K(1 - 2\sigma) \text{ and } Y = 2n(1 + \sigma)$$

$$\text{(or) } \frac{Y}{3K} = 1 - 2\sigma \quad (3)$$

$$\text{and } \frac{Y}{n} = 2(1 + \sigma) \quad (4)$$

Adding the two equations (3) and (4)

$$\frac{Y}{3K} + \frac{Y}{n} = 3 \quad \text{(or) } \boxed{Y = \frac{9Kn}{n + 3K}}$$

$$\text{(or) } \boxed{\frac{9}{Y} = \frac{1}{K} + \frac{3}{n}}$$

Now, Dividing equation (3) by (4)

$$\frac{n}{3K} = \frac{1 - 2\sigma}{2(1 + \sigma)} \quad \text{(or) } 2n(1 + \sigma) = 3K(1 - 2\sigma)$$

$$\text{(or) } 2n + 2n\sigma = 3K - 6K\sigma$$

$$\text{(or) } \boxed{\sigma = \frac{3K - 2n}{2n + 6K} = \frac{3K - 2n}{2(n + 3K)}}$$

1.7.1 Limiting values of poisson's ratio

We know $Y = 3K(1 - 2\sigma)$ & $Y = 2n(1 + \sigma)$

$$3K(1 - 2\sigma) = 2n(1 + \sigma) \quad (5)$$

(i) If σ is positive, RHS is positive.

\therefore LHS must be positive.

It will be only when

$$2\sigma < 1$$

$$\sigma < \frac{1}{2}$$

$$\boxed{\sigma < 0.5}$$

(ii) If σ is negative. LHS is positive.

\therefore RHS to be positive.

$$1 + \sigma > 0 \Rightarrow \boxed{\sigma > -1}$$

$$\boxed{-1 < \sigma < \frac{1}{2}}$$

1.8 Behaviour of a Wire under an Increasing Load

Let a wire be clamped at one end & loaded at the other end gradually from zero value until the wire break down.

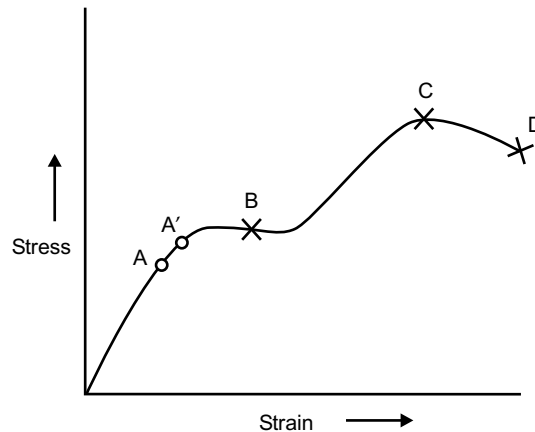


Fig. 9

A-Proportional limit; A'-Elastic limit

B-Yield point; C-Ultimate tensile strength D-Breaking stress

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1. The part OA of the curve is a straight line, where Hooke's law is obeyed. (i.e.,) Stress is proportional to strain.
2. A is called Proportional limit measured by the maximum stress that can be developed in the given material without causing a deviation from Hooke's law.
3. Vicinity of A lie another point A' , known as Elastic limit upto A' . The wire behaves as a perfectly elastic body, upto A' . In AA' of the curve the stress should be proportional to the strain is not necessary. A & A' – very nearer to each other, may coincide for some materials.
4. If wire is loaded beyond the elastic limit A' , the wire gets stretched & attains a permanent set (ie) there is a permanent deformation in the body after the removal of deforming forces.
5. On increasing the load still further a point B called yield point at which extension of the wire increases rapidly without an increase in the load. For a given material, the yield point is usually determined by the minimum value of stress for which the material begins to deform appreciably without an increase of load. The value of stress at the yield point is called **yield strength** of that material.
6. Elongation without addition in load is called creeping and this behaviour of the metal is called yielding.
7. If the wire is further loaded, a point represented by C is reached after which the wire begins to flow locally so that its cross sectional area gradually decreases. At point C the value of the developed stress is maximum and is called the **ULTIMATE TENSILE STRENGTH** or tensile strength of the given material.
8. Tensile strength is defined as the maximum value of tensile stress that a material can withstand before fracture under a steady load.

$$\text{Tensile strength} = \frac{\text{Maximum tensile load}}{\text{Original cross sectional area}}$$

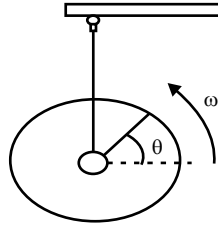
9. The stress corresponding to point D where the wire actually breaks down, is called **Breaking stress**. The normal value of the breaking stress is found to be less than that of the ultimate strength due to fracture formation. But the applied load at the breaking point is greater than the ultimate tensile strength point.

Normally working stress on a body is kept far below the ultimate tensile stress and is never allowed to cross the elastic limit. The above fact is practiced by all design engineers to get higher stability & reliability of the structures.

1.9 Torsional Pendulum

Consider a cylindrical wire clamped vertically at one end. The other end carries a disc of moment of inertia 'I' about the wire as its axis. Let the wire be twisted by

turning the body through a small angle ' θ ' and released. The disc executes Torsional Oscillations. This arrangement is called Torsional Pendulum.



Angle of twist and angle of shear

Imagine a cylindrical rod of length l and radius r fixed rigidly at its upper end. When a couple is applied at its lower free end in plane perpendicular to the length & whose axis coincides with the axis of the cylinder, the radius of each circular cross section of the rod gets rotated in its plane about the axis of the rod, by an amount proportional to the distance of the cross section from the fixed upper end.

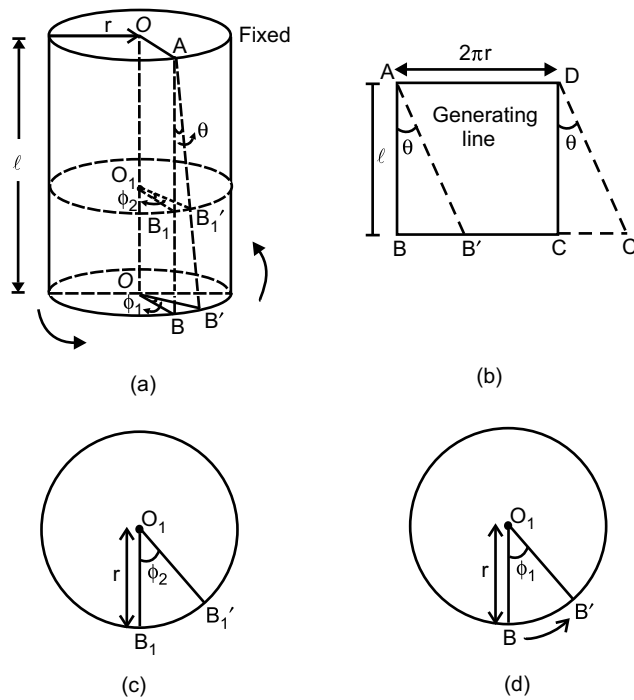


Fig. 10

The angle through which any cross section rotates is called the angle of twist (ϕ). Its value is zero at fixed end & maximum at the free end (Fig. 10(a)).

As the rod is twisted, a restoring couple is developed in it due to the elastic properties of the material which is equal & opposite to the external twisting couple.

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As the rod is twisted, a generating line AB (Fig. 10(a) and (b)) on its outermost layer is turned through an angle θ . Thus the line AB_1B moves into the position AB'_1B' , having been turned through an angle BAB' equal to θ . This angle is called Angle of shear.

From Fig. 10(a) angle in $\triangle BAB'$

$$\theta = \frac{BB'}{l} \Rightarrow BB' = l\theta \quad \left(\text{as angle} = \frac{\text{arc}}{\text{radius}} \right)$$

In $\triangle BOB'$

$$\phi = \frac{BB'}{r} \Rightarrow BB' = r\phi$$

$$l\theta = r\phi$$

$$\boxed{\theta = \frac{r}{l}\phi}$$

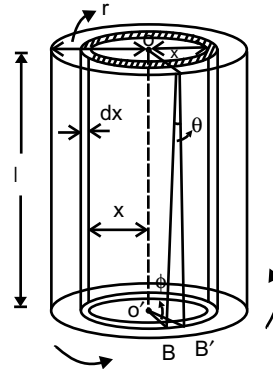


Fig. 11

1.10 Twisting Couple on a Cylindrical Rod

Consider a cylindrical rod of length l , radius r and modulus of rigidity n fixed at upper end and twisted at the lower free end by means of a couple of moment τ in a plane perpendicular to the length of the rod such that the axis of twisting couple coincides with the axis of the cylinder. As the rod is twisted, a restoring couple, equal & opposite to the twisting couple is set up in the cylinder due to elasticity of the material. To find the magnitude of this couple, consider the cylinder to be made up of large number of coaxial cylindrical shells.

Consider one shell of radius x and radial thickness dx .

$$\therefore BB' = l\theta = x\phi \quad (\text{or}) \quad \theta = \frac{x}{l}\phi \quad (6)$$

Let F be the tangential force acting over the base of the elementary cylindrical shell of radius x and width dx . Area of this annular shell = $2\pi x dx$.

Tangential stress acting on thin shell is

$$\text{Tangential stress} = \frac{\text{force}}{\text{area}} = \frac{F}{2\pi x dx} \quad (7)$$

Thus, Rigidity modulus n of the rod is

$$\begin{aligned} n &= \frac{\text{tangential stress}}{\text{shear}} = \frac{F/2\pi x dx}{\theta} \\ n &= \frac{F}{2\pi x dx} \cdot \frac{l}{\phi x} \\ F &= \frac{2\pi n \phi}{l} x^2 dx \quad (8) \end{aligned}$$

Moment of this force about the axis of the rod OO' is

$$Fx = \frac{2\pi n\phi}{l} x^3 dx \quad (9)$$

This is equal to the couple required to twist the elementary shell through an angle ϕ . The couple τ required to twist the whole rod is obtained by integrating Eq(9) within the limits $x = 0$ & $x = r$.

$$\tau = \int_0^r \frac{2\pi n\phi}{l} x^3 dx$$

$$= \frac{2\pi n\phi}{l} \left[\frac{x^4}{4} \right]_0^r$$

$$\tau = \frac{\pi n r^4}{2l} \phi$$

(i.e) couple required to twist the rod is proportional to the angle of twist ϕ . The couple required to twist the rod through an angle of one radian is (i.e., $\phi = 1$)

$$C = \frac{\pi n r^4}{2l}$$

where ' C ' is Torsional rigidity or Torsional constant of the rod

For hollow cylinder of inner radius r_1 and outer radius r_2

$$\tau = \int_{r_1}^{r_2} \frac{2\pi n\phi}{l} x^3 dx = \frac{2\pi n\phi}{l} \left(\frac{x^4}{4} \right)_{r_1}^{r_2}$$

$$\tau = \frac{2\pi n\phi}{l} \left(\frac{r_2^4 - r_1^4}{4} \right) = \frac{\pi n (r_2^4 - r_1^4)}{2l} \phi$$

1.10.1 To find time period of oscillation (T)

Consider an intermediate stage when the wire is under twist of angle α and the body is moving with an angular velocity ω . At this time

$$\left. \begin{array}{l} \text{P.E of the wire} \\ \text{(due to the twist)} \end{array} \right\} = \frac{1}{2} C \theta^2$$

$$\left. \begin{array}{l} \text{K.E of the wire} \\ \text{(due to its rotation)} \end{array} \right\} = \frac{1}{2} I \omega^2$$

$$\therefore \text{The total energy of system} = \frac{1}{2} C \theta^2 + \frac{1}{2} I \omega^2$$

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According to law of conservation of energy, the total energy of the system is constant

$$\therefore \frac{1}{2}C\theta^2 + \frac{1}{2}I \left(\frac{d\theta}{dt} \right)^2 = \text{constant}$$

Differentiating w.r to 't'

$$\frac{1}{2}C.2\theta \left(\frac{d\theta}{dt} \right) + \frac{1}{2}I.2 \frac{d\theta}{dt} \left(\frac{d^2\theta}{dt^2} \right) = 0$$

$$C\theta + I \frac{d^2\theta}{dt^2} = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$$

$$\text{i.e., } \frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta = 0$$

\therefore Angular acceleration displacement.

From the above equation, it is understood that torsional pendulum executes simple harmonic motion.

The time period of any SHM is $T = 2\pi \sqrt{\frac{1}{\left(\frac{\text{Acceleration}}{\text{Displacement}}\right)}}$

$$\frac{\left(\frac{d^2\theta}{dt^2}\right)}{\theta} \propto \frac{C}{I}$$

\therefore The time period of oscillation of torsional pendulum is $T = 2\pi \sqrt{\frac{I}{C}}$

1.10.2 Applications of torsional pendulum

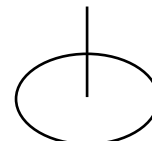
(i) Determination of moment of inertia of an irregular body:-

The moment of inertia of an irregular body can be determined using a torsional pendulum. First take a regular body and its moment of inertia is determined from its mass and dimensions.

The time period T_0 of the pendulum is determined without applying any mass. Take two equal masses, whose moment of inertia is known and place it on either side of the disc at a distance d and find the time period T_1 . Then, replace the regular body by an irregular body and keep it at same distance d_1 . Now the time period of the disc is calculated as T_2 .

Let I_0 be the moment of inertia of the disc and T_0 is the time period

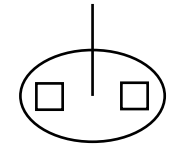
$$T_0 = 2\pi \sqrt{\frac{I_0}{C}} \quad (10)$$



Empty disc

For regular body whose moment of inertia is I_1

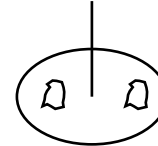
$$\therefore T_1 = 2\pi\sqrt{\frac{I_0 + I_1}{C}} \quad (11)$$



Disc with mass at equal distance

I_1 is calculated with the help of the dimensions. Irregular body of moment of inertia I_2 is to be determined.

$$T_2 = 2\pi\sqrt{\frac{I_0 + I_2}{C}} \quad (12)$$



Disc with irregular body at same distance

From Eq.(10) & Eq.(11)

$$\begin{aligned} T_1^2 - T_0^2 &= \frac{4\pi^2}{C}(I_0 + I_1) - \frac{4\pi^2}{C}I_0 \\ T_1^2 - T_0^2 &= \frac{4\pi^2}{C}I_1 \end{aligned} \quad (13)$$

From Eq.(10) & Eq.(12)

$$\begin{aligned} T_2^2 - T_0^2 &= \frac{4\pi^2}{C}(I_0 + I_2) - \frac{4\pi^2}{C}I_0 \\ T_2^2 - T_0^2 &= \frac{4\pi^2}{C}I_2 \\ \frac{T_2^2 - T_0^2}{T_1^2 - T_0^2} &= \frac{I_2}{I_1} \Rightarrow I_2 = \left(\frac{T_2^2 - T_0^2}{T_1^2 - T_0^2}\right) I_1 \end{aligned}$$

From the values T_0, T_1, T_2 and I_1 the value of I_2 (M.I of an irregular body) is calculated.

(ii) **Determination of rigidity modulus of the wire:-**

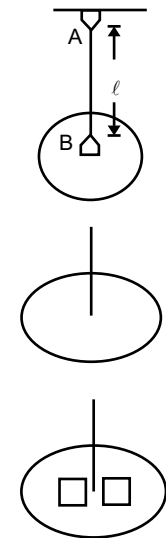
Consider a wire AB of length ℓ and radius r fixed at the upper end A. The lower end B is clamped to the disc. By adjusting the screw at B, the length of the wire ℓ can be varied. The rigidity modulus is determined by measuring the time period of the disc under the following conditions.

- (i) Let T_0 be the time period of pendulum without any masses

$$T_0 = 2\pi\sqrt{\frac{I_0}{C}} \quad (14)$$

- (ii) Let T_1 be the time period T_1 with equal masses at a distance d_1 placed on the disc

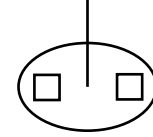
$$T_1 = 2\pi\sqrt{\frac{I_0 + 2md_1^2}{C}} \quad (15)$$



1.16 Applied Physics

- (iii) Let T_2 be the time period with equal masses at a distance d_2 placed on the disc

$$T_2 = 2\pi \sqrt{\frac{I_0 + 2md_2^2}{C}} \quad (16)$$



$$\text{Eq. (16)}^2 - (15)^2 \Rightarrow T_2^2 - T_1^2 = \frac{4\pi^2}{C} 2m(d_2^2 - d_1^2)$$

$$\text{We know couple per unit twist } C = \frac{\pi n r^4}{2\ell}$$

Substitute the value of C ,

$$T_2^2 - T_1^2 = \frac{4\pi^2}{\pi n r^4} \cdot 2\ell 2m(d_2^2 - d_1^2)$$

$$n = \frac{16\pi m \ell (d_2^2 - d_1^2)}{r^4 (T_2^2 - T_1^2)}$$

When 'n' is the rigidity modulus of material of the wire. The formula can be rewritten as

$$n = \frac{8\pi I \ell}{T_0^2 r^4} \quad \text{where } I = 2m \frac{(d_2^2 - d_1^2)}{(T_2^2 - T_1^2)} T_0^2$$

1.11 Shafts

A shaft is a component of a machine that transmits the power from the source to a load.

It is an arrangement for the transmission of a couple applied at one end to appear at the other end without any appreciable twist in it.

It can rotate on bearing about its own axis.

Thus any rotating member which is transmitting torque is called shaft.

A good shaft should have the following requirements:

1. It should transmit the couple without any appreciable twist in it. (i.e) couple / unit twist $C = \pi n r^4 / 2\ell$ must be large.
2. Even for large couples applied, the twist in the shaft should be very small.
3. Hence
 - it is preferable to use shafts of large radius.
 - made of high rigidity modulus material
 - avoid using longer shafts

(e.g) In flour mill the power transmission shaft is used to power (torque) developed in the prime mover (motor) to the load (machine) through a belt.

Comparison of hollow shaft with solid shaft

Consider a solid and hollow shaft of same mass, length and material. For hollow shaft of external and internal radii r_2 and r_1 respectively, subjected to an external couple, the couple/unit twist

$$C_h = \frac{\pi n}{2l}(r_2^4 - r_1^4) \quad (17)$$

For solid shaft made of same material having same length & mass.

$$C_s = \frac{\pi n}{2l} r^4 \quad (18)$$

Dividing (17) by (18)

$$\frac{C_h}{C_s} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{r^4}$$

Since the two have same mass

$$m = \pi(r_2^2 - r_1^2)l\rho = \pi r^2 l \rho \quad (19)$$

where ρ is density of the material of the two shafts.

$$\therefore r_2^2 - r_1^2 = r^2$$

$$\therefore \frac{C_h}{C_s} = \frac{r^2(r_2^2 + r_1^2)}{r^4} = \frac{r_2^2 + r_1^2}{r^2}$$

$$\text{(or)} \quad \frac{r_2^2 + r_1^2}{r^2} > 1$$

$$\therefore C_h > C_s$$

\therefore Hollow shaft is stronger than the solid shaft of same length, mass & material. (i.e.,) Hollow shaft can transmit large torque or moment of couple in an efficient manner without undergoing appreciable twist in itself.

1.11.1 Properties of shafts

The materials selected for making shafts should have the following properties:

- (i) It should have high strength
- (ii) A shaft should be of good machinability
- (iii) It should have low notch sensitivity factor
- (iv) Should transmit couple without appreciable twist
- (v) Should not be affected by environmental conditions
- (vi) Length of shaft should be greater than its other dimensions
- (vii) Efficiency of a shaft should be high
- (viii) It should have wear resistant & good heat treatment properties

1.12 Bending of Beam

Beams are most important for the constructions of bridges and fly overs or for supporting heavy loads. They are commonly used in the construction of multistoried buildings.

A beam is a bar of uniform rectangular or circular cross section whose length is much large as compared to its breadth and thickness. For such a structure the shearing stress for any given cross section is negligibly small. Beam may be regarded as made up of a large number of horizontal layers placed one over the other (Fig.12(a)).

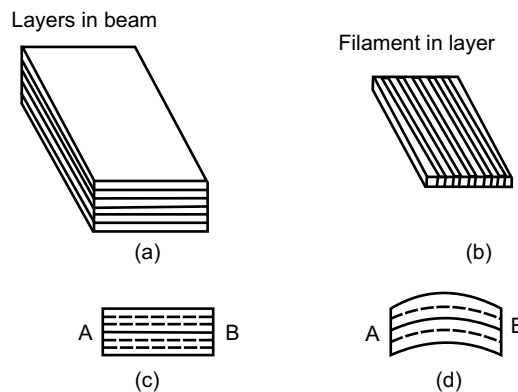


Fig. 12

Each layer may be considered as a collection of thin fibres parallel to the length of the beam. These fibres are called longitudinal fibres or filaments (Fig.16 (b)). When equal and opposite couples are applied at the two extreme ends of a beam in a plane parallel to its length, the beam bends into a circular arc. Initially, the various filaments are of equal length. When beam bends, filaments above the layer AB are expanded, while below AB are contracted. The length of the layer AB remains unaltered. Thus the layer in the beam, which is neither extended nor contracted is called neutral layer. The filaments constituting the neutral layer are called neutral filaments. The plane in which the beam bends is called plane of bending. Further, any change in the length of any filament is proportional to distance of the filament from neutral axis.

1.12.1 Bending moment

When a beam is bent under the action of a pair of two equal and opposite couples acting at its ends, to oppose this bending, an internal bending (restoring) couple is developed at each cross section of the beam, due to its elastic properties.

In equilibrium, the restoring couple is equal and opposite to the external bending couple. **The moment of the restoring couple is called bending moment or moment of resistance.**

1.12.2 Expression for bending moment

Assume weight of the beam is negligibly small as compared to the load suspended and its cross section remains unaltered by the action of applied forces.

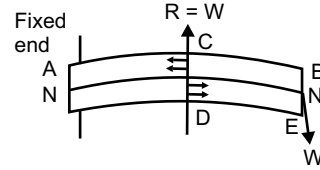


Fig. 13

The Fig.13, shows longitudinal section of a beam, whose one end is clamped and other end is depressed by a load W .

Let the beam be divided into two parts by a plane CD and NN' is neutral surface.

Consider the equilibrium of part CD . Filaments above neutral surface are extended and below are compressed. Filaments above are in tensile stress and their portions to the left of CD exerts a pulling force on their portions to the right of CD . Filaments below are in compressive stress and their portions to the left of CD exert a pushing force on their portions to the right of CD . The tensile stress developed in the upper half of beam and compressive stress in the lower half of the beam constitute a couple called restoring couple (in the direction opposite to the bending couple).

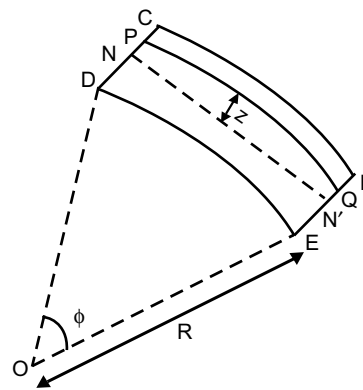


Fig. 14

To find the magnitude of this restoring couple or bending moment, consider the part CB of the bent beam and forces acting over the section D as shown in Fig.14. Let the portion $BCDE$ subtend an angle ϕ at the centre of curvature ' O ' of the arc and R be the radius of curvature of the neutral axis NN' .

Consider a filament PQ at a distance z above the neutral axis. Then from Fig.14,

$$PQ = (R + z)\phi$$

$$NN' = R\phi$$

Before bending, each filament was of the same length as neutral filament.

$$(i.e) PQ = NN' = R\phi$$

$$\therefore \text{Extension in } PQ \text{ on bending} = PQ - NN' = (R + z)\phi - R\phi = z\phi$$

Longitudinal strain for this filament

$$= \frac{\text{Increase in length}}{\text{Original length}} = \frac{z\phi}{R\phi} = \frac{z}{R}$$

1.20 Applied Physics

If f be the force acting longitudinally on the filament under consideration, a area of cross section and Y Young's modulus of the beam, then

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{f/a}{z/R}$$
$$f = \frac{Ya}{R} z$$

Moment of this force about the neutral axis is

$$fz = \frac{Ya}{R} z^2$$

Sum of moments of all these elementary forces of push and pull acting over entire cross section CD of the beam is the magnitude of the bending moment at this section.

∴ Bending moment

$$= \sum fz = \sum \frac{Ya}{R} z^2 = \frac{Y}{R} \sum az^2$$

The quantity $\sum az^2$ is analogous to $\sum mr^2$, known as geometrical moment of inertia I_g

$$\boxed{\text{Bending moment} = \frac{YI_g}{R}}$$

The quantity YI_g is known as flexural rigidity of the beam.

Geometrical moment of inertia for

$$\text{Rectangular cross section} = \frac{bd^3}{12} \quad \left(\begin{array}{l} \text{breadth} - b \\ \text{thickness of the beam} - d \end{array} \right)$$

$$\text{Bending moment} = \frac{Ybd^3}{12R}$$

For circular cross section of radius r

$$I_g = \frac{\pi r^4}{4}$$

$$\text{Bending moment} = \frac{Y\pi r^4}{4R}$$

1.13 Cantilever

A cantilever is a thin uniform bar fixed horizontally at one end and loaded at the other end (Fig.15).

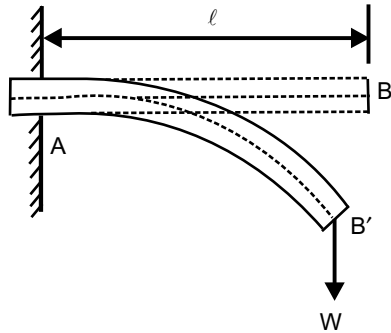


Fig.15

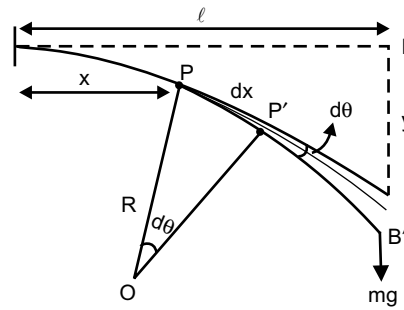


Fig.16

Consider a cantilever of length l , clamped rigidly at end A and loaded at the end B by a weight W . Due to the load W the end B is depressed downward. The end B is displaced to position B' . It is assumed that the weight of the beam is negligible as compared to load W and produce no bending. Due to load applied at the free end, a couple is created between two forces.

- (i) Force (load W) applied at the free end towards downward direction.
- (ii) Reaction (R) acting in the upward direction at the supporting end.

This external bending couple tends to bend the beam in clockwise direction, but since one end is fixed, the beam cannot rotate. \therefore External bending couple is balanced by another equal and opposite couple called internal bending moment, due to elastic nature of the body. Under equilibrium

$$\text{External bending moment} = \text{Internal bending moment}$$

1.13.1 Depression of a cantilever

Let ' l ' be length of the cantilever OA fixed at one end and loaded at the other.

Consider a section at P at a distance x from the fixed end (Fig.20). Neglecting the weight of the part between P and the free end, the moment of the external couple
 $= mg(l - x)$ (External bending moment)

The bending moment of the section P is $\frac{YI_g}{R}$ (Internal bending moment)

At equilibrium

$$mg(l - x) = \frac{YI_g}{R} \tag{20}$$

$$R = \frac{YI_g}{mg(l - x)} \tag{21}$$

As x increases R also increases. The maximum and minimum values being at the free and fixed ends respectively.

1.22 Applied Physics

Since the radius of curvature of the neutral axis (R) varies from point to point of the cantilever, the bending is non uniform.

Let P' be another point on the bent cantilever at a distance dx from P .

$\therefore P$ & P' are very near, we can assume that the radius of curvature R is practically same and O , the center of curvature. Let angle between the tangents at P & P' be $d\theta$.

$$\begin{aligned} \text{Then } \angle POP' &= d\theta \\ \therefore dx &= R d\theta \\ R &= \frac{dx}{d\theta} \end{aligned} \quad (22)$$

Substituting Eq.(21) in Eq.(22)

$$\begin{aligned} mg(l-x) &= YI_g \frac{d\theta}{dx} \\ \text{(or)} \\ d\theta &= \frac{mg(l-x)dx}{YI_g} \\ \therefore \theta &= \int_0^l \frac{mg(l-x)dx}{YI_g} = \frac{mgl}{YI_g} \int_0^l dx - \frac{mg}{YI_g} \int_0^l x dx \\ &= \frac{mgl^2}{YI_g} - \frac{mgl^2}{2YI_g} = \frac{mgl^2}{2YI_g} \end{aligned} \quad (23)$$

If dy is the depression of B due to curvature PP' , then

$$\begin{aligned} dy &= (l-x)d\theta \\ dy &= \frac{mg(l-x)^2}{YI_g} dx \\ y &= \int dy = \int_0^l \frac{mg(l-x)^2}{YI_g} dx \\ &= \frac{mg}{YI_g} \int_0^l (l-x)^2 dx \\ &= \frac{mg}{YI_g} \left[l^2x - \frac{2lx^2}{2} + \frac{x^3}{3} \right]_0^l \\ &= \frac{mg}{YI_g} \left[l^3 - l^3 + \frac{l^3}{3} \right] \\ \boxed{y} &= \frac{mgl^3}{3YI_g} \end{aligned} \quad (24)$$

Case (i): For rectangular cross section

$$I_g = \frac{bd^3}{12}$$

$$y = \frac{mgl^3}{3Ybd^3} = \frac{4mgl^3}{Ybd^3}$$

Case (ii) : Circular cross section

$$I_g = \frac{\pi r^4}{4}$$

$$y = \frac{4mgl^3}{3Yr^4\pi}$$

1.13.2 Experimental determination of young's modulus by depression of a cantilever

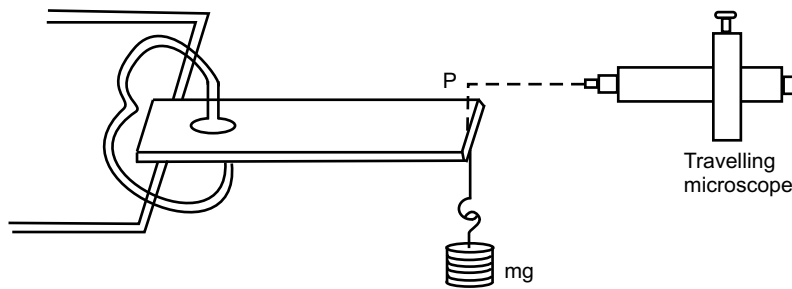


Fig. 17

It consists of a beam with one end clamped to the edge of the table. A tall pin P is stuck vertically to the free end. A hook is attached to suspend the weight hanger. A travelling microscope is focussed on the tip of the pin.

A minimum load is attached to the hook. Microscope is adjusted such that the horizontal cross wire coincides with the tip of the image of the pin and reading on the vertical scale is taken. Loads are added to the hanger in steps of 50 gm and every time, the readings are noted.

S.No	Load gms	Microscope reading while		Mean	Depression (y) for a load of m kg	m/ y
		loading	unloading			
1	W					
2	$W + 50$					
3	$W + 100$					
⋮	⋮					

The experiment is repeated for decreasing loads also and the mean depression (y) for a load m kg is found.

1.24 Applied Physics

The depression y produced for an addition of m kg is thus found. Breadth b and thickness d are found out. Young's modulus for rectangular cross section is,

$$Y = \frac{4mgl^3}{bd^3y}$$

$$\text{(or) } Y = \frac{4gl^3}{bd^3} \left(\frac{m}{y} \right)$$

Knowing l, b, d and m/y Young's modulus can be calculated.

1.14 Non Uniform Bending - Depression at the Mid Point of a Beam Loaded at the Middle

Consider a beam of length ' l ' (distance between two knife edges) supported on the two knife edges A and B . A load of weight ' W ' is suspended at the center ' C '. It is found that the beam bends and the maximum displacement is at the point ' D ', where the load is given. Due to load (W) applied, at the middle of the beam the reaction $W/2$ is acted vertically upwards at each knife edges - The bending is called non uniform bending.

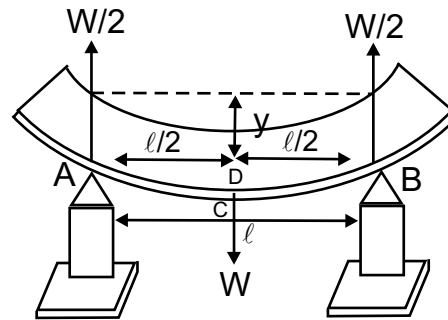


Fig. 18

The beam may be considered as two cantilevers, whose free end carries a load $W/2$ each of length $l/2$ and fixed at the point ' D '.

Hence we can say the elevation of A above D as the depression of D below A .

We know, Depression of cantilever

$$y = \frac{Wl^3}{3YI_g} \quad (25)$$

Therefore, substituting $l = l/2$ & $W = W/2$ in Eq.(25)

$$y = \frac{\frac{W}{2} \left(\frac{l}{2} \right)^3}{3YI_g} = \boxed{\frac{Wl^3}{48YI_g}}$$

1.14.1 Experimental determination of Young's modulus – Non uniform bending

It consists of a beam, symmetrically supported on the two knife edges A and B . A weight hanger is suspended at the center (C) of the beam by means of a loop. A pin

is fixed vertically at 'C' by some wax. The tip of the pin is focussed by a travelling microscope. Taking weight hanger as a dead load (W) the microscope is adjusted & the tip is made to coincide with horizontal cross wire. The readings is noted from the vertical scale of the microscope. Weights are added in steps of $m, 2m, 3m$ kg and corresponding readings are taken. Same procedure is repeated for unloading and readings are tabulated.

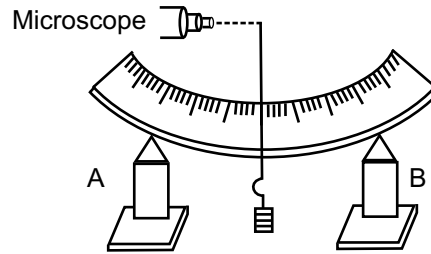


Fig. 19

The mean depression y is found for a load of m kg.

Load (m) kg	Microscope readings			Depression (y) m	m/y kg/m
	Increasing load $\times 10^{-2}$ m	decreasing load $\times 10^{-2}$ m	mean $\times 10^{-2}$ m		
W			y_0		
$W + m$			y_1		
$W + 2m$			y_2		
$W + 3m$			y_3	$y_3 - y_0$	
$W + 4m$			y_4	$y_4 - y_1$	
$W + 5m$			y_5	$y_5 - y_2$	

$$\text{We know } y = \frac{Wl^3}{48YI_g}$$

where ' l ' in length of the beam,

For rectangular beam

$$I_g = \frac{bd^3}{12}, \quad W = mg$$

$$y = \frac{mgl^3}{48Y \left(\frac{bd^3}{12} \right)} = \frac{mgl^3}{4Ybd^3}$$

$$Y = \frac{gl^3}{4bd^3} \left(\frac{M}{y} \right) \text{ N/m}^2$$

Substituting all the values, Young'd modulus of the given beam is found out.

1.15 Uniform Bending - Elevation at the Center of the Beam Loaded at Both Ends

Consider a beam of negligible mass, supported symmetrically on two knife edges A and B . Length between A and B be ' l '. Let equal weights W , be added to either end of beam C and B .

Let the distance $CA = BD = a$.

Due to load applied the beam bends from position F to E into an arc of a circle and produces an elevation ' x ' from position F to E . Let W be the reaction produced at A & B , which acts vertically upwards as shown in Fig.20.

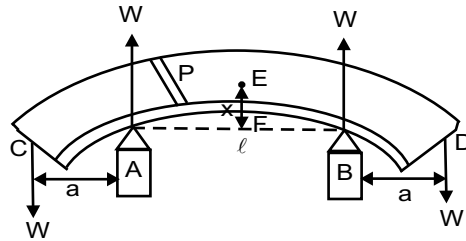


Fig.20

Consider a point ' P ' on the cross section of the beam. Then the forces acting on the part PC of the beam are

Force W at ' C ' and Reaction W at A as shown in Fig.21.

Let $PC = a_1$; $PA = a_2$ then, external bending moment about ' P ' is

$$M_p = W \times a_1 - W \times a_2$$

Here clockwise moment is taken as negative and anticlockwise moment is taken as positive.

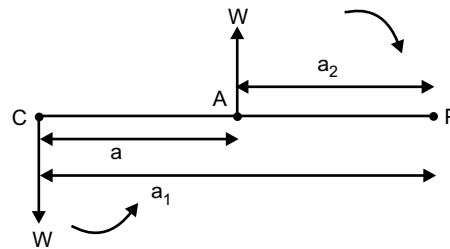


Fig.21

$$M_p = W(a_1 - a_2) = Wa \tag{26}$$

$$\text{Internal bending moment} = \frac{Y I_g}{R} \tag{27}$$

Under equilibrium

$$\text{External bending moment} = \text{Internal bending moment}$$

Equating Eq.(26) and Eq.(27)

$$Wa = \frac{Y I_g}{R} \tag{28}$$

[∵ For a given load (W) Y, I_g, a & R are constant, the bending is called as uniform bending].

Here it is found that the elevation ' x ' from an arc of the circle of radius ' R ' as shown in Fig.22.

From the Fig.22, we have

$$(2R - x)x = \frac{l}{2} \cdot \frac{l}{2} \quad (\text{Rule of sagita})$$

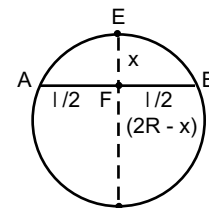


Fig.22

$$2Rx - x^2 = \frac{l^2}{4}$$

Neglecting higher powers of x ,

$$2Rx = \frac{l^2}{4}$$

(or) $R = \frac{l^2}{8x}$ (29)

Substituting Eq.(29) in Eq.(28)

$$Wa = \frac{Y I_g}{l^2/8x} = \frac{8Y I_g x}{l^2}$$

Elevation $x = \frac{Wal^2}{8Y I_g}$

Young's modulus $Y = \frac{Wal^2}{8x I_g}$ (30)

1.15.1 Experimental determination of Young's modulus by uniform bending

A beam is symmetrically supported on two knife edges with two weight hangers at its ends. A pin is fixed at the center, whose tip is focussed through a microscope. For dead load, the tip of the pin is made to coincide with the horizontal cross wire and readings are noted. The same procedure is repeated for various loads and elevation x for a given load m kg is found.

Load m kg	Microscopic readings			Elevation x	m/x
	↑ load	↓ load	mean		
W			x_0		
$W + m$			x_1		
$W + 2m$			x_2		
$W + 3m$			x_3		
$W + 4m$			x_4	$x_4 - x_0$	
$W + 5m$			x_5	$x_5 - x_1$	
$W + 6m$			x_6	$x_6 - x_2$	
$W + 7m$			x_7	$x_7 - x_3$	

$$x = \frac{Wal^2}{8Y I_g}$$

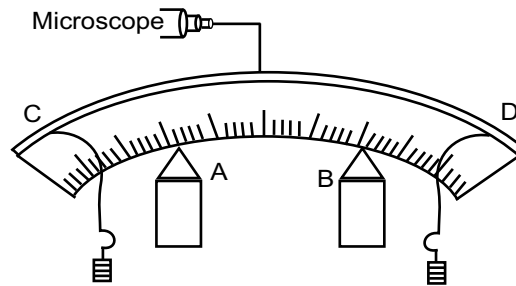


Fig. 23

For rectangular beam

$$I_g = \frac{bd^3}{12}$$

$$W = mg$$

$$x = \frac{mgal^2}{8Ybd^3/12} = \frac{3}{2} \frac{mgal^2}{Ybd^3}$$

$$Y = \frac{3gal^2}{2bd^3} \left(\frac{m}{x} \right)$$

Difference between uniform and nonuniform bending

S.No	Uniform bending	Non Uniform bending
1.	When uniform load is acting on the beam, the envelope of the bent beam forms an arc of a circle & bending is called uniform bending	When the beam is loaded only at a point or non - uniformly the bending is called non uniform bending.
2.	Load is applied at the ends of the beam	Load is applied at the centre.
3.	Centre of the beam is elevated	Centre of the beam is depressed.

1.16 Girders

The girders with upper & lower section broadened and the middle section tapered, so that it can withstand heavy loads over it is called as *I* shaped girders.

[Since the girder look like letter *I*, they are named so] In general any girder supported at its two ends as on the opposite walls of a room, bends under its own weight and a small depression is produced at the middle portion. This may also be caused when loads are applied to the beams.

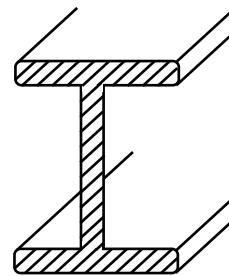


Fig. 24

Due to the depression produced, the upper parts above neutral axis expands while lower parts contracts. (i.e) stresses have a maximum value at the top and bottom, which decreases as it approaches neutral axis.

∴ Upper & lower surfaces of the girder must be stronger than the intervening part. Thus they are made of *I* shape.

The depression of its mid point is given by

$$y = \frac{4mgl^3}{Ybd^3}$$

When a beam is used as a girder, it should have minimum depression under its own weight. Further depression of the girder should be small for a given load also.

This can be achieved by decreasing its length, increasing *Y* and increasing *b* or *d*. Decreasing '*l*' is not economical in many aspects. By selecting the girder material with high '*Y*' one can get small depression.

Since '*d*' occurs in the equation as d^3 a small increase in '*d*' produces the same effect as a larger change in *b*. The corresponding increase in volume of the girder will be much smaller when *d* is increased than when *b* is increased so as to have the same value of depression. It is therefore more economical to have a large depth and a small breadth.

For stability, the upper and lower parts of the cross section will be broader so that the section will have the shape of *I*.

When a girder is supported at its two ends, its middle part is depressed and the surfaces above and below its neutral surface are compressed and extended respectively. Compression is maximum at the upper face and extension is maximum at the lower face, as stresses are maximum. As we proceed towards the neutral surface, stresses starts decreasing. It follows therefore that upper and lower faces of the girder should be much stronger than its middle portions. In other words, the middle portions may be made of smaller breadth than upper and lower faces, thus saving a good amount of material with no loss in strength and hence the shape of *I*.

Applications

1. Used in construction of bridges over the river.
2. Production of iron rails.
3. Supporting beams for ceilings in construction of buildings.

□ Solved Problem 1

Find the stress, strain and Young's modulus in the case of wire 1.5 m long and 1 sq mm in cross section, if it increases by 1.55 mm, when a weight of 10 kg is suspended.

1.30 *Applied Physics*

► **Solution**

$$\text{stress} = \frac{F}{A}; \quad \text{strain} = \frac{\Delta l}{l}$$

$$Y = \text{Young's modulus} = \frac{\text{stress}}{\text{strain}}$$

$$F = 10 \text{ kg} = 10 \times 1000 \times 980 \text{ dynes} \qquad l = 1.5 \text{ m}$$

$$A = 1 \text{ sq.mm} = 0.01 \text{ sq.cm} \qquad \qquad \qquad = 150 \text{ cm}$$

$$\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \Delta l = 1.55 \text{ mm}$$

$$\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad = .155 \text{ cm}$$

$$\text{stress} = \frac{98 \times 10^5}{0.01} = 98 \times 10^7 \text{ dynes/cm}^2$$

$$\text{strain} = \frac{\Delta l}{l} = \frac{0.155}{150} = 10.33 \times 10^{-4}$$

$$Y = \frac{98 \times 10^7}{10.33 \times 10^{-4}} = 9.49 \times 10^{11} \text{ dynes/cm}^2$$

□ **Solved Problem 2**

Find the force which must act tangentially over the surface of a body of surface area 18 cm^2 in order to produce a shear of 2° . $n = 9 \times 10^{11} \text{ dynes/cm}^2$.

► **Solution**

$$n = \frac{F/A}{\phi} \Rightarrow F = nA\phi$$

$$n = 9 \times 10^{11}; \quad A = 18 \text{ cm}^2; \quad \phi = 2^\circ = 2 \times \frac{\pi}{180}$$

$$F = 9 \times 10^{11} \times 18 \times \frac{2\pi}{180} = 56.52 \times 10^{10} \text{ dynes}$$

□ **Solved Problem 3**

A brass bar 1 cm square in cross section is supported on 2 knife edges 100 cm apart. A load of 1 kg at the centre of the bar depresses that point by 2.51 mm. What is Young's modulus for the bars?

► **Solution**

$$y = \frac{WL^3}{48YI_g}; \quad I_g = \frac{bd^3}{12}$$

$$b = d = 1 \text{ cm} \qquad \qquad \qquad y = 2.51 \text{ mm}$$

$$l = 100 \text{ cm} \qquad \qquad \qquad = 0.251 \text{ cm}$$

$$W = 1 \text{ kg} = 100 \times 981 \text{ dynes}$$

$$\begin{aligned}
 Y &= \frac{12WL^3}{48ybd^3} = \frac{WL^3}{4yd^3b} \\
 &= \frac{1000 \times 981 \times 100^3}{4 \times 0.251 \times 1} = 9.77 \times 10^{11} \text{ dynes/cm}^2
 \end{aligned}$$

□ **Solved Problem 4**

A solid cylinder of 2 cm radius weighing 200 g is rigidly connected with its axis vertical to the lower end of the fine wire. The period of oscillation of the cylinder under the influence of the torsion of the wire is 2 sec. Calculate the couple necessary to twist it through 4 complete turns.

► **Solution**

Period of the cylinder executing torsional vibrations is

$$\begin{aligned}
 T &= 2\pi\sqrt{\frac{I}{c}} \\
 I &= \frac{MR^2}{2} \\
 &= \frac{200 \times 10^{-3} \times (2 \times 10^{-2})^2}{2} = 400 \times 10^{-7} \text{ kgm}^2 \\
 C &= \frac{4\pi^2 I}{T^2} \\
 &= \frac{4 \times \pi^2 \times 400 \times 10^{-7}}{2^2} \text{ Nm}
 \end{aligned}$$

4 complete turns are exactly equal to $4 \times 2\pi$ radian angle of twist.

$$\theta = 8\pi \text{ radian}$$

∴ required twisting couple = $C\theta$

$$\begin{aligned}
 &= \frac{4 \times \pi^2 \times 400 \times 10^{-7}}{2^2} \times 8\pi \\
 &= 9.9 \times 10^{-3} \text{ Nm}
 \end{aligned}$$

□ **Solved Problem 5**

Calculate the density of lead under a pressure $2 \times 10^8 \text{ N/m}^2$. Density of lead is $11.4 \times 10^3 \text{ kg/m}^3$. Bulk modulus of elasticity = $8 \times 10^9 \text{ Nm}^2$.

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► **Solution**

$$K = \frac{-PV}{dV}$$

mass $m = V\rho$

$$Vd\rho + \rho dV = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$K = \frac{P}{d\rho/\rho}$$

$$ds = \frac{P\rho}{K}$$

$$= \frac{2 \times 10^8 \times 11.4 \times 10^3}{8 \times 10^9}$$

$$= 0.285 \times 10^3 \text{ kg/m}^3$$

Density under applied pressure = $\rho + d\rho$

$$= 11.4 \times 10^3 + 0.285 \times 10^3$$

$$= 11.685 \times 10^3 \text{ kg/m}^3$$

□ **Solved Problem 6**

Calculate the maximum length of the steel rod that can hang vertically without breaking. The breaking stress for steel is $8 \times 10^6 \text{ N/m}^2$ and $\rho_{\text{steel}} = 8 \times 10^3 \text{ kg/m}^3$.

► **Solution**

$$\text{stress} = \frac{mg}{\text{area}} = \frac{\text{vol} \times \rho \times g}{\text{area}}$$

$$= \frac{\text{area} \times \text{length} \times \rho \times g}{\text{area}}$$

$$S_{\text{max}} = l_{\text{max}} \times \rho \times g$$

$$l_{\text{max}} = \frac{S_{\text{max}}}{\rho g} = \frac{8 \times 10^6}{8 \times 10^3 \times 9.8} = 102.04 \text{ m}$$

□ **Solved Problem 7**

The couple / unit twist for a certain solid cylinder of radius r is 100 Nm. Calculate the contribution to this couple due to the central part up to radius $r/4$ and due to the outer most part between radii $3r/4$ and r .

► **Solution**

$$C = \frac{\pi nr^4}{2l} = 100 \text{ Nm}$$

Couple required / unit twist for an elementary cylindrical shell of radius x and thickness dx is

$$dc = \frac{2\pi n}{l} x^3 dx$$

Couple in central part (ie.) 0 to $\frac{r}{4}$

$$\begin{aligned} C' &= \int dc = \frac{2\pi n}{l} \int_0^{r/4} x^3 dx \\ &= \frac{2\pi n r^4}{4l(4)^4} = 100 \times \frac{1}{256} = 0.39 \text{ Nm} \end{aligned}$$

Similarly for outer most part between $\frac{3r}{4}$ and r

$$\begin{aligned} C'' &= \int dc = \frac{2\pi n}{l} \int_{3r/4}^r x^3 dx = \frac{2\pi n}{l} \left(\frac{x^4}{4} \right)_r \\ &= \frac{2\pi n}{4l} \left(r^4 - \left(\frac{3r}{4} \right)^4 \right) = \frac{\pi n r^4}{2l} \left(\frac{175}{256} \right) \\ &= 68.3 \text{ Nm} \end{aligned}$$

□ **Solved Problem 8**

If the cross section of a cantilever is rectangular with sides of length a and b and if the maximum depressions of the end of the beam for a given load and y_a and y_b respectively, when a and b are vertical show that $y_a/y_b = b^2/a^2$.

► **Solution**

$$y = \frac{wl^3}{3YI}; \quad y_a = \frac{wl^3}{3YI_a}; \quad y_b = \frac{wl^3}{3YI_b}$$

$$\frac{y_a}{y_b} = \frac{I_b}{I_a}$$

$$I_a = \frac{ba^3}{12}; \quad I_b = \frac{ab^3}{12}$$

$$\frac{y_a}{y_b} = \frac{ab^3}{12} / \frac{ba^3}{12} = \frac{b^2}{a^2}$$

□ **Solved Problem 9**

The modulus of rigidity and poisson's ratio of the material of a wire are 2.87×10^{10} N/m and 0.379 respectively. Find the Young's modulus of the material of the wire.

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► **Solution**

$$\begin{aligned}\sigma &= \frac{Y}{2n} - 1 \\ \frac{Y}{2n} &= 1 + \sigma \Rightarrow Y = 2n(1 + \sigma) \\ &= 2 \times 2.87 \times 10^{10}(1 + 0.379) \\ &= 7.915 \times 10^{10} \text{ N/m}^2\end{aligned}$$

□ **Solved Problem 10**

For the same cross section area, show that the beam of a square cross section is stiffer than one of circular cross section of the same material. Find the ratio of the depressions for a given load.

► **Solution**

For square cross section, let depression be y_s and for circular let it be y_c . Since depression is inversely proportional to I_g

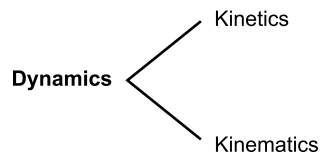
$$\begin{aligned}\frac{y_s}{y_c} &= \frac{I_{gc}}{I_{gs}} \\ &= \frac{\pi r^4/4}{b^4/12} = \frac{3\pi r^4}{b^4}\end{aligned}$$

∴ C.S.A are same,

$$\begin{aligned}\pi r^2 &= b^2 \Rightarrow \frac{r^2}{b^2} = \frac{1}{\pi} \\ \frac{y_s}{y_c} &= \frac{3\pi r^4}{b^4} = \frac{3\pi r^2 \cdot r^2}{b^2 \cdot b^2} \\ &= \frac{3r^2}{b^2} = \frac{3}{\pi}\end{aligned}$$

(or) $y_s : y_c = 3 : \pi$

The science which deals with the action of force on bodies in motion is called dynamics.



The two branches of dynamics will be discussed later.

1.17 Centre of Gravity and Moment of Inertia

Definition of centre of gravity Every particle of a body is attracted by the earth towards its centre. The force of attraction which is proportional to the mass of the particle, acts vertically downwards and is known as weight of the body $W = mg$. The **centre of gravity** of a body is a point through which the line of action of the weight of the body always passes in whatever position the body is held. A body may be considered to be made up of a number of very small particles. The weights of these particles constitute a system of like parallel forces. The resultant of this parallel system is the weight of the body, and the point of application of this resultant is the centre of gravity of the body.

A body may be considered to be made up of very small masses. If each such mass is acted on by a force proportional to the respective mass, and if these forces form a parallel system, the point of application of this resultant is called the '**centre of mass**' of the body.

Thus the centre of gravity and centre of mass are at the same point. In universe, there are zones where the influence of earth's gravitational attraction is completely absent. In these places, the body will have no centre of gravity, but has centre of mass.

The centre of gravity of a uniform rod lies at its middle, the centre of gravity of a triangle lies at a point where three medians of the triangle meet and the centre of gravity of a rectangle is the point where the diagonal meet and of circle is at its centre.

1.17.1 Centre of gravity of plane figures by method of moments

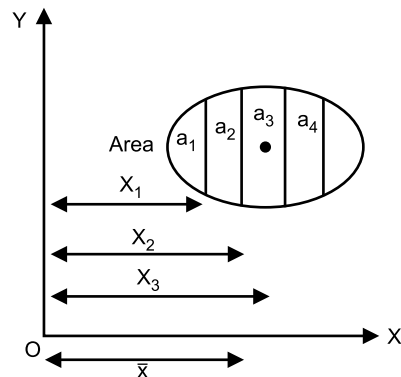
Let us consider a plane figure of total area A whose centre of gravity is to be determined. The area A is composed of a number of small areas a_1, a_2, a_3, \dots

$$A = a_1 + a_2 + a_3$$

Let x_1, x_2, x_3 and x_4 be the distance of the centre of gravity of area a_1, a_2, a_3 , and a_4 from axis OY .

The moments of all small areas about the axis OY

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots$$



Let G be the centre of gravity of the area A whose distance from the axis OY is \bar{x} .

The moment of total area about $OY = A\bar{x}$

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Moment of all small areas about OY = Moment of total area about OY

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots = A\bar{x}$$
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots}{A}$$

Moment of the small areas about OX = moment of total area about OX

$$\therefore \bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4 + \dots}{A}$$

Where \bar{y} is the distance of G from OX and $y_1, y_2, y_3,$ are the distance of centre of gravity of area $a_1, a_2, a_3,$ from axis OX respectively

Integration Method - Centre of gravity of area

$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i}$$
$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i}$$

where $i = 1, 2, 3, 4, \dots$

x_i and y_i are the distances of C.G of area a_i from axis OY and OX respectively. If the value of i is large in number then the summation can be replaced by integration (i.e) large number of small area.

$$\bar{x} = \frac{\int x dA}{\int dA} \text{ and } \bar{y} = \frac{\int y dA}{\int dA}$$

x and y are the distance of C.G of area dA from axis OY and OX respectively.

Centre of gravity of a line

$$\bar{y} = \frac{\int y dL}{L} \text{ and } \bar{x} = \frac{\int x dL}{L}$$

$L \rightarrow$ length and $dL \rightarrow$ length of an element. The centre of gravity of the line is similar to the centre of gravity of the wire.

Centre of gravity of volume

Consider a cone of volume V .

$$\text{Then } \bar{x} = \frac{\int x dV}{V}; \bar{y} = \frac{\int y dV}{V}; \bar{z} = \frac{\int z dV}{V}$$

Centre of gravity of some shapes

Shape	Area(A)	Centre of gravity (\bar{x}, \bar{y})
Plane figure	$A = \int dA$	$\bar{y} = \int ydA/A; \bar{x} = \int xdA/A$
Rectangle	$A = bd$	$\bar{x} = \frac{b}{2}; \bar{y} = \frac{b}{2}$
Triangle (right angled)	$A = \frac{1}{2}bh$	$\bar{x} = b/3; \bar{y} = h/3$
Triangle (iso celes)	$A = \frac{1}{2}bh$	$\bar{x} = b/2; \bar{y} = h/3$
Circle	$A = \pi R^2$	$\bar{x} = 0; \bar{y} = 0$
Semicircle	$A = \frac{\pi R^2}{2}$	$\bar{x} = 0; \bar{y} = \frac{4R}{3\pi}$
Ellipse	$A = \pi ab$	$\bar{x} = 0; \bar{y} = 0$

Centre of gravity of a right – angled triangle

Consider a triangle of height h and base b .
 Consider one elementary strip of thickness dy
 at a distance ' y ' from the X-axis

Area of the strip $dA = xdy$

From similar triangles

$$\frac{b}{h} = \frac{x}{(h-y)}$$

$$\therefore x = \frac{b}{h}(hy)$$

$$dA = \frac{b}{h}(h-y)dy$$

(31)

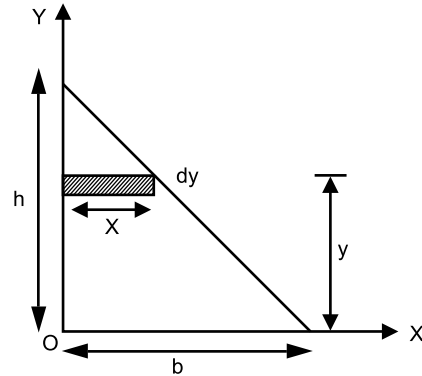
$$A = \int_0^h (b/h)(h-y)dy = \frac{b}{h} \left[hy - \frac{y^2}{2} \right]_0^h = \frac{bh}{2}$$

$$\int_0^h ydA = \int_0^h y(b/h)(hy)dy$$

$$\frac{b}{h} \int_0^h hydy - y^2dy = \frac{b}{h} \left(\frac{h^3}{2} - \frac{h^3}{3} \right)$$

$$b/h = \left[\frac{h^3}{6} \right] = \frac{bh^2}{6}$$

$$\therefore \bar{y} = \frac{\int ydA}{\int dA} = \frac{bh^2/6}{bh/2} = \frac{h}{3}$$



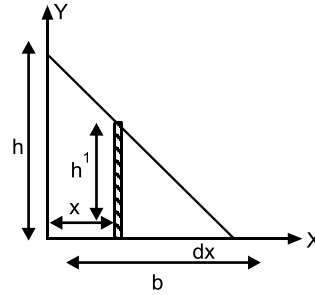
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To find \bar{x} : consider a strip of depth h' and width dx .

$$dA = h' dx$$

From similar triangles

$$\begin{aligned} \frac{b}{h} &= \frac{h'}{b-x} \text{ or } h' = \frac{h(b-x)}{b} \\ \therefore dA &= \frac{b}{h}(b-x)dx \\ \bar{A} &= \frac{b}{h} \int_0^b (b-x)dx \\ &= \frac{b}{h} \left[b^2 - \frac{b^2}{2} \right] = \frac{bh}{2} \\ \int_0^b x dA &= \frac{h}{b} \int_0^b x(b-x)dx \\ &= \frac{h}{b} \left[\frac{b^3}{2} - \frac{b^3}{3} \right] = \frac{hb^2}{6} \\ \bar{x} &= \frac{\int x dA}{dA} = \frac{hb^2}{6} / \frac{bh}{2} = \frac{b}{3} \end{aligned}$$



1.17.2 Centre of gravity of a rectangle

To find \bar{y} : Consider a strip of thickness dy of width b .

Area of the strip $dA = bdy$

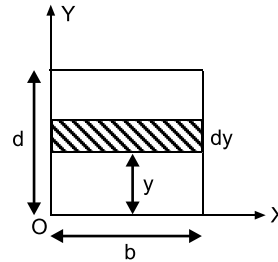
$$\int dA = \int_0^d bdy = bd$$

First moment of dA about X axis

$$y dA = y bdy$$

Total first moment of area about X-axis

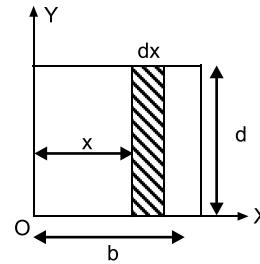
$$\begin{aligned} \int_0^d y dA &= \int_0^d y bdy = \frac{bd^2}{2} \\ \therefore \bar{y} &= \frac{bd^2}{2} / bd = \frac{d}{2} \end{aligned}$$



Consider a strip of thickness dx and depth d as shown.

$$dA = ddx$$

$$A = \int_0^b ddx = bd.$$



First moment of dA about Y-axis = $x dA = x ddx$.

$$\text{Total first moment of area about Y-axis} = \int_0^b x ddx = \frac{db^2}{2}$$

$$\therefore \bar{x} = \frac{db^2}{2} / bd = b/2.$$

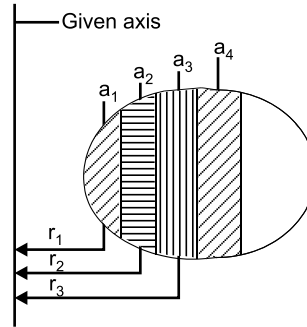
1.17.3 Moment of inertia

Consider a plane area which is split into small areas a_1, a_2, a_3, \dots . Let the C.G of the small areas from a given axis be at a distance of $r_1, r_2, r_3 \dots$

Moment of inertia of the plane area about the given axis is given by

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \dots$$

or $I = \Sigma ar^2$



The moment of inertia is the sum of the products of the area (or mass) and the square of the distances from the axis of rotation.

1.18 Radius of Gyration

Suppose the body consists of n particles of mass m

$$I = \Sigma mr^2 = m[r_1^2 + r_2^2 + r_3^2 + \dots r_n^2]$$

$$I = mn \frac{[r_1^2 + r_2^2 + r_3^2 + \dots r_n^2]}{n}$$

$$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}$$

where $K^2 = \frac{r_1^2 + r_2^2 + r_3^2 + \dots r_n^2}{n}$

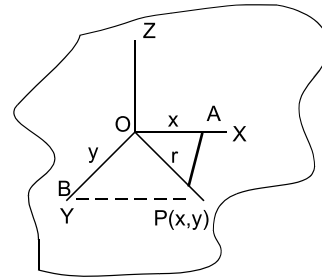
or $K = \sqrt{\frac{[r_1^2 + r_2^2 + r_3^2 + \dots r_n^2]}{n}}$

K is called the **radius of gyration** and is equal to the root mean square distance of the particles from the axis of rotation.

Suppose the whole mass of the body is concentrated at a single point such that M.I of this concentrated point mass is same as the M.I of the whole body about the axis, the distance of that single point from the axis is called the radius of gyration of the body about the axis.

1.18.1 Theorem of perpendicular axis

This theorem states that the moment of inertia of a plane lamina body about an axis perpendicular to the plane is equal to the sum of moment of inertia about two mutually perpendicular axis in the plane of the lamina such that the three mutually perpendicular axis have a common point of intersection. Consider a plane lamina having the axis OX and OY in the plane of the lamina. The axis OZ passes through O and is perpendicular to the plane of the lamina. Let the lamina be divided into a large number of particles each of mass m . Let a particle of mass m be at P with coordinates (x, y) and situated at a distance r from the point of intersection of the axis.



$$r^2 = x^2 + y^2$$

Moment of inertia of the particle about the axis $OZ = mr^2$.

Moment of inertia of the whole lamina about the axis,

$$OZ = \Sigma mr^2 = I_z$$

Moment of inertia of the whole lamina about the axis OX

$$I_x = \Sigma my^2$$

Similarly, $I_y = \Sigma mx^2$

$$I = I_x + I_y$$

$$I_z = \Sigma mr^2$$

$$= \Sigma m(x^2 + y^2)$$

$$I_z = \Sigma mx^2 + \Sigma my^2$$

$$I_z = I_x + I_y$$

1.18.2 Theorem of parallel axis

It states that the moment of inertia of a plane area about an axis in the plane of area through the C.G of the plane area be represented by IG, then the moment of inertia

of the given plane area about a parallel axis AB at a distance h from C-G of the area is given by,

$$I_{AB} = I_G + Ah^2$$

I_{AB} - Moment of inertia of the given area about AB.

I_G - Moment of inertia of the given area about C.G.

A - Area of the section

h - Distance between C-G of the section and the axis AB.

Proof A lamina of plane area A is shown in the fig.

X - X The axis in the plane of area A and passing through the C.G of the area.

AB The axis that is parallel to axis X X

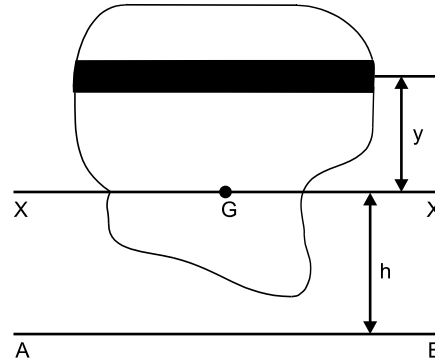
Consider a strip parallel to X X at a distance y from the X X axis.

Let the area of the strip = d A

Moment of inertia of the strip about X X = $y^2 dA$.

Moment of inertia of the total area about X X

$$I_G = \int y^2 dA$$



Moment of inertia of the area dA about AB

$$= dA(h + y)^2$$

Moment of inertia of the total area A about AB

$$\begin{aligned} I_{AB} &= \int dA[h^2 + y^2 + 2hy] \\ &= \int h^2 dA + \int y^2 dA + \int 2hy dA \end{aligned}$$

As h or h^2 is a constant

$$= h^2 \int dA + \int y^2 dA + 2h \int y dA.$$

But $\int dA = A$ and $\int y^2 dA = I_G$

$$= h^2 A + I_G + 2h \int y dA.$$

Moment of area of the strip about X-X = $y dA$ and $\int y dA$ represents moment of total area about X-X axis. Moment of total area about X-X axis is equal to the product of the total area A and the distance of C.G of the total area from X-X axis. Since distance of C.G of the total area from X-X axis is zero $\int y dA = 0$.

$$\therefore I_{AB} = I_G + Ah^2.$$

The theorem of parallel axis states that the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis and the product of the area of the body and the square of distance between the two parallel axis.

1.19 Moment of Inertia of Rigid bodies

Moment of Inertia of simple bodies can be determined as follows:

1. Take a general element.
2. Write down the expression for mass of the element and its distance from the axis.
3. Integrate the term between suitable limits such that the entire mass of the body is covered.

1.19.1 Moment of inertia of a thin uniform bar (Rod)

Consider a thin uniform bar AB of mass m and length l rotating about an axis passing through its centre and perpendicular to its length (axis YY_1)

Mass of the bar = M

Length of the bar = l

Mass per unit length = $\frac{M}{l}$

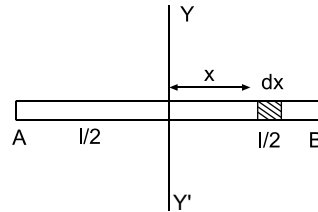
Take an element of length dx at a distance x from the axis

Mass of the element = $\left(\frac{M}{l}\right) dx$

Moment of inertia of the element about YY' = $\left(\frac{M}{l}\right) dx x^2$

Moment of inertia of the bar AB about the axis YY'

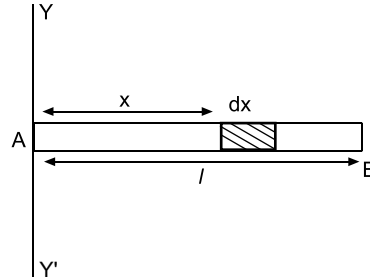
$$\begin{aligned} I &= \int_{-l/2}^{l/2} \left(\frac{M}{l}\right) x^2 dx \\ &= \frac{M}{l} \left[\frac{x^3}{3}\right]_{-l/2}^{l/2} \\ &= \frac{M}{l} \left[\frac{l^3}{24} + \frac{l^3}{24}\right] \\ &= \frac{M}{l} \frac{l^3}{12} = \frac{Ml^2}{12} \end{aligned}$$



1.19.2 About an axis at the end of the rod and normal to it

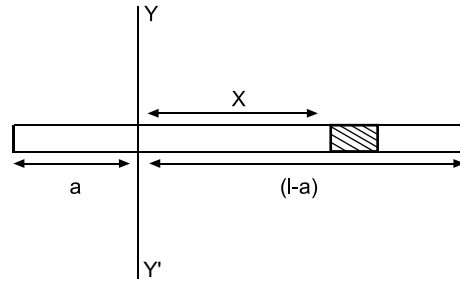
Moment of inertia of the bar about the axis YY'

$$\begin{aligned}
 I &= \int_0^l \left(\frac{M}{l}\right) x^2 dx \\
 &= \frac{M}{l} \left[\frac{x^3}{3}\right]_0^l \\
 &= \frac{M}{l} \frac{l^3}{3} = \frac{Ml^2}{3}
 \end{aligned}$$



1.19.3 Moment of inertia of a bar about an axis perpendicular to its length at a distance a from one end

$$\begin{aligned}
 I &= \frac{M}{l} \int_{-a}^{l-a} x^2 dx \\
 &= \frac{M}{l} \left[\frac{x^3}{3}\right]_{-a}^{l-a} \\
 &= \frac{M}{l} \left[\frac{(l-a)^3}{3} + \frac{a^3}{3}\right] \\
 &= \frac{M}{l} \left[\frac{l^3 - a^3 - 3l^2a + 3la^2}{3} + \frac{a^3}{3}\right] \\
 &= \frac{M}{3} [l^2 3la + 3a^2]
 \end{aligned}$$



1.19.4 Moment of inertia of a ring

Consider a thin uniform ring of mass M and radius R . The ring rotates about an axis YY' passing through its centre.

Mass of the ring = M

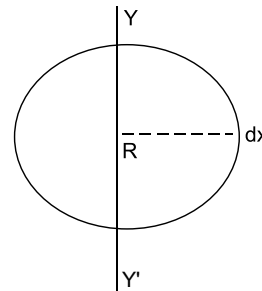
Length of the ring = $2\pi R$.

Mass per unit length = $\frac{M}{2\pi R}$

Take an element of length dx . Its distance from the axis is R .

Mass of the element = $\left(\frac{M}{2\pi R}\right) dx$

Moment of inertia of the element about the axis = $\left(\frac{M}{2\pi R}\right) R^2 dx$.



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$$\text{Moment of inertia of the ring } I = \frac{MR}{2\pi} \int_0^{2\pi R} dx = \frac{MR}{2\pi} [x]_0^{2\pi R} = MR^2$$

1.19.5 Moment of inertia of a solid sphere

About the diameter

Consider a solid sphere of radius R and mass M .

$$\text{Volume of the sphere} = \frac{4}{3}\pi R^3$$

$$\text{Mass per unit volume} = \frac{M}{\frac{4}{3}\pi R^3}$$

Consider an element of thickness dx at a distance x from the centre.

$$\text{Radius of the element } r = \sqrt{R^2 - x^2}$$

$$\text{Volume of the element} = \pi r^2 dx$$

$$= \pi(R^2 - x^2) dx$$

$$\text{Mass of the element} = \frac{M}{\frac{4}{3}\pi R^3} \pi(R^2 - x^2) dx$$

$$= \frac{3M}{4R^3} (R^2 - x^2) dx$$

Moment of inertia of the element about the axis XX^1

$$= \frac{1}{2} \times \text{mass} \times \text{square of radius.}$$

$$= \left[\frac{3M}{4R^3} (R^2 - x^2) \right] \frac{r^2}{2} dx$$

$$= \left(\frac{3M}{8R^3} \right) (R^2 - x^2)(R^2 - x^2) dx$$

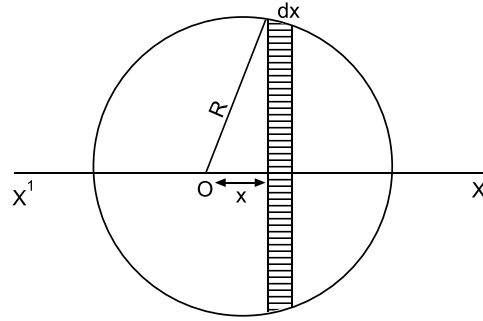
$$= \frac{3M}{8R^3} (R^2 - x^2)^2 dx$$

Moment of inertia of the whole sphere about the axis XX^1 .

$$I = 2 \int_0^R \frac{3M}{8R^3} (R^2 - x^2)^2 dx$$

$$= \frac{3M}{4R^3} \int_0^R (R^4 + x^4 - 2R^2 x^2) dx$$

$$= \frac{3M}{4R^3} \left[R^4 x + \frac{x^5}{5} - \frac{2R^2 x^3}{3} \right]_0^R$$

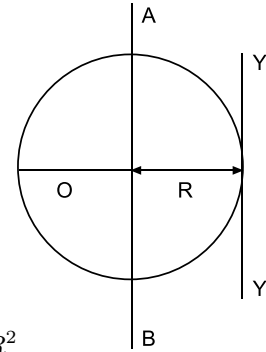


$$\begin{aligned}
 &= \frac{3M}{4R^3} \left[R^5 + \frac{R^5}{5} - \frac{2R^5}{3} \right] \\
 &= \frac{3MR^2}{4} \times \frac{8}{15} \\
 \frac{2MR^2}{5} &= \frac{2}{5}MR^2
 \end{aligned}$$

About a tangent

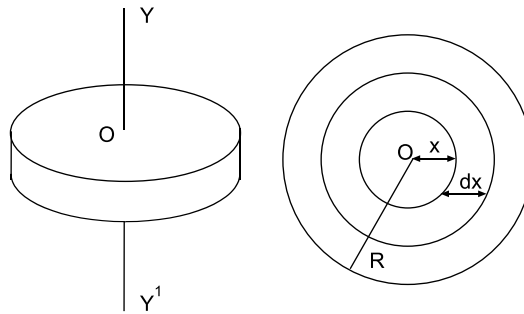
Moment of inertia of a solid sphere about a tangent is given by,

$$\begin{aligned}
 I_y &= I_{AB} + MR^2 \\
 &= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2
 \end{aligned}$$



1.19.6 Moment of inertia of a uniform circular disc

Consider a uniform circular disc of mass M and radius R rotating about an axis passing through its centre.



Mass of the disc = M

Area of the disc = πR^2

Mass per unit area = $\frac{M}{\pi R^2}$

Consider a thin element of the disc of radius x and radial thickness dx.

Area of the element = $2\pi x dx$.

Mass of the element = $\frac{M}{\pi R^2} 2\pi x dx$
 $= \frac{2M}{R^2} x dx$

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Moment of inertia of the element about the axis of rotation

$$= \text{mass} \times x^2$$

$$= \frac{2M}{R^2} x^3 dx$$

Moment of inertia of the whole disc about the axis of rotation = $\int_0^R \frac{2M}{R^2} x^3 dx$

$$I = \frac{2M}{R^2} \frac{R^4}{4}$$

$$= \frac{1}{2} MR^2$$

Moment of Inertia of a disc about its diameter

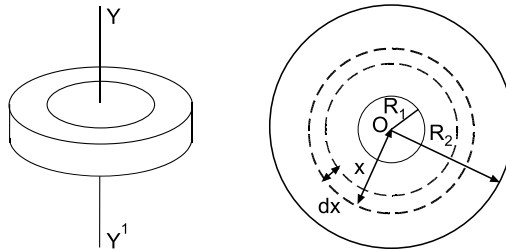
$$I = \frac{MR^2}{4}$$

where $I = I_x + I_y = 2I_x$.

$$\text{or } I_x = \frac{1}{2} \frac{MR^2}{2} = \frac{MR^2}{4}$$

1.19.7 Moment of inertia of an annular disc

Consider a uniform circular (annular) disc of inner radius R_1 , and outer radius R_2 . Let mass of the disc be M .



$$\text{Area of the disc} = \pi (R_2^2 - R_1^2)$$

$$\text{Mass per unit area} = \frac{M}{\pi (R_2^2 - R_1^2)}$$

Consider an element of radius x and radial thickness dx

$$\text{Area of the element} = \frac{M}{\pi (R_2^2 - R_1^2)} 2\pi x dx$$

$$= \frac{2Mx dx}{R_2^2 - R_1^2}$$

Moment of inertia of the element about an axis passing through its centre and perpendicular to the plane of the disc.

$$= \frac{2Mx dx}{R_2^2 - R_1^2} x^2$$

Moment of inertia of the whole disc about YY'

$$\begin{aligned} I &= \int_{R_1}^{R_2} \frac{2Mx^3 dx}{R_2^2 - R_1^2} \\ &= \frac{2M}{R_2^2 - R_1^2} \left[\frac{x^4}{4} \right]_{R_1}^{R_2} \\ &= \frac{2M}{(R_2 - R_1)(R_2 + R_1)} \frac{(R_2^4 - R_1^4)}{4} \\ &= \frac{2M}{R_2^2 - R_1^2} \frac{(R_2^2 - R_1^2)(R_2^2 + R_1^2)}{4} \\ I &= \frac{M}{2}(R_2^2 + R_1^2) \end{aligned}$$

b) About the diameter $I = I_x + I_y = 2I_x \{I_x = I/2\}$

$$= \frac{M}{4}(R_2^2 + R_1^2)$$

c) About a tangent in the plane of the disc.

$$\begin{aligned} I &= I_X + MR_2^2 \\ &= \frac{M(R_2^2 + R_1^2)}{4} + MR_2^2 \end{aligned}$$

1.19.8 Moment of inertia of a spherical shell

Consider a spherical shell of radius R and mass M . Consider an element between two planes P and Q . The distance between the two planes is dx .

$$\angle GOE = \delta\theta$$

$$\text{Then } EG = R\delta\theta.$$

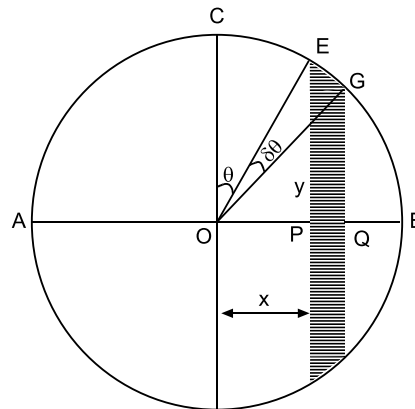
Radius of thin element

$$y = R \cos \theta$$

$$x = R \sin \theta dx = R \cos \theta d\theta$$

$$\text{Surface area of the element} = 2py(EG)$$

$$= 2\pi R \cos \theta R d\theta$$



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$$= 2\pi R^2 \cos \theta d\theta$$

$$= 2\pi R dx$$

$$\text{Mass per unit area of the shell} = \frac{M}{4\pi R^2}$$

$$\text{Mass of the element} = \frac{M}{4\pi R^2} 2\pi R dx$$

$$= \frac{M dx}{2R}$$

Moment of inertia of the element about the diameter AB

$$\frac{M dx}{2R} y^2 = \frac{M dx}{2R} (R^2 - x^2)$$

Moment of inertia of the whole shell about the diameter

$$\begin{aligned} &= 2 \int_0^R \frac{M dx}{2R} (R^2 - x^2) \\ &= \frac{M}{R} \left[R^2 x - \frac{x^3}{3} \right]_0^R \\ &= \frac{M}{R} \left[R^3 - \frac{R^3}{3} \right] = \frac{2}{3} MR^2 \end{aligned}$$

About the tangent

$$I + MR^2 = \frac{2}{3} MR^2 + MR^2 = \frac{5}{3} MR^2$$

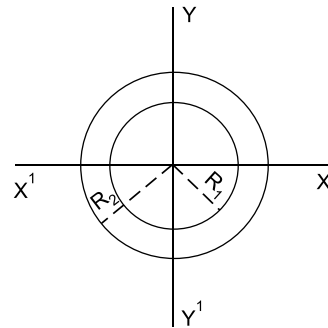
1.19.9 Moment of inertia of a hollow sphere

Consider a hollow sphere of inner radius R_1 and outer radius R_2

Mass of the sphere = M

Let the density of the material = ρ

$$M = \frac{4}{3}\pi(R_2^3 - R_1^3)\rho \quad (32)$$



Moment of inertia of the hollow sphere

$$\begin{aligned} I &= \frac{2}{5}(M_1 R_2^2 - M_2 R_1^2) \\ I &= \frac{2}{5} \left(\frac{4}{3}\pi R_2^3 \rho R_2^2 - \frac{4}{3}\pi R_1^3 \rho R_1^2 \right) \\ &= (R_2^5 - R_1^5) \pi \rho \\ \rho &= \frac{M}{\frac{4}{3}\pi(R_2^3 - R_1^3)} \text{ from equation (1)} \end{aligned}$$

$$I = \frac{\frac{2}{5}M(R_2^5 - R_1^5)}{(R_2^3 - R_1^3)}$$

when $R_1 = 0; R_2 = R$

$$I = \frac{2}{5}MR^2$$

1.19.10 Moment of inertia of a rectangular plate

Consider a rectangular plate of uniform thickness, length and breadth be l and b respectively.

Consider an element of length dx at a distance x from YY' .

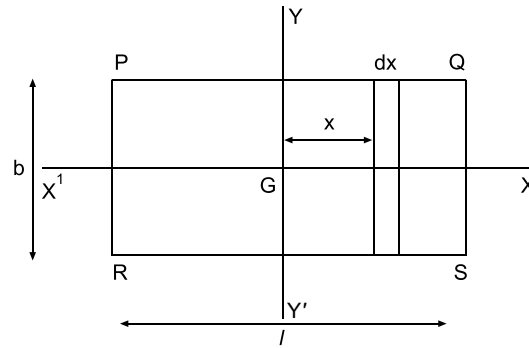
Mass of the plate = M

Area of the plate = $l \times b$

Mass per unit area = $\frac{M}{l \times b}$

Area of the element = $b dx$

Mass of the element = $\frac{M}{lb} b dx$
 $= \frac{M}{l} dx$



Moment of Inertia of the element about the axis $YY' = \frac{M}{l} dx x^2$

Moment of inertia of the whole plate about the axis YY'

$$I_y = \frac{M}{l} \int_{-l/2}^{l/2} x^2 dx$$

$$= \frac{M}{l} \left[\frac{l^3}{24} + \frac{l^3}{24} \right] = \frac{Ml^2}{12}$$

Similarly about XX'

$$I_x = \frac{Mb^2}{12}$$

Moment of Inertia of the plate about an axis ZZ' passing through the centre of gravity (perpendicular axis theorem).

$$I_z = I_x + I_y$$

$$I_z = \frac{M}{12}(l^2 + b^2)$$

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Moment of Inertia of the plate about an axis PR

$$I_{PR} = I_y + M \left(\frac{l}{2} \right)^2$$

$$= \frac{Ml^2}{12} + \frac{Ml^2}{4} = \frac{Ml^2}{3}$$

Moment of Inertia of the plate about an axis PQ.

$$I_{PQ} = I_x + M \left(\frac{b}{2} \right)^2$$

$$= \frac{Mb^2}{3}$$

1.19.11 Moment of inertia of hollow rectangular section

Moment of Inertia of solid rectangular about X-X axis

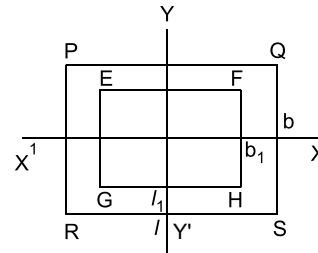
$$I_x = \frac{Mb^2}{12}$$

Moment of Inertia of section EFGH about X-X axis

$$= \frac{Mb_1^2}{12}$$

Moment of Inertia of hollow rectangular section about X-X axis

$$= \frac{Mb^3}{12} - \frac{Mb_1^3}{12}$$



1.19.12 Moment of inertia of a circular section

Let us consider a circular section of radius R and O as centre. Consider an elementary circular ring of radius r and thickness dr.

Area of elementary circular ring = $2\pi r dr$

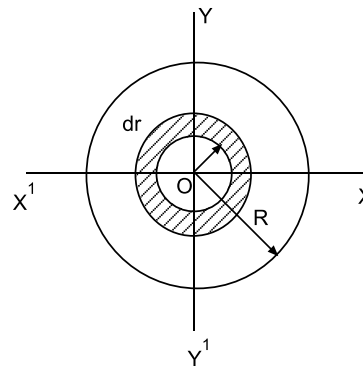
Moment of inertia of the elementary circular ring about an axis passing through O and perpendicular to the plane of the paper

$$\text{Moment of Inertia} = 2\pi r dr \cdot r^2$$

$$= 2\pi r^3 dr.$$

Moment of Inertia of the whole circular section about an axis passing through O and perpendicular to the plane of the paper is given by

$$\int_0^R 2\pi r^3 dr = 2\pi \left[\frac{r^4}{4} \right]_0^R$$



$$= \frac{\pi}{2} R^4$$

But $R = \frac{D}{2}$

D Diameter of the circular section

$$I = \frac{\pi}{2} \left(\frac{D}{2}\right)^4 = \left(\frac{\pi D^4}{32}\right)$$

But from the theorem of perpendicular axis is given by $I_{xx} = I_{yy}$

$$I = I_{xx} + I_{yy}$$

$$I = 2I_{yy}$$

$$\frac{I}{2} = I_{xx} \text{ (or) } I_{yy}$$

$$= \frac{\pi D^4}{32} \times \frac{1}{2}$$

$$= \frac{\pi D^4}{64}$$

1.19.13 Moment of Inertia of a hollow circular section

D → Diameter of the outer circle

d → Diameter of the inner circle

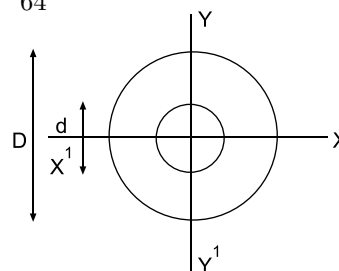
Moment of Inertia of outer circle about XX axis = $\frac{\pi D^4}{64}$

Moment of Inertia of inner circle about XX axis = $\frac{\pi d^4}{64}$

.5 Moment of Inertia of hollow circular section about XX axis

$$I_{xx} = \frac{\pi}{64} [D^4 - d^4]$$

Similarly $I_{yy} = \frac{\pi}{64} [D^4 - d^4]$



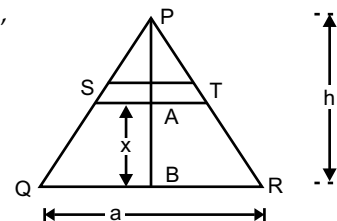
1.19.14 Moment of inertia of triangular plate

Consider a triangular plate PQR of mass 'M', base 'a' and height 'h' as shown in the figure.

From the above figure

QR = a, the base of the triangular plate

PB = h, the height of the plate.



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Area of the plate

$$\begin{aligned}
 &= \frac{1}{2} \times \text{base} \times \text{height} \\
 &= \frac{1}{2} \times QR \times PB \\
 &= \frac{1}{2}ah \qquad (33)
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Mass per unit area (m) of the triangular plate} &= \frac{M}{\frac{1}{2}ah} \\
 &= \frac{2M}{ah}
 \end{aligned}$$

\therefore Mass of the plate for the given area PQR is

$$\begin{aligned}
 M &= \text{Area} \times \text{Mass per unit area} \\
 M &= \frac{1}{2}amh \qquad (34)
 \end{aligned}$$

To find moment of inertia of the plate about base QR

Consider a small strip 'ST' of thickness 'dx' at a distance 'x' from the base QR. (Many such strips combine together to constitute the given triangular plate)

$$\text{Area of the strip} = ST.dx \qquad (35)$$

Mass of the strip

$$\begin{aligned}
 &= \text{Area of the strip} \times \text{Mass per unit area of the plate.} \\
 &= ST.dx.m \qquad (36)
 \end{aligned}$$

To find 'ST'

$$ST = SA + AT$$

From the similar triangles ΔPQB & ΔPSA

$$\begin{aligned}
 \frac{SA}{QB} &= \frac{PA}{PB} = \frac{h-x}{h} \\
 \therefore SA &= \left(\frac{h-x}{h}\right).QB \qquad (37)
 \end{aligned}$$

From the similar triangles ΔPRB & ΔPTA

$$\begin{aligned}
 \frac{AT}{BR} &= \frac{PA}{PB} = \frac{h-x}{h} \\
 \therefore AT &= \left(\frac{h-x}{h}\right).BR \qquad (38)
 \end{aligned}$$

Adding Eq.(37) and Eq.(38) we get

$$\begin{aligned}
 ST &= SA + AT = \left(\frac{h-x}{h}\right)(QB + BR) \\
 \therefore ST &= \left(\frac{h-x}{h}\right)QR \\
 ST &= \left(\frac{h-x}{h}\right)a \tag{39}
 \end{aligned}$$

Substituting Eq.(39) in Eq.(36) we get

$$\begin{aligned}
 \text{Mass of the strip} &= ST \cdot dx \cdot m \\
 &= \left(\frac{h-x}{h}\right)a \cdot m \cdot dx \tag{40}
 \end{aligned}$$

M.I. of the strip about base BC

$$\begin{aligned}
 &= \text{Mass of the strip} \times (\text{distance from base})^2 \\
 &= \left(\frac{h-x}{h}\right)a \cdot m \cdot dx \cdot x^2 \\
 &= am \left(\frac{h-x}{h}\right)x^2 dx \tag{41}
 \end{aligned}$$

\therefore M.I. of the triangular plate about base axis QR is

$$\begin{aligned}
 &= \int_0^h am \left(\frac{h-x}{h}\right)x^2 dx \\
 &= \frac{am}{h} \int_0^h (h-x)x^2 dx \\
 &= \frac{am}{h} \left[\frac{hx^3}{3} - \frac{x^4}{4} \right]_0^h \\
 &= \frac{am}{h} \frac{h^4}{12} \\
 &= \frac{amh^3}{12} \\
 \therefore M.I. &= \frac{Mh^2}{6} \tag{42}
 \end{aligned}$$

Where $M = \frac{1}{2} amh$, is the mass of the triangular plate. Eq.(41) gives the moment of inertia of the triangular plate about its base axis.

Moment of inertia of a triangular section about an axis passing through C.G and parallel to the base

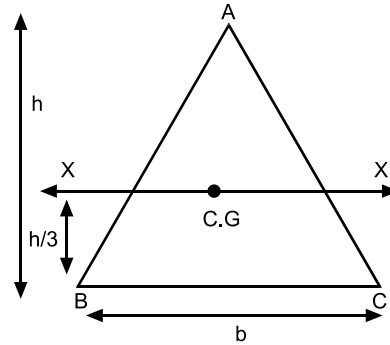
Height = h

Base width = b

Distance between the base and C.G of the triangular section = $\frac{h}{3}$

From the theorem of parallel axis

Moment of Inertia about BC = Moment of Inertia about C.G + Area \times (Distance between XX and BC)²



$$I_{BC} = I_G + A \left(\frac{h}{3} \right)^2$$

$$I_G = I_{BC} \frac{Ah^2}{9}$$

$$= \frac{bh^3}{12} - \frac{b \times h}{2} \left(\frac{h}{3} \right)^2$$

$$= -\frac{bh^3}{12} - \frac{bh^3}{18}$$

$$I_G = \frac{bh^3}{36}$$

1.19.15 Moment of inertia of a hollow cylinder

About its own axis

Consider a cylinder of length l and mass M . Its inner radius is R_1 and outer radius R_2 .

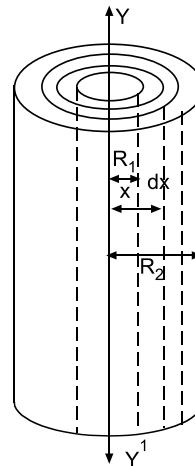
$$\text{Mass per unit volume} = \frac{M}{\pi (R_2^2 - R_1^2) l}$$

Consider an element of length l , radius x and thickness dx .

$$\text{Volume of the element} = 2\pi x dx l$$

$$\text{Mass of the element} = \frac{M(2\pi x dx)l}{\pi (R_2^2 - R_1^2) l}$$

Moment of Inertia of the element about the axis $YY^1 = \frac{2Mx dx}{R_2^2 - R_1^2} x^2$



Moment of Inertia of the whole cylinder about the axis YY^1

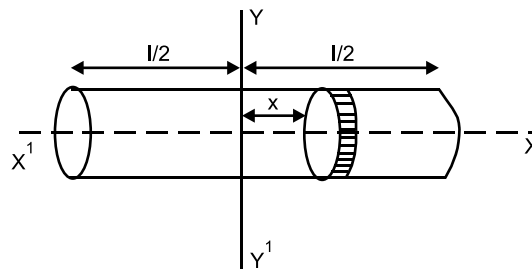
$$\begin{aligned}
 I &= \int_{R_1}^{R_2} \frac{2M}{(R_2^2 - R_1^2)} x^3 dx \\
 &= \frac{2M}{(R_2^2 - R_1^2)} \left[\frac{x^4}{4} \right]_{R_1}^{R_2} \\
 I &= \frac{M(R_2^2 + R_1^2)}{2}
 \end{aligned}$$

For solid cylinder

$$\begin{aligned}
 R_1 &= 0 \\
 R_2 &= R \\
 I &= \frac{MR^2}{2}
 \end{aligned}$$

Moment of Inertia of a solid cylinder about an axis passing through its centre and perpendicular to its own axis of cylindrical symmetry

Let l be the length, R the radius and M is the mass of the cylinder, then the mass per unit length of the cylinder will be $\frac{M}{l}$. Let YY^1 be the axis about which the moment of inertia has to be determined.



Consider a small disc of width dx at a distance x from the axis YY^1 .

Moment of inertia of the disc about its diameter = $\frac{MR^2}{4}$

Moment of inertia of this small disc about an axis YY^1

$$\left(\frac{M}{l} dx \right) \left(\frac{R^2}{4} \right) + \left(\frac{M}{l} dx \right) (x^2)$$

Moment of inertia of the cylinder about YY^1

$$I = 2 \int_0^{l/2} \left[\frac{M}{l} \cdot \frac{R^2}{4} dx + \frac{M}{l} x^2 dx \right]$$

$$\begin{aligned}
&= \frac{2M}{l} \int_0^{l/2} \left[\frac{R^2}{4} + x^2 \right] dx \\
&= \frac{2M}{l} \left[\frac{R^2}{4}x + \frac{x^3}{3} \right]_0^{l/2} \\
&= \frac{2M}{l} \left[\frac{R^2}{4} \cdot \frac{l}{2} + \frac{l^3}{24} \right] \\
&= \frac{M}{l} \left[\frac{R^2}{4}l + \frac{l^3}{12} \right] \\
&= \frac{M}{12l} [3R^2l + l^3] \\
&= M \left[\frac{R^2}{4} + \frac{l^2}{12} \right]
\end{aligned}$$

1.20 Oscillations of Rigid Bodies

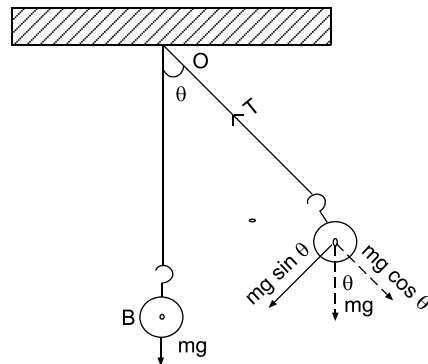
A mechanical vibration is the motion of a particle or a body which oscillates about a position of equilibrium. A mechanical vibration generally results when a system is displaced from a position of stable equilibrium. The time interval required for the system to complete a full cycle of motion is called the period of vibration. The number of cycles per unit time defines the frequency, and the maximum displacement of the system from its position of equilibrium is called the amplitude of vibration.

When a body vibrates under the action of restoring forces only, the motion is called a free vibration. If resisting forces are also present, the motion is called damped free vibration. If the vibrations are caused by a periodic force applied to the body, the motion is called a forced vibration which may or may not be damped.

1.20.1 Simple pendulum

A simple pendulum displaced slightly from its equilibrium position executes oscillation.

Figure shows a simple pendulum displaced to a position A by a small angle θ from its mean position B. At the position A, the weight mg of the bob acts vertically downwards and tension T in the string acts along the string upwards. The weight mg can be reduced into two mutually perpendicular components.



1. The component $mg \cos \theta$, along the string, is balanced by the tension T in the string.
 $\therefore mg \cos \theta = T$.
2. The component $mg \sin \theta$, along the direction perpendicular to the string is unbalanced. This force on the bob is directed towards the mean position and under the influence of this force the bob moves towards the mean position.

The displacement $AB = x = l\theta$ where l is the effective length of the pendulum and θ is very small.

The restoring force acting on the oscillating particle = $mg \sin \theta$

For small angular displacement θ , $\sin \theta = \theta$.

\therefore Restoring force $F = mg \theta$.

(ve sign indicates restoring force is opposite to the direction of displacement).

Substituting the value for θ

$$F = \frac{-mg}{\ell}x$$

Since the restoring force is directly proportional to the displacement and is directed opposite to the displacement, the oscillations of simple pendulum are simple harmonic.

$$\begin{aligned} \therefore \text{Force constant } K &= \frac{\text{Force}}{\text{displacement}} \\ &= \frac{F}{x} = \frac{mg\theta}{l\theta} = \frac{mg}{l} \end{aligned}$$

Time period of the pendulum

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ &= 2\pi\sqrt{\frac{m}{mg/\ell}} \\ &= 2\pi\sqrt{\frac{\ell}{g}} \end{aligned}$$

Frequency of oscillation of simple pendulum

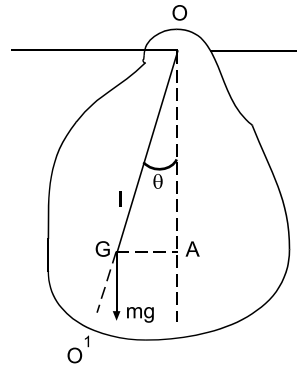
$$\begin{aligned} v &= \frac{1}{T} \\ &= \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \end{aligned}$$

The time period does not depend on the mass of the body but only on the length ℓ of the pendulum.

1.20.2 Compound pendulum

A compound pendulum is a rigid body, capable of oscillating freely about a horizontal axis passing through it.

Let m be the mass of the pendulum and it makes small oscillation under gravity about an axis through a point O . Let G be the centre of gravity of the pendulum and l be the distance OG . The pendulum is displaced from its position of rest so that OG makes an angle θ with OA the vertical through O . The weight of the pendulum mg acting vertically downwards produces equal and opposite reaction acting at O . These two forces together form a couple which tends to bring the pendulum to its initial position.



Now let us take moment about the axis of rotation.

The moment of weight mg is $= mgl \sin \theta$.

Here θ is small angular displacement from the position of equilibrium.

If I is the moment of inertia of the rigid body about the axis of rotation, then the equation of moment.

$$I \frac{d^2\theta}{dt^2} = mgl \sin \theta$$

The negative sign indicates as θ increases the couple tries to reduce it.

We know $I = MK^2$

where K is radius of gyration about O

$$\therefore mk^2 \frac{d^2\theta}{dt^2} + mgl \theta = 0$$

when θ is small $\sin \theta \approx \theta$

$$\begin{aligned} \therefore mk^2 \frac{d^2\theta}{dt^2} + mgl \theta &= 0 \\ k^2 \theta &= gl\theta \\ \therefore \theta &= \frac{-gl\theta}{k^2} \end{aligned}$$

From the above relation it is clear that the motion is S.H.M, as $\frac{d^2\theta}{dt^2}$ is proportional to angular displacement θ .

$$\text{Time period of the pendulum} = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$T = 2\pi \sqrt{\frac{\theta}{\frac{lg}{k^2} \cdot \theta}} = 2\pi \sqrt{\frac{k^2}{lg}}$$

$$\text{Frequency } f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{lg}{k^2}}$$

Torsional Pendulum

A torsional pendulum is a pendulum performing torsional oscillations and consisting of a metal wire clamped to a rigid support at one end and carries a heavy circular disc at the other end. When the disc is subjected to slight rotation and left free, it starts oscillating periodically about the wire as axis. During its oscillations, its angular acceleration 'a' = $\frac{d^2\theta}{dt^2}$ and angular velocity, ' ω ' = $\frac{d\theta}{dt}$.

Potential energy confined to the wire (equal to the work done in twisting it through θ) = \int_0^θ moment of the couple x $d\theta$.

$$= \int_0^\theta c \theta \cdot d\theta = \frac{1}{2} c\theta^2$$

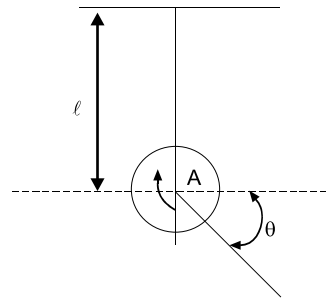
where 'c' is the couple per unit twist.

$$\text{Kinetic energy confined to the rotating disc} = \frac{1}{2} \sum m_i v_i^2 = \frac{1}{2} I \omega^2.$$

where $r_i \omega = v_i$ and $I = \sum m_i r_i^2$ = moment of inertia of the circular disc about the axis of suspension.

$$\text{Total energy of the torsional pendulum} = \frac{1}{2} c \theta^2 + \frac{1}{2} I \omega^2$$

According to the principle of conservation of energy $\frac{1}{2} c \theta^2 + \frac{1}{2} I \omega^2 = \text{constant}$.



Diff. w.r.to time

$$\frac{1}{2} c \cdot 2 \cdot \theta \frac{d\theta}{dt} + \frac{1}{2} I \cdot 2 \cdot \omega \frac{d\omega}{dt} = 0$$

$$\text{Sub: } \omega = \frac{d\theta}{dt}; \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\text{we get, } \frac{1}{2} c \cdot 2 \cdot \theta \frac{d\theta}{dt} + \frac{1}{2} I \cdot 2 \cdot \frac{d\theta}{dt} \frac{d^2\theta}{dt^2}$$

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$$\frac{d^2\theta}{dt^2} = \frac{-c\theta}{I}$$

This eqn represents a simple harmonic motion eqn.

Then,

$$\omega^2 = \frac{-c}{I}$$

$$\omega = \sqrt{\frac{c}{I}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{c}{I}}$$

Time period, $T = 2\pi \sqrt{\frac{I}{c}}$

Frequency, $f = \frac{1}{2\pi} \sqrt{\frac{c}{I}}$

Short Questions

1. Define 'Elasticity'
2. Define 'Strain' and 'stress'
3. State Hooke's law.
4. Represent the relation between stress & strain by a graph and indicate the important point.
5. What do you mean by elastic limit.
6. What is breaking stress.
7. Explain yield point and yielding stress
8. Differentiate perfect elastic and plastic materials
9. Define Tensile strength
10. Define Young's modulus, bulk modulus and rigidity modulus
11. Define poisson's ratio
12. Derive the limiting value of σ ?
13. What is a shaft?
14. What are the requirements of a good shaft?
15. Define neutral surface?
16. What is a cantilever ?
17. Differentiate uniform and non uniform bending
18. What are I shaped girders.
19. What are the three moduli of elasticity? Give the relation between them?

20. What is a torsion pendulum?
21. Why do we prefer I shaped girders rather than solid girders?
22. What are the basic assumptions for theory of bending.
23. **Define moment of inertia**

The moment of inertia is the sum of the products of the area (or mass) and the square of the distance from the axis of rotation.

$$I = \sum ar^2 \text{ or } I = \sum mr^2.$$

24. **What is a centroid?**

The plane figures (like triangle, quadrilateral, circle etc) have only area but no mass. The centre of area of such figures is known as centroid.

25. **What do you mean by centre of gravity?**

It is defined as the point through which the whole weight of the body acts, irrespective of the position of the body

26. **Define radius of gyration?**

Suppose the whole mass of the body is concentrated at a single point such that moment of inertia of this concentrated point mass is same as the moment of inertia of the whole body about the axis, the distance of that single point from the axis is called the radius of gyration of the body about the axis

or

Radius of gyration is equal to the root mean square distance of the particles from the axis of rotation.

$$k = \sqrt{r_1^2 + r_2^2 + \dots + r_n^2 / n}$$

27. **State theorem of perpendicular axis**

This theorem states that the moment of inertia of a plane lamina about an axis perpendicular to the plane is equal to the sum of moment of inertia about two mutually perpendicular axis in the plane of the lamina such that the three mutually perpendicular axis have a common point of intersection.

28. **State theorem of parallel axis**

Parallel axis theorem states that the moment of inertia of a plane area about an axis in the plane of area through the centre of gravity of the plane area be represented by I_G , then the moment of inertia of the given plane area about a parallel axis AB in the plane of the area at a distance h from centre of gravity of the area is given by

$$I_{AB} = I_G + Ah^2 \text{ where } A \text{ is area of the section.}$$

$$I_{AB} \longrightarrow \text{Moment of inertia of the given area about AB}$$

$$I_G \longrightarrow \text{Moment of inertia of the given area about centre of gravity}$$

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29. Give the moment of inertia of a rectangular plate, triangular plate and cylinder?

$$\text{Moment of inertia of a rectangular plate} = \frac{M}{12} (l^2 + b^2)$$

$$\text{Moment of inertia of a triangular plate} = \frac{bh^3}{12}$$

(about its base)

$$\text{Moment of inertia of a solid cylinder} = \frac{MR^2}{2}$$

$$\text{Moment of inertia of a hollow cylinder} = \frac{M(R_2^2 + R_1^2)}{2}$$

Review Questions

1. What is a shaft? Give examples? What are the requirements of a good shaft? Explain why hollow shaft are stiffer than solid shaft.
2. Define Torsion pendulum. With necessary theory, derive the expression for the time period of oscillation for a torsion pendulum.
3. With necessary explanation, derive the expression for couple / couple unit twist on a cylinder fixed at one end.
4. Explain the determination of rigidity modulus of the given wire, by Torsional oscillations.
5. Derive an expression for the internal bending moment of a beam.
6. Define a cantilever? If 'y' be the depression of the cantilever when loaded at its free end, with necessary theory, derive an expression for 'y' in terms of Young's modulus of the beam.
7. Describe an experimental method for determining the Young's modulus of a material of a cantilever.
8. Derive an expression for the time period of oscillation of cantilever.
9. Derive an expression of a beam supported at two ends and loaded at the middle. Also describe an experiment to determine the Young's modulus using this.
10. Define uniform bending and obtain an expression for the elevation at the centre of the beam subjected to the loads at its ends Also give experimental determination of young is modulus using uniform bending.
11. State parallel axis theorem? Derive on expression for moment of inertia and radius of gyration?
12. State perpendicular axis theorem? Derive the expression for moment of inertia of rectangular plate?

13. Obtain an expression for the moment of inertia of a triangular plate about its base and about an axis passing through centre of gravity and parallel to the base?
14. Deduce an expression for the moment of inertia of a hollow and solid cylinder about its own axis?
15. Derive the moment of inertia of a solid cylinder about an axis passing through its centre and perpendicular to its own axis?
16. Obtain the expression for time period of oscillation of simple pendulum? Derive the expression for frequency of oscillation of compound pendulum?
17. Describe with a neat diagram the construction of torsional pendulum and deduce an expression for the time period of oscillation?

Exercise Problems

1. Calculate Young's modulus of a rubber tube 0.4 m long, whose external and internal diameter are 0.01 m and 0.004 m respectively extends 0.0006 m when stretched by a force of 5 kg wt. **(Ans : $4.9 \times 10^8 \text{ N/m}^2$)**
2. A spherical ball contracts in volume by 0.01%. When subjected to a normal uniform pressure of 100 atmospheres. Calculate the bulk modulus of the material of ball. **(Ans: 10^{10} N/m^2)**
3. A material has Poisson's ratio of 0.20. If a uniform rod of it suffers a longitudinal strain 4.0×10^{-3} , deduce the percentage change in its volume. **(Ans: 0.24%)**
4. A wire of length 1m and diameter 10^{-3}m , is stretched by 6×10^{-4} m by a load 10 kg. The wire is twisted by 70° by a force of 5×10^{-3} kg applied to each end of 0.2 m length rod soldered at its mid point to the end of the wire. Calculate y, n, k & σ . **(Ans: $n = 8.17 \times 10^{10} \text{ N/m}^2$; $Y = 20.806 \times 10^{10} \text{ N/m}^2$;
 $k = 15.2 \times 10^{10} \text{ N/m}^2$; $\sigma = 0.27$)**
5. A metallic wire of length 1 m and radius 0.5 mm is twisted through 0.6 radian by applying a torque 0.006 Nm. Calculate the corresponding modulus of elasticity of the material of the wire. **(Ans: $12.07 \times 10^{10} \text{ pa}$)**
6. A steel transmission shaft 2 cm in diameter and 5 m long rotates at 1500 rpm. The torsion in shaft is 0.02 radian. Rigidity modulus = $8 \times 10^{10} \text{ N/m}^2$. Find power transmitted by the shaft. **(Ans: 790.2 watts)**
7. The restoring couple/unit twist in a solid cylinder of radius 5 cm in 10^{-1} Nm . Find the restoring couple / unit twist in a hollow cylinder of the same material, mass and length but the internal radius 12 cm. **(Ans: 1.25 Nm)**

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8. Calculate the % saving in material if a hollow coupling shaft with internal radius one half of the external radius, replaces a solid shaft of same material and length, the 2 shafts having equal torsional rigidities. **(Ans: 22%)**
9. A thin walled circular tube of mean diameter 20 cm and thickness 0.5 mm is melted up and recast with a solid rod of same length. Compare the torsional rigidities in the 2 cases. **(Ans: 200)**
10. Forces of 10^5 N each are applied in opposite directions on upper and lower faces of a cube of side 10 cm, shifting the upper face parallel to itself by 0.5 m. If the side of the cube was 20 cm, what would be the displacement?
(Ans: 0.25m)
11. Two solid cylinders of same material having lengths l , $2l$ and radii r , $2r$ respectively are joined coaxially. Under a couple applied between the free ends, the shorter shows a twist of 30° . Calculate the twist of the larger cylinder.
(Ans: 3.75°)
12. If the values of Young's modulus and rigidity modulus are 20×10^{10} N/m² and 8×10^{10} N/m². Find bulk modulus and poisson's ratio.
(Ans: $\sigma = 0.25$; $k = 13.3 \times 10^{10}$ N/m²)
13. A gold wire 0.32 mm in diameter, elongates by 1 mm, when stretched by a force of 330 gmwt and twists through 1 radian. When equal and opposite torques of 145 dynes cm, are applied at its ends. Find the values of Poisson's ratio for gold.
(Ans: 0.429)
14. How much force is required to punch a hole $\frac{1}{2}$ in, in diameter in a steel sheet $\frac{1}{8}$ in thick whose shearing strength is 4×10^4 lb/in². **(Ans: 7840 lb)**
15. An Aluminium wire 3 mm in diameter and 4 m long is used to support a mass of 50 kg. What is the elongation of the wire? $Y = 7 \times 10^{10}$ Pa.
(Ans: 3.96 mm)
16. A sheet of Indian rubber of 10 cm sq and 2 cm thick has one face fastened to a vertical wall and to the other face a piece of wood is cemented when a load of 30 kg is hung from the wood, the wood is found to be finally lowered by 0.03 cm. Find the co-efficient of rigidity of rubber. **(Ans: 1.96×10^7 dynes/cm²)**
17. A uniform glass tube is hung from a support and stretched by a weight. It is found that 1 m of the tube stretches by 0.6 cm, but that a column of water 1 m long contained within the tube lengthens by only 0.4 cm. Find the Poisson's ratio of glass.
(Ans: 0.3332)

18. Two cylindrical shafts have the same length and mass and are made of the same material. One is solid, which the other, which is hollow, has an external radius twice the internal radius. Compare their torsional rigidities and the maximum strains produced by equal twisting torques. **(Ans: $3/5$ & $5/2\sqrt{3}$)**
19. A 10 cm wide and 0.2 mm thick metal sheet is bent to form a cylinder of 10 cm length and 50 cm radius. If the Young's modulus of the metal is 1.5×10^{12} dynes/cm², calculate (i) the stress and strain on the convex surface (ii) bending moment. **(Ans: 3×10^8 dynes/cm²; 2×10^{-4} ; 2×10^5 dyne cm)**
20. A cantilever of length 0.5 m has a depression of 15 mm at its free end. Calculate the depression at a distance of 0.3 m from the fixed end. **(Ans: 6.48×10^{-3} m)**

CHAPTER

2

Technical Acoustics

2.1 Doppler Effect

It is commonly observed that the pitch of a note apparently changes when either the source or the observer are in motion relative to each other. When the source approaches the observer or when the observer approaches the source or when both approach each other the apparent pitch is higher than the actual pitch of the sound produced by the source. Similarly, when the source moves away from the observer or when the observer moves away from the source or when both move away from each other, the apparent pitch is lower than the actual pitch of the sound produced by the source.

Suppose a person is standing on a platform. The apparent pitch of the whistle of the engine increases, when the engine is approaching the person. When the engine moves away from the person, the apparent pitch of the whistle of the engine decreases. This apparent change in the pitch due to the relative motion between the source and the observer is called Doppler Effect.

Doppler effect in sound is asymmetric. When the source moves towards the observer with a certain velocity, the apparent pitch is different to the case when the observer is moving towards the source with the same velocity. But it is not so in the case of light. Doppler effect in light is symmetric. The apparent pitch in different cases is calculated as shown below.

2.1.1 Observer at rest and source in motion

(a) When the source moves towards the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ . The velocity of sound is v (Fig.1).

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Let the source move with a velocity a towards the observer. In one second, n waves will be contained in a length $(v - a)$ and the apparent wavelength,

$$\lambda' = \frac{(v - a)}{n}$$

The apparent pitch,

$$n' = \frac{v}{\lambda'}$$

$$\therefore n' = \left(\frac{v}{v - a} \right) n \quad (1)$$

Thus the apparent pitch of the note increases when the source moves towards a stationary observer.

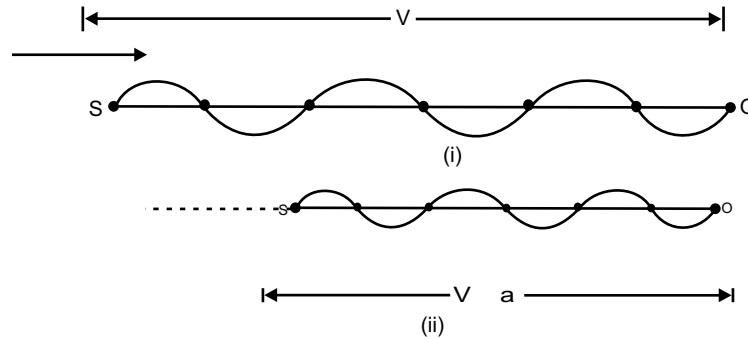


Fig. 1

(b) When the source moves away from the stationary observer

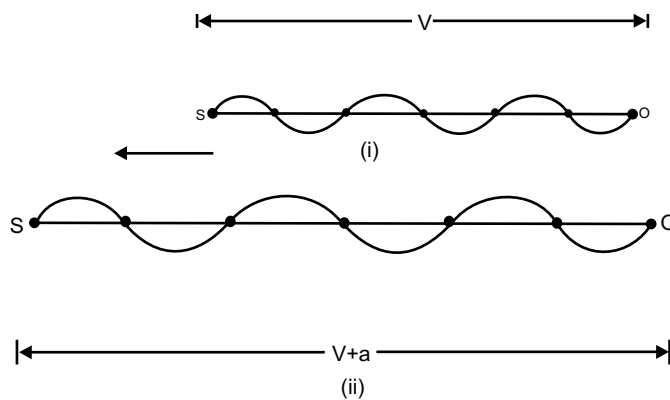


Fig. 2

Suppose a source S is producing sound of pitch n and wavelength λ . The velocity of sound is v (Fig.2). Let the source move with a velocity a away from the observer.

In one second, n waves will be contained in a length $(v + a)$ and the apparent wavelength,

$$\lambda' = \frac{v + a}{n}$$

The apparent pitch,

$$\begin{aligned} n' &= \frac{v}{\lambda'} \\ \therefore n' &= \left(\frac{v}{v + a} \right) n \end{aligned} \quad (2)$$

Thus the apparent pitch of the note decreases when the source moves away from a stationary observer.

Note: For the same values of v , a and n , the values of n' in equations (1) and (2) will be different.

2.1.2 Source at rest and observer in motion

(a) When the observer moves towards a stationary source

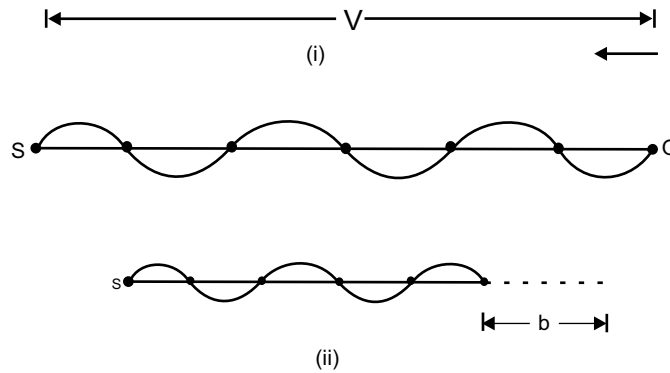


Fig. 3

Suppose a source S is producing sound of pitch n and wavelength λ . The velocity of sound is v (Fig.3). Let the observer move with a velocity b towards a stationary source. In the case the observer receives more number of waves in one second. The apparent wavelength remains the same. The apparent frequency

$$\begin{aligned} n' &= n + \frac{b}{\lambda} \\ n' &= \frac{v}{\lambda} + \frac{b}{\lambda} \\ n' &= \left(\frac{v + b}{\lambda} \right) \end{aligned}$$

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$$\begin{aligned} \text{But} \quad \lambda &= \frac{v}{n} \\ \therefore n' &= \left(\frac{v+b}{v} \right) n \end{aligned} \quad (3)$$

Thus the apparent pitch of the note increases when the observer moves towards the stationary source.

(b) When the observer moves away from a stationary source

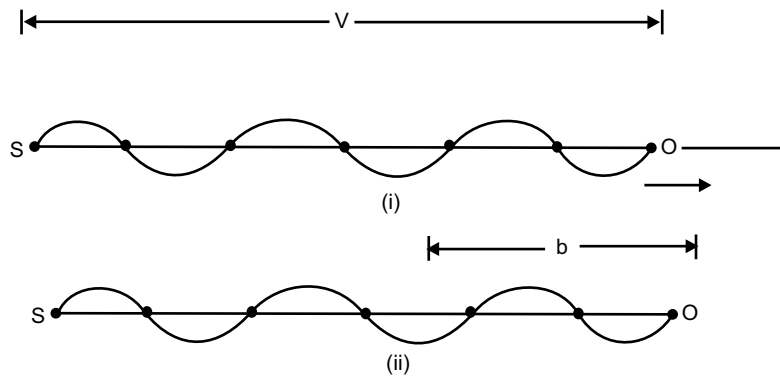


Fig.4

Suppose a source S is producing sound of pitch n and wavelength λ (Fig.4). The velocity of sound is v . Let the observer move with a velocity b away from a stationary source. In this case, the observer receives less number of waves in one second. The apparent wavelength remains the same, The apparent frequency

$$\begin{aligned} n' &= n - \frac{b}{\lambda} \\ n' &= \frac{v}{\lambda} - \frac{b}{\lambda} \\ n' &= \left(\frac{v-b}{\lambda} \right) \end{aligned}$$

$$\begin{aligned} \text{But} \quad \lambda &= \frac{v}{n} \\ \therefore n' &= \left(\frac{v-b}{v} \right) n \end{aligned} \quad (4)$$

Thus the apparent pitch of the note decreases when the observer moves away from a stationary source.

Note: For the same values of v, b and n , the values of n' in equations (3) and (4) will be different.

2.1.3 When both the source and the observer are in motion

When the source moves towards the observer and the observer moves away from the source

Suppose a source S is producing a sound of pitch n and wavelength λ . The velocity of sound is v (Fig.5). The velocity of the source is a and the velocity of the observer is b .

Let the source move towards the observer with a velocity a and the observer move away from the source with a velocity b .

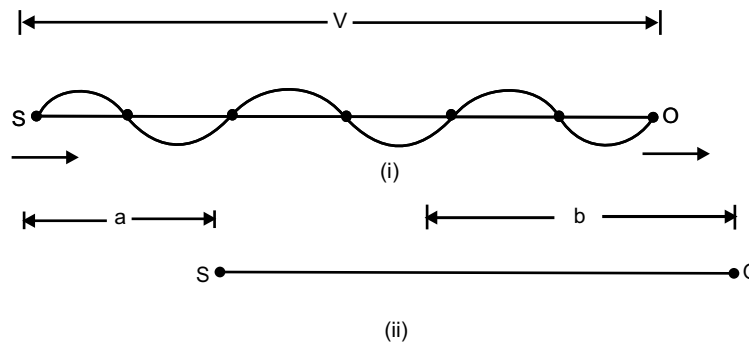


Fig.5

The apparent wavelength

$$\lambda' = \frac{v - a}{n} \tag{5}$$

and
$$n' = \left(\frac{v - b}{\lambda'} \right)$$

$$\therefore n' = \left(\frac{v - b}{v - a} \right) n \tag{6}$$

Special Cases

(a) When the source and observer move towards each other in equation (5), taking b to be negative,

$$n' = \left[\frac{v - (-b)}{v - a} \right] n$$

$$n' = \left(\frac{v + b}{v - a} \right) n \tag{7}$$

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(b) When the source and observer move away from each other

In equation (5), taking a to be negative

$$n' = \left[\frac{v - b}{v - (-a)} \right] n$$
$$n' = \left(\frac{v - b}{v + a} \right) n \quad (8)$$

(c) Source moving away from the observer and the observer moving towards the source

In equation (5), taking both a and b negative,

$$n' = \left[\frac{v - (-b)}{v - (-a)} \right] n$$
$$n' = \left(\frac{v + b}{v + a} \right) n \quad (9)$$

Note: While solving numerical problems, the general formula

$$n' = \left(\frac{v - b}{v - a} \right) n$$

should be applied.

The general relation refers to the case when the source moves towards the observer and the observer moves away from the source. When any of these two directions changes, the signs of a and b have to be changed. The modified formula should be used for calculating the apparent pitch.

2.1.4 Effect of wind velocity

Suppose the wind is moving with a velocity w in the direction of propagation of sound. The apparent velocity of sound will be $(v + w)$. In all relations, in place of v , $(v + w)$ should be used. If the wind is blowing in a direction opposite to the direction of propagation of sound, velocity of sound will be $(v - w)$. In all relations, in place of v , $v - w$ should be used.

The general relation will be

$$n' = \left[\frac{(v + w) - b}{(v + w) - a} \right] n \quad (10)$$

Here the wind direction is the same as the direction of propagation of sound.

When the direction of wind is opposite to the direction of propagation of sound,

$$n' = \left[\frac{(v - w) - b}{(v - w) - a} \right] n \quad (11)$$

2.1.5 Applications of Doppler effect

1. The broadening of spectral lines is due to Doppler effect, due to the fact that the emitting atoms or molecules move in all directions & with varying speeds relative to observing instruments.
2. Using the Doppler shift in frequency, it is found that the sun is rotating from west to east about its own axis.
3. The theory of expanding universe is based on red shift of spectrum, moving away from the earth.
4. When a star is moving toward the earth, it shows a shift of spectral lines towards violet. Double stars were discovered by application of Doppler effect.
5. Doppler effect is employed in radar to determine the velocity of a moving object.
6. It is used to determine the blood flow velocity in the blood vessels.
7. It is also used to monitor the heart beat rate of fetus.

2.2 Mach Number 'M'

It is used to indicate the velocities of supersonic jets or aeroplanes in terms of the velocity of sound in air. Thus Mach number is defined as "Ratio between the velocity of the body moving in a medium and the velocity of sound in the medium". (No dimensions).

$$M = \frac{V}{C} = \frac{\text{Velocity of supersonic plane or jet in air}}{\text{Velocity of sound in air}}$$

- Case(i) If $V < C$ then, $M < 1$ Sub sonic
 Case(ii) If $V = C$ then, $M = 1$ Sonic
 Case(iii) If $V > C$ then, $M > 1$ Supersonic

(e.g: $M = 2$, aeroplane can move with the velocity equal to twice the velocity of sound in air)

2.3 Sonic Boom

If a source of sound (aeroplane) is moving towards a stationary detector at a speed equal to speed of sound.

- (i.e) $V = C$, then Doppler shifted sound frequency becomes infinite.

This means that the source is moving so fast that it keeps pace with its own spherical wavefront. When speed of source exceeds speed of sound, the wavefronts originated at various points of the moving source bunch along the surface of Mach cone. "Mach cone is the cone with the vertex at source point 'S' & is formed by the tangents drawn to the different circles which represent the propagated spherical wavefronts lagging behind the source of sound". A shock wave is said to exist along the surface

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of Mach cone because the bunching of wavefront cause an abrupt rise & fall of air pressure as the surface passes through any point. The half angle ' θ ' is called Mach angle.

$$\sin \theta = \frac{1}{M} = \frac{C}{V}$$

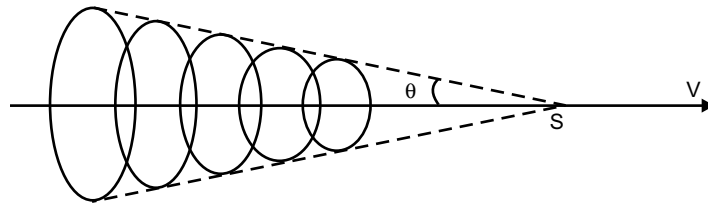


Fig. 6

C – Velocity of sound in air

V – Velocity of aeroplane or jet.

The shock wave generated by a supersonic aircraft produces a burst of sound called sonic boom.

Comparison of Reynold number and Mach number

S.No	Reynold number	Mach number
1.	It is used to indicate the fluid flow.	It is used to indicate velocity of supersonic aeroplanes in air
2.	$R = \frac{\text{Inertia force of fluid}}{\text{Viscous force on the fluid}}$	$M = \frac{\text{Velocity of body in air}}{\text{Velocity of sound in air}}$
3.	$R < 2000$ for stream line flow of water for a pipe of diameter 2 cm.	$M = 1$; Velocity of aeroplane = velocities of sound in air

2.4 Acoustics

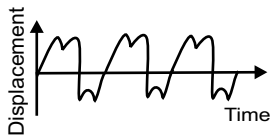

2.4.1 Introduction

Acoustics is the science of sound. Acoustics is a branch of physics which deals with the properties of sound waves, laws of their excitation, propagation and action of sound waves at an obstacle. Originally, acoustics was limited to stimulation of human ear by the incident sound wave. Modern acoustics deals with all sorts of sound which have no relation to human ear (e.g seismological disturbances). A new branch, namely "Architectural acoustics" deals with the design of a good auditorium and buildings with less noise & clear sound, to avoid confusion to the listener.

Basic acoustics may be divided into three categories namely, production, transmission and detection of sound. Sound is a mechanical wave (Elastic waves). It propagates through a medium by means of wave motion. They are mainly transmitted by vibrations of the constituent particles of a medium and hence they cannot travel in free space. Since they propagate as a wave, characterised by various properties such as wave length, frequency, velocity etc, sound waves can be classified based on their frequencies.

Classification	frequency ν	Characteristics
(1) Infrasonics	$\nu < 20\text{Hz}$	Inaudible to human ear
(2) Ultrasonics	$\nu > 20\text{kHz}$ (or) $20,000\text{Hz}$	Inaudible to human ear
(3) audible sound	$20\text{Hz} < \nu < 20\text{kHz}$	audibility range of human ear.

Audible sound are further classified as Musical sound & Noise.

Musical Sound	Noise
<p>They produce pleasing effect to the ear.</p> <p>Musical sounds are produced when a series of similar impulses follow each other regularly at equal intervals of time</p> <p>Absence of any sudden changes in their intensities</p> <p>Wave form</p> 	<p>They producing a jarring and unpleasant effect to the ear.</p> <p>Noises are the sounds of complex nature having irregular periods and amplitudes.</p> <p>undergo sudden changes in their intensities</p> 

2.5 Characteristics of Musical Sound

1. Pitch (or) frequency
2. Quality (or) Timbre
3. Intensity (or) Loudness.

2.5.1 Pitch or Frequency

- (i) Pitch is that characteristic of sound which distinguishes between a shrill sound and a grave sound.

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- (ii) It is the sensation conveyed to human brain.
- (iii) Pitch of a musical sound is determined by its frequency but it is also a function of intensity and waveform.
- (iv) Greater is the frequency of a musical note, higher is its pitch and vice versa.
- (v) Frequency is a physical quantity, that can be measured. Whereas pitch is a physiological quantity, that depends on the mental sensation experienced by the observer.
- (vi) The change in pitch with loudness is most pronounced at frequencies of about 100 Hz.
- (vii) In the frequency range 20 Hz to 10,000 Hz the pitch varies in a parabolic manner with frequency.
- (viii) Frequency of a sound is defined as the number of vibrations per second.

2.5.2 Loudness

- (i) Loudness of the sound is defined as the degree of sensation produced on the human ear.
- (ii) Loudness depends upon the listener.
- (iii) It is not a purely physical quantity but it is subjective in nature.
- (iv) Loudness signifies how far and to what extent, the sound is audible.
- (v) Loudness (L) is related to intensity of a sound (I) as

$$L \propto \log I$$

(or) $L = k \log I$

This is known as **Weber Feschner's law**. Here k is a constant that depends on the sensitivity of the ear, quality of sound and other factors.

- (vi) Loudness varies with frequency also.

2.5.3 Intensity

Intensity of a sound is a physical quantity and does not depend on the listener.

Intensity of sound is defined as the amount of sound energy flowing across unit area held normally to the direction of propagation of the sound waves per second.

$$\text{unit} = \text{watt/m}^2$$

(a) Factors affecting intensity or loudness of sound

Intensity of sound waves is given by

$$I = 2\pi^2\nu^2 a^2 v\rho$$

where

- ν - frequency of sound wave
- a - amplitude of sound wave
- v - velocity of sound and
- ρ - density of the medium.

From the above equation the following points are noted.

- (i) Intensity and hence loudness are directly proportional to square of the amplitude.
- (ii) Intensity of sound is directly proportional to the density of medium.
- (iii) Intensity and loudness vary with frequency of sound.
- (iv) With increase in the size of a body, larger quantity of air is set into vibration. So larger the size of the body, greater is the intensity and louder is the sound produced.
- (v) Loudness decreases with increase of distance from the source of sound. Thus loudness varies inversely with the square of the distance from the source.
- (vi) Loudness is greater in the direction of wind motion than in the opposite direction.

(b) Measurement of intensity

Intensity of a sound wave is measured in watt/m². Just as there is a minimum value of frequency below which human ear cannot hear the sound, a minimum intensity of sound exists below which human ear cannot detect the sound. This lowest intensity of sound to which a normal human ear can respond is 10⁻¹² watt/m². This value is chosen as standard intensity. Intensity of any sound is measured with reference to this standard intensity only. Hence it is only the relative intensity that is measured, which indicates how much the intensity of sound is higher than the standard intensity.

The ratio of the intensity of sound wave to the standard intensity is known as intensity level (or relative intensity of sound).

$$\text{Intensity level} = \frac{\text{Intensity } (I)}{\text{Standard intensity } (I_0)}$$

Relation between loudness (L) and intensity (I) is

$$L = k \log I$$

Let L_1 be the loudness corresponding to intensity I and L_0 , the standard loudness corresponding to standard intensity (I_0).

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Then,

$$L_1 = k \log I \quad \text{and} \\ L_0 = k \log I_0$$

Now difference in the loudness is

$$L = L_1 - L_0 = k \log I - k \log I_0 \\ = k \log \frac{I}{I_0}$$

where L now is called intensity level or loudness level and the above relation is true for all frequencies.

The unit of intensity level is Bel.

We know $L = k \log \frac{I}{I_0}$

If $k = 1$ and $I/I_0 = 10$ then

$$L = 1 \times \log_{10}(10) = 1 \text{ bel}$$

In practice bel is a larger unit and hence decibel ($\frac{1}{10}$ of a bel) is used. (i.e.,)

If $k = 10$ and $I/I_0 = 10$ then

$$L = 10 \log_{10}(10) = 10 \text{ decibel (dB)} \\ \text{(i.e.,)} \quad 1 \text{ bel} = 10 \text{ decibel} = 10 \text{ dB}$$

(c) Significance of intensity level

Let a sound have an intensity of say $I = 10^{-5}$ watt/m².

$$\text{Intensity level} = \frac{I}{I_0} = \frac{10^{-5}}{10^{-12}} = 10^7$$

This means that the sound intensity is 10^7 times more than the standard intensity. **1 dB change in intensity level of sound corresponds to an increase of 26% of the intensity of sound.**

Proof: Consider two sounds of intensity I_1 and I_2 . Expressing them with reference to the standard intensity,

Intensity level $dB_1 = 10 \log \frac{I_1}{I_0}$
 $dB_2 = 10 \log \frac{I_2}{I_0}$
Hence $dB_2 - dB_1 = 10 \left(\log \frac{I_2}{I_0} - \log \frac{I_1}{I_0} \right)$
(or) $1 = 10 \log \frac{I_2}{I_1}$
(or) $\log \frac{I_2}{I_1} = \frac{1}{10} = 0.1$
(or) $\frac{I_2}{I_1} = 10^{0.1} = 1.26$

(or) change in intensity level

$$\frac{I_2 - I_1}{I_1} = 0.26 \text{ (or) } 26\%$$

(i.e) An increase of one decibel in intensity level of sound corresponds to an increase of 26% of the source intensity

Note: Whenever intensity of sound is expressed in dB, it represents intensity level of sound only.

Intensity level of some sound signals

Sound Signal	dB	Sound Signal	dB
General noise level	10	Busy Traffic	70-80
Whispering	10-20	Runining Train	100
Passing Motor Car	30	Passing aeroplane	100
Soft Music	40	Thunder	100-110
Average Conversation	60-70	Painful sound	7120

Difference between loudness and intensity

S.No	Loudness	Intensity
1.	It is the amount of sensation produced in the ear & hence it depends upon the listener.	It is quantity of sound energy flowing across unit area / sec & hence it depends on source of sound.
2.	It is not a purely physical quantity but is subjective in nature.	It is pure physical quantity
3.	Measured in sones.	Measured in W/m^2
4.	It is difficult to measure.	It can be easily & accurately measured.

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We know that an intensity of 10^{-12} watt/m² is the minimum intensity that a human ear can detect. Similarly there exist an upper limit of hearing corresponding to 120 dB (as indicated in the above table). This upper limit is called threshold of feeling or pain threshold. The lower limit of hearing (10^{-12} watt / m²) is called threshold of audibility which is also zero level intensity corresponding to sound level of 0 dB.

(d) Phon

For measurement of sound in decibels, it was assumed that zero intensity level I_0 is same for sound of all frequencies. But actually, sound of same intensities may have different frequencies and also in loudness. Hence a standard frequency of 1000 Hz is chosen for comparing sounds. This gives another unit called phon for measuring intensity level.

The loudness level or equivalent loudness of a sound is said to be 1 phon, if the intensity level of standard tone with equal loudness is 1 decibel above the standard intensity.

(i.e.,) For a standard frequency of 1000 Hz

$$\text{Loudness (in phon)} = L_p = 10 \log \frac{I}{I_0}$$

(e) Sone

The unit of loudness is a sone which is defined as being the loudness of a 1000 Hz tone of 40 dB intensity level. (or 40 phons of intensity level).

Empirically,

$$\log L(\text{sone}) = 0.033(L_p - 40)$$

where L is loudness in sone.

L_p is loudness level in phon.

We know

$$L_p = 10 \log \frac{I}{I_0}$$

$$I_0 = 10^{-12} \text{ watt/m}^2$$

$$\begin{aligned} L_p &= 10 \log \frac{I}{10^{-12}} \\ &= 10 \log I - 10 \log 10^{-12} \\ &= 10 \log I + 120 \end{aligned}$$

$$\begin{aligned} \therefore \log L &= 0.033(10 \log I + 120 - 40) \\ &= 0.033(10 \log I + 80) \\ &= 0.33 \log I + 2.64 \end{aligned}$$

$$\text{(or)} \quad \log \frac{L}{I^{0.33}} = 2.64$$

$$\text{(or)} \quad \frac{L}{I^{0.33}} = e^{2.64}$$

$$\text{(or)} \quad L = \text{const } I^{0.33}$$

$$\text{(or)} \quad L \propto \sqrt[3]{I}$$

(i.e.,) Loudness level in sone is proportional to cube root of intensity of sound at a frequency of 1000 Hz.

(f) Sound pressure level

The intensity of sound is also measured in terms of sound pressure level. As discussed earlier, since sound is a form of energy, and the rate of flow of energy is power, one can associate this concept in measuring the intensity of sound. But generally the power associated with a sound is extremely small. Hence sound measuring devices actually respond to pressure exerted by sound.

$$\text{Sound pressure level (SPL)} = 20 \log \left(\frac{P}{P_0} \right) \text{ dB}$$

P_0 = standard effective pressure

$$= 2 \times 10^{-5} \text{ N/m}^2$$

Sound level meters are used to measure the intensity level of sounds and loudness. It consist of a high sensitive microphone of good stability, a linear amplifier with uniform frequency response, a set of frequency weighting networks and an indicating meter. The frequency weighting network is to make the reading of sound level meter correspond as closely as possible to observed loudness levels. The microphone converts sound energy into electrical energy (voltage), which is then amplified and passed through frequency weighting network and then calculated using indicating meter, which reads the sound level in dB above the standard reference intensity.

2.5.4 Quality or Timber

The third characteristics of a musical note is quality or Timbre. The note emitted by musical instruments are never simple, but are composed of several frequencies. Normally they consist of a large number of tones of different frequencies of varying intensity. The tone of the smallest frequency is the loudest and is called the fundamental, which is dominant and defines the pitch of a note. The frequencies accompanying the fundamental are called overtones. The intensities of the overtones diminishes with increase of their frequencies. Each musical instrument produces different overtones and they are generally characteristics of those instruments. Even if two different musical instruments produce notes of same pitch, the overtones accompanying will be different. This differences defines the quality of the note emitted by one instrument and distinguishes the note produced by that instrument from others.

Thus, quality of the note is marked by the nature of the composition of the frequencies of the note. It depends upon the presence or absence of particular overtones and their relative intensities.

2.6 Acoustic of Buildings

2.6.1 Reverberation

The persistence of sound in a room, hall auditorium etc due to successive reflections from the walls, ceilings, flooring & materials present in them, even after the source ceases to produce sound is called reverberation.

2.6.2 Reverberation time

The time interval required by the reverberant sound to fall to one millionth of its original intensity is called reverberation time.

$$I = I_0 \times 10^{-6} \quad \frac{I}{I_0} = 10^{-6}$$

2.6.3 Absorbtion of sounds

Amount of absorbtion of sound depends upon the nature of the materials and their surface area.

Absorbtion coefficient (a)

Absorbtion coefficient = $\frac{\text{Sound energy absorbed}}{\text{Sound energy absorbed by an equal area of an open window}}$

An open window is a perfect absorber. Absorbtion coefficient of open window is taken as unity and is called sabine. Absorbtion coefficient of a surface is also defined as the reciprocal of its area which absorbs the same amount of sound energy as absorbed by a unit area of an open window.

Absorbing power is product of surface area of absorbtion and its absorbtion coefficient

$$\boxed{A = aS} \quad \text{unit : } m^2 - \text{Sabine}$$

For more than one surface, $A = a_1S_1 + a_2S_2 + a_3S_3 + \dots = \Sigma aS$

2.6.4 Principles to be observed in acoustically designing an auditorium

Acoustically designing an auditorium refers to the design of the auditorium with necessary following acoustical parameter determining factors:

1. Shape
2. Volume
3. Sound absorption
4. Sound distribution

The prime factor in designing the auditorium is the selection. The size should be selected well off from industries, airports, railway stations, etc. to avoid external noise. In such place the auditorium can be air conditioned to avoid external noises under unavoidable circumstances.

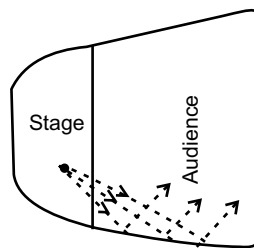
1. Shape

Shape of auditorium is one of the prime factor in yielding good acoustics for the auditorium. A proper shape can effectively distribute sound throughout the auditorium.

Some of the preferred shape are:

1. Fan shape
2. Rectangular shape
3. Horse shoe shape
4. Oval or circular shapes

For illustrations a fan shaped auditorium outline is shown below in the fig..



The above diagram explain even **distribution** of sound through out the auditorium.

2. Volume

Volume of the auditorium can be decided in such a way that the intensity of sound throughout the auditorium is of optimum level. The volume of hall or auditorium is directly proportional to the intensity of sound produced. For cinema theatre or *concert* hall, the volume is maximum compare to lecture hall or class rooms where a minimum volume serves better for better intensity and distribution of sound.

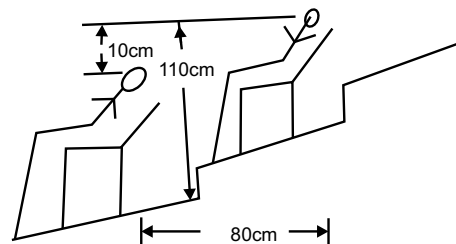
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In considering the volume of auditorium, its height is a prime deciding factor for uniform sound and intensity distributions.

The normal ratio of ceiling height to breadth is 2:3 for better acoustics and the ratio between volume and number of seats must not be more than 5.7. As the audience also contribute for absorption of sound, volume per seat may be allocated in 3.74 to 4.2 m³ range.

3. Sound absorption

In an auditorium the sitting audience contribute for considerable absorption of sound. Therefore proper arrangement of seats and its number can considerably alter the reverberation time which decides better acoustics. A proper and better seating arrangement is shown below.



In the above figure the sound can not be blocked or absorbed by the audience as their heads are not in line.

4. Sound distribution

The shape and volume decide the sound distribution. In addition to these parameters, intensity of sound also plays a vital role. The implementation of digital theatre sound system, stereo loud speakers etc, in different places of auditorium or theatre attribute for better sound distribution.

Further points to keep the reverberation time in desired level are:

1. Sound absorbing curtains can be used at the back side wall of theatre.
2. Floor carpet can be used for better absorption.
3. Audience seat can be fixed with good sound absorbing materials.

2.7 Sabine's Formula

Sabine found experimentally that the time of reverberation of a hall is directly proportional to its volume and is inversely proportional to its absorbing power.

$$T = \frac{0.163V}{A}$$

Derivation

Let a_1, a_2, a_3 etc be the absorption coefficients of the surfaces S_1, S_2, S_3 etc of the given hall.

Average absorption coefficient

$$\bar{a} = \frac{a_1 S_1 + a_2 S_2 + a_3 S_3 + \dots}{S_1 + S_2 + S_3} \quad (12)$$

$$= \frac{\sum aS}{S}$$

$$\Rightarrow \bar{a}S = \sum aS \quad (13)$$

Let S be total internal surface area of hall.

By statistical method, Jager showed that, the sound travels an average distance of $\frac{4V}{S}$ between two successive reflections, where V is Volume and S is total surface area of the hall.

If ' v ' is the velocity of sound, then

$$\text{Time taken between two successive reflections} = \frac{4V}{Sv} \quad \left[t = \frac{\text{distance}}{\text{velocity}} \right]$$

Average number of reflections in time ' t '

$$n = \frac{Svt}{4V} \quad (14)$$

Sound absorbed at a single reflection = \bar{a}

Sound energy reflected for a single reflection = $(1 - \bar{a})$

Sound energy reflected after two reflections = $(1 - \bar{a})(1 - \bar{a})$
 $= (1 - \bar{a})^2$

(From Fig.7)

similarly for ' n ' reflections = $(1 - \bar{a})^n$

Let I_0 be initial intensity of sound and I_t be intensity after time ' t '.

(i.e) after $\frac{Svt}{4V}$ reflections

$$I_t = I_0(1 - \bar{a})^n \quad (15)$$

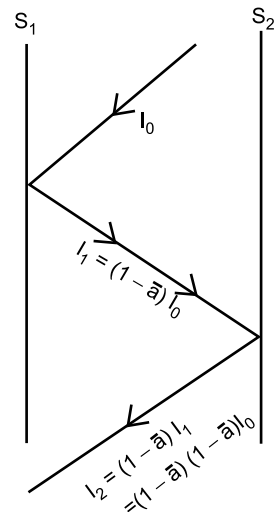


Fig.7

Substituting for n from Eq.(13), Eq.(14) becomes

$$I_t = I_0(1 - \bar{a})^{Svt/4V} \quad (16)$$

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According to the definition of reverberation time,

When $t = T$, reverberation time.

$$\frac{I_t}{I_0} = 10^{-6}$$

Substituting from Eq.(15)

$$10^{-6} = (1 - \bar{a})^{SvT/4V}$$

Taking log on both sides

$$\begin{aligned} \log 10^{-6} &= \frac{SvT}{4V} \log_e(1 - \bar{a}) \\ \text{(or)} \quad T &= \log_e 10^{-6} \frac{4V}{Sv \log_e(1 - \bar{a})} \\ &= 2.303 \times \log_e 10^{-6} \times \frac{4V}{Sv \log_e(1 - \bar{a})} \\ \text{but, } v &= 330 \text{ m/s (Velocity of sound)} \\ &= \frac{2.303 \times (-6)}{330} \times \frac{4V}{S \log_e(1 - \bar{a})} \\ &= \frac{-0.1625V}{S \log_e(1 - \bar{a})} \end{aligned} \tag{17}$$

Eq.(16) is known as Eyring's formula. Approximation of this formula leads to Sabine's formula.

$\therefore \bar{a} < 1$, $\log_e(1 - \bar{a})$ can be expanded in logarithmic series

$$\log_e(1 - \bar{a}) = \left(-\bar{a} - \frac{\bar{a}^2}{2} - \frac{\bar{a}^3}{3} - \dots \right)$$

Neglecting higher power of \bar{a}

$$T = \frac{-0.1625V}{S(-\bar{a})} = \frac{+0.1625V}{S\bar{a}}$$

From Eq.(12) $\bar{a}S = \Sigma aS$

$$\boxed{T = \frac{0.1625V}{A}}$$

$$T = \frac{0.162V}{\Sigma aS}$$

2.7.1 Sabine's Formula (Alternate methods)

The relation between

- Reverberation time (t)
- Volume of the hall (V)
- Area of the surface and its absorption coefficient

is given by Sabine's formula.

The standard reverberation time is defined as the time taken by the sound wave to fall in its intensity to one million of its original value.

Absorption coefficient for the material of the surface is defined as the ratio of the sound energy absorbed by the surface to the sound energy absorbed by perfect absorber such as an open window. It is denoted by α . The unit of absorption coefficient is Sabine.

Sabine is the sound energy absorbed by one square feet of an open window.

Let I be the average intensity of the sound at time 't' and let dI be the fall in intensity during the time dt .

$$dI = -\alpha n I dt \tag{18}$$

where

n - no. of reflections of the sound wave.

α - absorption coefficient of the absorbing surface.

(-ve sign indicates that as the time increases the intensity of the sound decreases.)

By statistical method it is shown that the sound wave travels an average distance $\frac{4V}{S}$ between successive reflections.

Where

V → Volume of the hall

S → Total area of the reflecting surface

Let C be the velocity of sound wave.

Time taken for two successive reflection is

$$t = \frac{\text{Distance travelled between 2 successive reflections}}{\text{Velocity of sound wave}}$$

$$t = \frac{\frac{4V}{S}}{C} = \frac{4V}{SC}$$

$$n = \frac{1}{t} = \frac{SC}{4V} \tag{19}$$

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Sub (2) in (1)

$$dI = -\alpha \frac{SC}{4V} I dt$$

$$\frac{dI}{I} = -\alpha \frac{SC}{4V} dt$$

For identifying the limits of integration

Let I_o - Steady intensity of sound wave when source is cutoff.

I_t - Intensity of sound wave after t sec

$$\int_{I_o}^{I_t} \frac{dI}{I} = -\alpha \frac{SC}{4V} \int_0^t dt$$

$$\log [I]_{I_o}^{I_t} = -\alpha \frac{SC}{4V} [t]_0^t$$

$$\log I_t - \log I_o = -\alpha \frac{SC}{4V} [t - 0]$$

$$\log \frac{I_t}{I_o} = -\alpha \frac{SCt}{4V}$$

$$\frac{I_t}{I_o} = e^{-\alpha \frac{SCt}{4V}} \quad (20)$$

From the definition of standard reverberation time.

$$\frac{I_t}{I_o} = 10^{-6} \quad (21)$$

Comparing (3) & (4)

$$e^{-\alpha \frac{SCt}{4V}} = 10^{-6}$$

$$-\frac{\alpha SCt}{4V} = \log e^{(10^{-6})}$$

Converting to logarithm to the base 10,

$$\frac{\alpha SCt}{4V} = 6 \times 2.303 \times \log_{10} 10$$

$$t = \frac{6 \times 2.303 \times \log_{10} 10 \times 4 \times V}{\alpha \times S \times C}$$

$$C \simeq 330 \text{ m/s}$$

$$t \simeq \frac{0.16 V}{\alpha S} \quad (22)$$

2.8 Measurement of Absorption Coefficient

Determination of Absorption Coefficient:

The reverberation time in hall is determined by

- Without absorbing material in hall (t_1)
- With absorbing material in hall (t_2)

$$t_1 = \frac{0.16 V}{\alpha S}$$

$$t_2 = \frac{0.16 V}{\alpha S + \alpha_a S_a}$$

(α_a - absorption coefficient of the material of the area S_a).

$$\begin{aligned} \frac{1}{t_2} - \frac{1}{t_1} &= \frac{\alpha S + \alpha_a S_a}{0.16V} - \frac{\alpha S}{0.16V} \\ &= \frac{\alpha_a S_a}{0.16V} \\ \alpha_a &= \frac{0.16V \left[\frac{1}{t_2} - \frac{1}{t_1} \right]}{S_a} \end{aligned} \quad (23)$$

2.9 Factors Affecting Acoustics of Buildings

(a) Reverberation

As we know, the persistence of sound in the hall even after the source is cut off is called reverberation. This is due to successive reflections taking place on the walls of the hall. Because of this, the previous note of sound is heard for short interval of time, even though the second note of sound is emitted. Too much of reverberation may cause booming sound while too low reverberation causes flat sound. Therefore, the reverberation must be at optimum level for continuous hearing of sound. This can be obtained by using sound absorbing materials in walls, ceilings, floor etc.

(b) Reflections and Echoes

The walls of the hall should scatter the sound and not reflect it. To avoid reflections, the walls should be rough. When sound is reflected from walls and other surfaces, they produce echoes which causes a nuisance effect and a change in the original sound.

An echo is heard when the direct and reflected sound waves coming from the same source reach the listener with a time interval of about 1/7 second. The reflected sound reaching earlier than this helps in raising the loudness (Reverberation) while those arriving later, produces echoes and cause confusion. These echo effects can

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be avoided by using sound absorbing materials and by providing more number of doors and windows.

Echelon effect: Sometimes a separate musical note with regular phase difference is produced due to combination of echoes. (e.g) If there is a stair case, then reflections from equally spaced steps at regular intervals of time, produces a separate sound, which may interfere with original sound and cause confusion.

Thus a set of railings or any regular spacing of reflecting surfaces may produce a separate musical note due to regular succession of echoes of the original sound to the listener. This effect is called echelon effect.

This can be avoided if any such regular spacing available, is covered with sound absorbing materials.

(c) Focussing and interference effects

If there is any concave surface present in the hall, then sound is concentrated at its focus region and hence dead space at some other region is created. Hence such surfaces may be avoided. If present, they can be covered with sound absorbing materials. Similarly if there is interference of direct and reflected waves, interference patterns create maximum intensities at some places and minimum intensities at some other places in the hall. All these can be avoided by using good sound absorbing material.

(d) Resonance effect

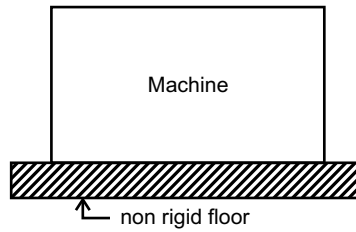
Hollows, crevices, window panes etc select their natural frequencies from the sound produced in the hall and reinforce them there by producing a resonance effect, which can disturb the original sound. Hence they may produce a jarring effect. These can be reduced by convex cylindrical segments on the walls and ceilings, which diverge the reflected sound and provide uniform distribution.

(e) Noises from exterior

Any noise outside the hall produces a distributing effect inside. To avoid these, the opening like windows etc, can be covered using screens etc.

2.10 Noise Control in a Machine

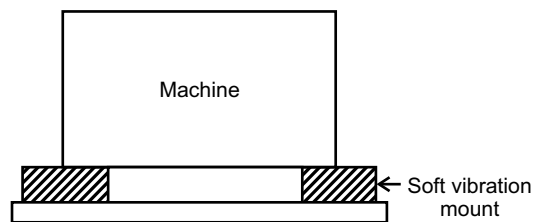
In factories the noise produced by the machineries are very high. This produces harmful effects to the employees. This noise can be controlled by modifying the design of the machine. For example instead of toothed cutting an abrasive cutting action can be used for noise control. The noise can also be reduced by properly insulating the machine with sound proof materials. The following are the steps involved in controlling the noise in a machine.



The machine is mounted on non rigid floor which produces a noise level of 85 dB.

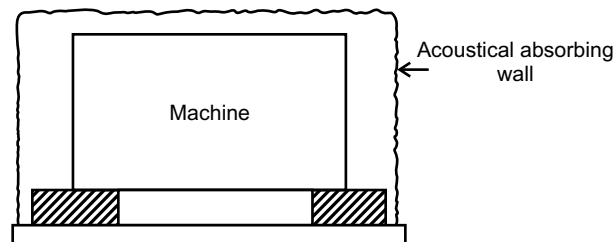
Step 1

The machine is first mounted with soft vibration mount which decreases the noise at lower frequencies.



Step 2

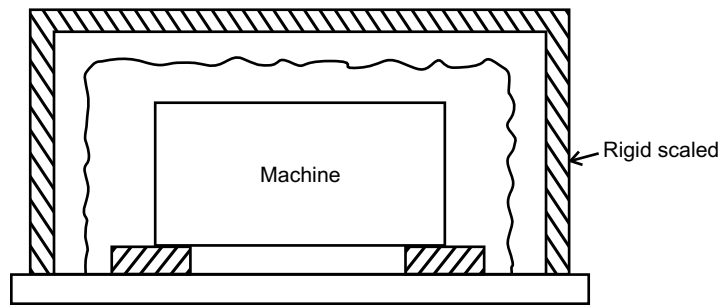
A porous acoustical blanket is covered over the machine. This blanket controls the reverberation time and also reduces the amplitude of high frequency components.



Step 3

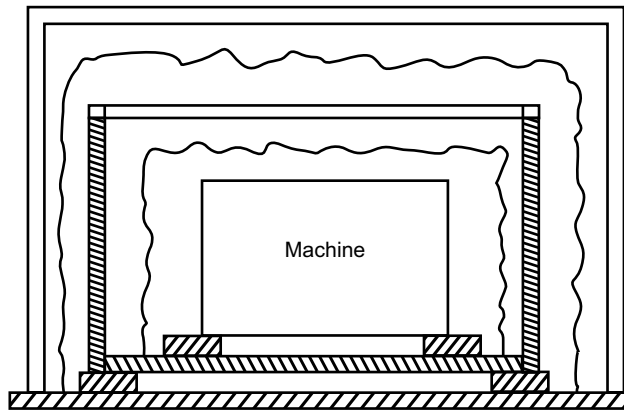
Now the machine is kept in an air tight sealed enclosure which reduces a large amount of higher and lower frequency noise.

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Step 4

Finally the machine resting on the vibration mount enclosed by the air tight box enclosure is further enclosed by an air tight box so that the noise is controlled and can be reduced to a large extent. This method is called noise reduction by double enclosure with double vibration mounts. Thus the noise level is very much reduced.



2.10.1 Noise Pollution

We know that noise can be classified into air borne, structure borne and internal noises. All these three noises pollute the area at which it has been produced and create harmful effects to human beings, especially air borne noise produced in rural areas.

Effects

1. It produces mental fatigue and irritation.
2. It divides the concentration on work and hence reduces the efficiency of work.
3. It sometimes affects the nervous system and lowers the restorative quality of sleep.

4. Some strong noises lead to the damage of the ear drum and make the working people hearing impaired.
5. The noises which are produced regularly will even retard the normal growth of infants and young children.

Noise Reduction

The following methods can be adopted to control the noise and thereby reducing it.

- (i) It is controlled by using special construction to reduce transmission of sound the walls and the partitions (i.e.,) providing double sealed walls.
- (ii) It can be reduced by keeping the sound proof near the sources from which noises are produced.
- (iii) It is further reduced by isolating the source of sound from supporting structures.
We can keep the source of sound inside some special mountings.
- (iv) The noise can also be reduced by properly choosing the site for constructing the building. For example hospitals, colleges, temples, etc. should be constructed away from airport, rail rods and busy streets so that noise can be avoided.
- (v) In some working places the noises are reduced by wearing ear muffs and ear plugs. Also in many industries the employees are conducted hearing test periodically to ensure the hearing capacity.
- (vi) Finally the noise can be controlled by decreasing the directivity index in the direction in which the listener is located.

2.11 Ultrasonics

Introduction

A very important type of sound waves is the Ultrasonic wave. They are sound waves characterised by very high frequency greater than 20,000 Hz. They have properties that are common to all sound waves. Ultrasonics find a greater range of application in the field of Engineering and Medicine.

Apart from sound waves that can be heard by human ear (20 Hz - 20 kHz) there are sound waves of frequencies less than 20Hz (infrasonics) and greater than 20 kHz (ultrasonics).

Ultrasonic waves are called high frequency waves and are inaudible to human ear.

Ultrasonic applications are based on the principle "sound waves are reflected at the boundary of two surfaces that have different acoustical parameter".

Properties

1. They are acoustical waves with frequency > 20 kHz.
2. They are highly energetic.
3. They show negligible diffraction due to their small wavelength. Hence they can be transmitted over long distances without any appreciable loss of energy.
4. They are reflected at boundaries. These reflected ultrasonic waves are called echoes.

2.11.1 Production of Ultrasonic Waves

2.11.1.1 Magnetostriction oscillator

Principle

Magnetostriction effect: When a magnetic field is applied parallel to the length of a ferromagnetic rod of shorter length, a small change in length occurs (ie) elongation or contraction. This phenomenon is known as magnetostriction and the effect is known as magnetostriction effect.

This change in length depends on the intensity of the applied magnetic field and the nature of the ferromagnetic material.

When alternating field is applied the rod suffers change in length for every half cycle of the applied field, which indicates that the rod vibrates (alternate expansion and compression). If the rod is made to vibrate with its natural frequency (ie) if the frequency of the alternating current coincides with the frequency of the rod, then Resonance occurs. At resonance, the amplitude of vibration increases enormously. The natural frequency of vibration of the rod is given by

$$f = \frac{n}{2l} \sqrt{\frac{Y}{\rho}}$$

where Y – Young's modulus of the material of the rod

ρ – density of the rod

l – length of the rod

n – integer (1,2,3.....)

Construction

It consists of a tuned oscillator constructed with a NPN transistor which is used to generate the alternating field. Tank circuit is connected to the collector. Another coil L_1 is connected to and is kept close to L_2 . The ferromagnetic rod which is clamped in the middle is surrounded by L_1 & L_2 .

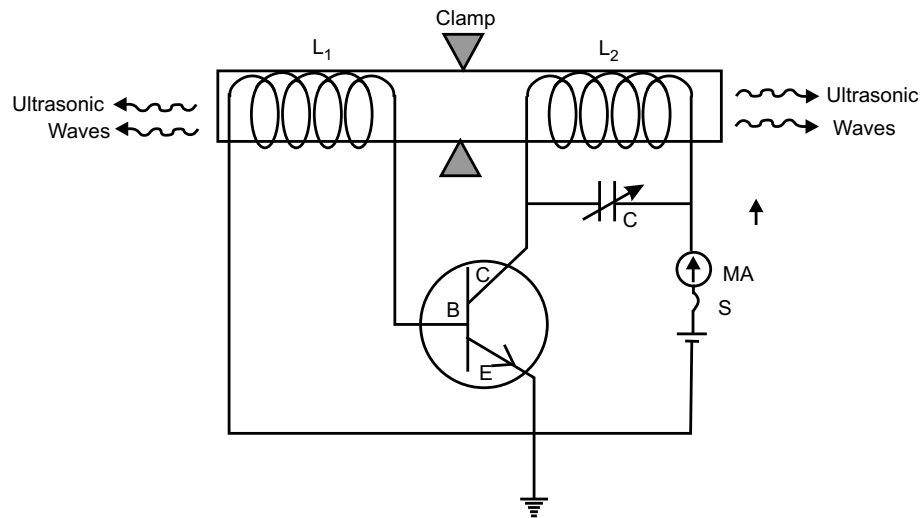


Fig. 8

Working

When the supply is switched on, collector current starts rising and oscillations start in L-C circuit (Tank circuit). To maintain sustained oscillations, changes of current in L are fed back to the Base, through mutual inductance between L_2 and L_1 . The frequency of oscillation of LC circuit is given by

$$f = \frac{1}{2\pi\sqrt{L_2C}}$$

By varying ' C ' this frequency can be adjusted to be equal to the natural frequency of vibration of the ferromagnetic rod. When the frequency of vibration of the rod equals the frequency of the ac circuit, resonance occurs and the rod is thrown into vigorous vibration, thus emitting ultrasonic waves.

Since the tank circuit is connected to the collector of the transistor, the circuit is also known as collector tuned circuit.

Advantages

- Magnetostrictive materials are easily available & inexpensive.
- Oscillatory circuit is simple to construct.
- Large output power can be generated.

Disadvantages

- Generation of frequencies beyond 300kHz is not possible.
- Single frequency oscillation is not possible, since elastic constant varies with magnetisation.

2.11.1.2 Piezo electric generator

Principle

Piezo electric effect

This effect refers to the development of an emf across one pair of faces of certain crystals like quartz, if mechanical force is applied perpendicular to other pair of faces.

When a pair of opposite faces of certain crystal such as quartz is subjected to pressure, electric charges are developed in the other pair of opposite faces. When subjected to tension instead of pressure, the sign reverses.

The converse is also true (i.e)

Inverse Piezo electric effect

When high frequency alternating current is applied to the opposite faces, certain crystals like quartz (cut with their faces perpendicular to its optic axis), expands and contracts periodically.

When the frequency of vibration of the rod and alternating current frequency are made equal, resonance occurs and the crystal vibrates vigorously, emitting ultrasonic waves.

Optic axis, mechanical axis and preparation of the crystal

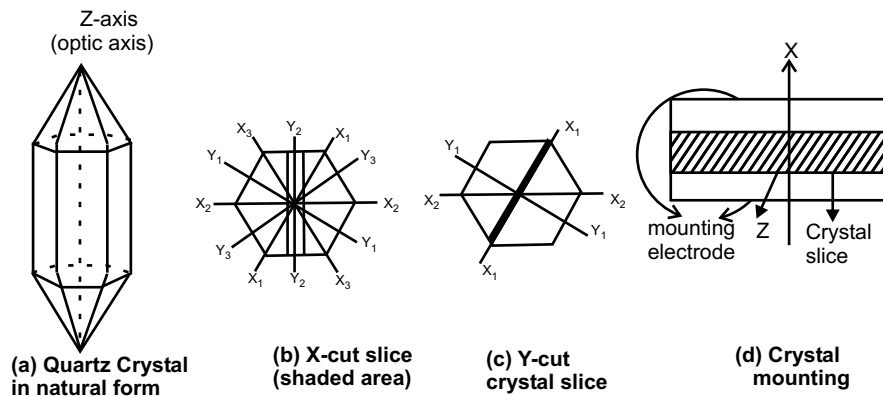


Fig. 9

Natural crystal has hexagonal prism shape with a pyramid attached at the end (Fig.9). The line joining the end points of these pyramids is called the optic axis or Z axis. The three lines which pass through the opposite corners of the crystal constitute its electrical axes or X axes and the three lines perpendicular to the sides of the hexagon forms the mechanical or Y axes.

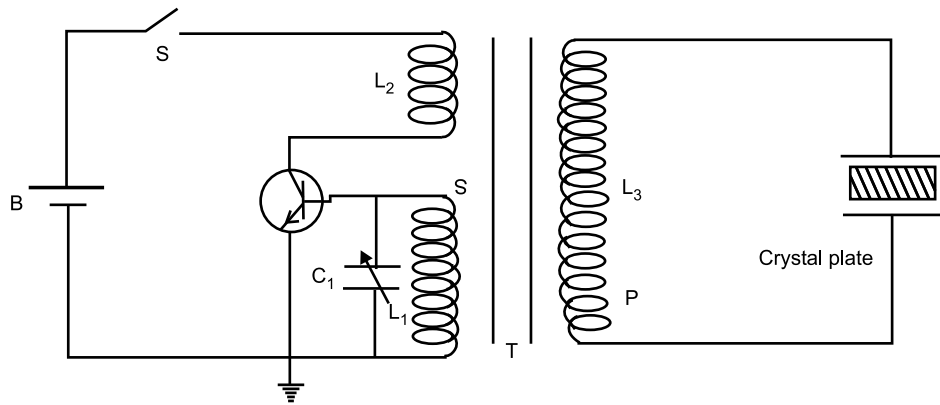


Fig. 10 Piezoelectric ultrasonic generator

Thin plates cut perpendicular to X axes are called X cut plates, which can generate longitudinal mode of ultrasonic vibrations. Plates cut perpendicular to Y axes are known as Y cut plates, which can generate transverse ultrasonic waves.

Consider a plate of thickness t and length ' l '. Its thickness is parallel to X axis, the length is parallel to Y axis and breadth is parallel to Z axis. When an alternating voltage is applied along electrical axis, then alternating compression and expansion are set up both in its thickness and length.

Hence frequency of vibration is given by

$$n = \frac{P}{2t} \sqrt{\frac{Y}{\rho}} \quad \text{and} \quad n = \frac{P}{2l} \sqrt{\frac{Y}{\rho}}$$

where

Y – Young's modulus

ρ – density of the material of the plate

Both these forms of oscillations are used for generation of ultrasonic waves.

Construction

The crystal plate is sandwiched between two metal sheets forming a parallel plate capacitor, which is coupled to an electronic oscillator through primary coil L_3 of the transformer T .

Coils L_2 and L_1 of oscillator circuit are taken from the secondary of a transformer T . Coils L_2 and L_1 are inductively coupled. The coil L_1 and variable capacitor C form the tank circuit. Since the tank circuit is connected to the base of the transistor, the circuit is called base tuned circuit.

Working

When the battery is switched on, the oscillator produces high frequency oscillation, which can be varied using the variable capacitor. By transformer action, emf is induced in L_3 which is oscillating in nature. Now the crystal is subjected to high frequency alternating voltage. The value of C_1 is varied until the frequency of oscillation matches with the natural frequency of vibration of the crystal (i.e)

$$f = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{P}{2l}\sqrt{\frac{Y}{\rho}}$$

- P - Order of harmonics
 P - 1, 2, 3,
 Y - Y's modulus of the crystal
 ρ - Density of material of the crystal

Now the crystal is thrown into resonance, vibrate vigorously and emits high power ultrasonic waves.

Advantages

- (i) High frequency of about 500 MHz can be generated
- (ii) Stable and constant frequencies can be achieved.
- (iii) Synthetic materials like $LiNbO_3$ etc can be used.
- (iv) It is insensitive to temperature and humidity.

Disadvantages

- (i) Cost of piezo electric quartz plate is very high.
- (ii) Cutting and shaping process is very complicated.

Comparison of Piezo electric and magnetostriction method

S.No	Piezo electric method	Magnetostriction method
1.	Generates very high frequency (500 MHz)	Generates low frequency (3 MHz)
2.	We can obtain constant frequency	Constant frequency cannot be obtained
3.	Peak of resonance curve is narrow	peak of resonance curve is broad.
4.	Frequency of oscillation is independent of temperature	frequency of oscillation depends on temperature.

2.12 Applications of Ultrasonics

(a) Industrial application

(i) Non destructive testing

- Ultrasonic waves are used to detect surface and internal faults, such as cracks, cavities, foreign objects in materials.
- They are also used to assess the continuity of composite materials such as reinforced plastics etc.
- Using ultrasonics, deep seated defects in metals can be located.
- Even minute flaws, their location, nature and size can be determined.

(ii) Ultrasonic Welding(cold welding) Properties of some metals changes on heating and thus they cannot be welded by electric or gas welding. In such cases metals can be welded together at room temperature using ultrasonic waves.

A hammer is made to vibrate ultrasonically. The tip of that hammer is made to press the two metal sheets very rapidly and the molecules of one metal diffuse into other, and thus gets welded together without heating.

- (iii) Ultrasonic Soldering** This type of soldering is used in aluminium foil condensers, aluminium wires and plates without using any fluxes.
- (iv)** Ultrasonics are also useful in drilling holes in hard materials with high impact brittleness such as glass, diamond, gems, ceramics etc. They can be used for cutting or machining also.
- (v)** Ultrasonic cleaners are extensively employed for cleaning a wide range of articles like parts of motors, airplanes, machines and electronic assemblies etc. This sort of acoustic drying process is appreciably faster than infra red drying.
- (vi)** Many metals such as Iron, lead, Aluminium, Cadmium etc, which cannot be alloyed in their liquid state can be brought together when subjected to high intensity ultrasonic waves.
- (vii)** High frequency ultrasonic waves can be used to study surface structure and their homogeneity with higher resolution
- (viii)** Acoustic holograms are used to study the surface structures of various Engineering materials used for space applications.

(b) Communication

- (i)** Due to high frequencies of ultrasonics, and their capability to travel long distance (in narrow beams, with less absorption) through water or air, they are used for direction signalling in submarines.
- (ii)** For depth sounding, echo sounding technique is used. This system is known as SONAR (sound navigation and ranging) which is similar to RADAR in air, which is described as follows.

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A beam of ultrasonic waves is directed towards the bottom of the sea from where it is reflected back to a suitable receiver. The time taken by the waves to go and come back is recorded. Knowing the velocity of ultrasonic waves in water, depth of sea is calculated.

(c) Physical applications

- (i) Adiabatic compressibility of liquids can be studied using ultrasonics
- (ii) They are used to determine the grain size of an alloy or metal
- (iii) Elastic constants (Y , n and K) and poisson's ratio can be determined using longitudinal and transverse ultrasonic wave velocities in solids.

(d) Chemical applications

- (i) They are used for forming stable emulsions of immiscible liquids like water and oil.
- (ii) They are used to remove gas or air bubble in liquid metals when they are converted into fused metals.
- (iii) They are used to improve the homogeneity and stability of photographic emulsions.
- (iv) Proper acceleration is given to super saturated liquids during their crystallisation, using ultrasonics.

(e) Medical applications

- (i) Ultrasonic blood flow meters are used to study the blood flow velocities in blood vessels.
- (ii) They are used in painless dental cutting
- (iii) These waves are also used for guiding the blind who carries a walking stick with ultrasonic transmitter and receiver.
- (iv) They are also used for detecting tumors and abnormal growths in human body.
- (v) Similarly twins or any defect in growth of fetus before delivery can be detected.
- (vi) They are also used to remove kidney stones and brain tumors, selectively tissue cut, without blood shed.
- (vii) They are also used in treating diseases like bursitis, lumbago etc.

2.13 Acoustical Grating

When ultrasonic waves propagate in a liquid, due to their periodic vibration of the transducer, pressure varies periodically resulting in density and hence refractive index variation. This is called Acoustical grating.

When monochromatic light is passed through such a grating diffraction takes place. Measuring angle of diffraction, velocity of acoustical waves in liquids can be measured.

In ultrasonic cell (glass cell) liquid under study is taken. Ultrasonic transducer is fixed at one side wall inside the cell and ultrasonic waves are generated [Fig.11].

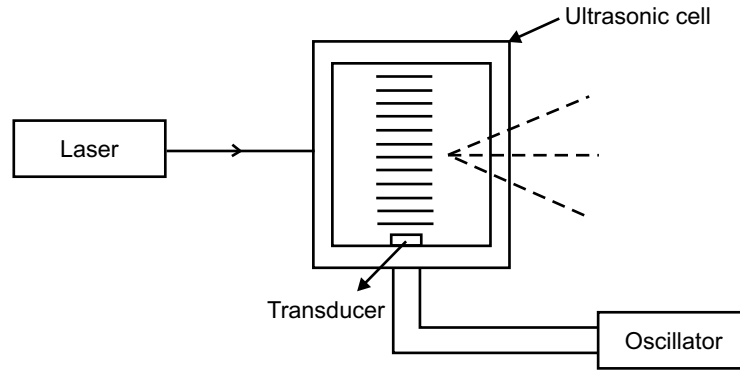


Fig. 11

The waves travelling from the transducer get reflected from the opposite wall and standing wave pattern is generated. The liquid particles align themselves according to the wave pattern (ie) they reorganise as regions of maximum density (nodal points) and regions of minimum densities (anti nodal points). Thus the liquid now acts as a grating, in which the loci of nodal points act as opaque region and loci of antinodal points acts as transparent region. Thus acoustic grating is formed. When light falls on such an arrangement, through the liquid in a direction perpendicular to the grating, diffraction takes place and an image is seen.

The angular separation between the direct ray and diffracted rays of different order can be calculated by measuring the distance of diffracted beams from the central beam.

Applying the theory of diffraction,

if d is distance between two adjacent nodes or antinodal planes

$$d \sin \theta_n = n\lambda$$

where

- n - order of diffraction
- λ - wave length of light
- θ_n - angle of diffraction for n^{th} order.
- λ_a - wavelength of ultrasonic waves

But, $d = \frac{\lambda_a}{2}$ (Fig.12)

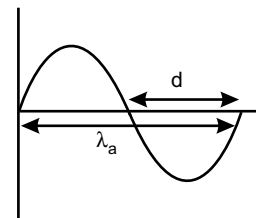


Fig. 12

Hence

$$\frac{\lambda_a}{2} \sin \theta_n = n\lambda$$

$$\lambda_a = \frac{2n\lambda}{\sin \theta_n}$$

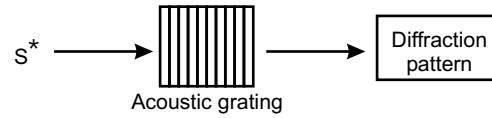


Fig. 13

If N is frequency of ultrasonic waves and v is Velocity of the waves in liquid, then,

$$v = N\lambda_a$$

from which velocity of ultrasonic waves can be calculated.

2.14 Sound Navigation and Ranging (SONAR)

Principle

It is based on the principle of ECHO-sounding. When ultrasonic wave is transmitted through water, it is reflected by objects in the water and will produce an echo signal. The change in frequency of the echo signal, due to doppler effect helps us to determine the velocity and direction of object.

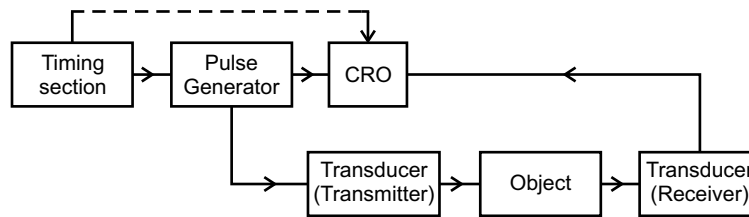


Fig. 14

Description

It consists of a timing section which triggers the electric pulse from pulse generator. This pulse generator is connected to the transducer (transmitter and receiver), so that ultrasonics can be produced. The transducer is further connected to CRO for display. The timing section is also connected to CRO display for reference of the timing at which the pulse is transmitted as shown in the figure. The transducer is mounted on the ship hull without any air gap between them.

Working

The timing at which pulse generated is recorded at the CRO for reference and this electrical pulse triggers the transducer which is kept in the hull of ship to produce ultrasonic wave due to principle of inverse piezoelectric effect. These ultrasonic waves

are transmitted through the water in sea. On striking the object the ultrasonic waves (echo pulses) are reflected in all directions. These waves are picked by the receiver (transducer) and is again converted into electrical pulse due to piezoelectric effect. These pulses will be weak and hence amplified and are recorded in cathode ray oscilloscope.

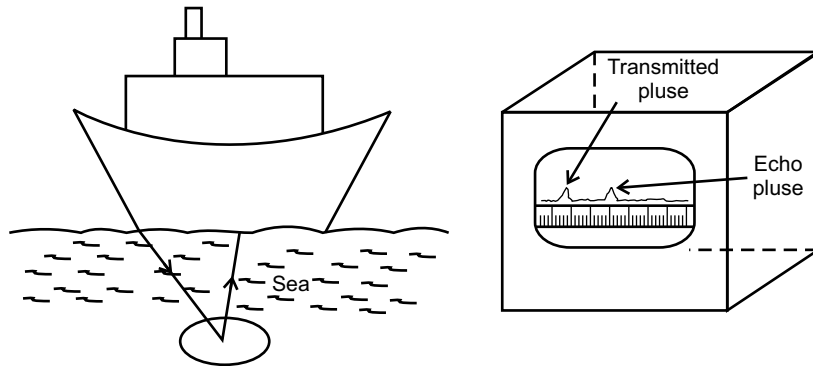


Fig. 15

Hence both the transmitted pulse and echo pulse are recorded from the time interval and from pulse height between the transmitted and received pulse, the position, distance and direction of moving object can be calculated.

2.15 Phono Cardiography(PCG)

Acoustic events of heart are divided into (i) Heart sounds (ii) Murmurs.

- (i) **Heart sounds:** These are due to the opening and closing of heart valves. They have low frequency and high amplitude.
- (ii) **Murmurs:** Noisy characters which are of longer duration, that is due to turbulent flow of blood in the heart. They have high frequency and small amplitude.

We know that ECG(Electro Cardiography) in which the activity of heart such as rhythmic disturbance of myocardial activity can be found with electric techniques. But valvular defects cannot be identified by using ECG. Hence PCG can be used for defecting these defects.

- Phono cardiograph records the sounds produced by pumping action of the heart.
- It provides information on heart rate, blood pumping, valve action, etc.,

Principle of working

The block diagram of PCG is as shown.

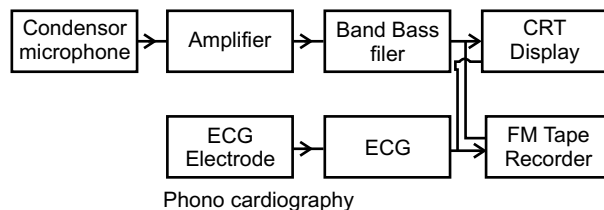


Fig. 16

A condenser microphone is placed on the chest of the patient. Ultrasonic waves are passed into the patient and reflected wave is received. The received sound signals are converted to electrical signals by microphone. These electrical signals are amplified by the amplifier. The amplified signals are driven to band pass filter, which separates the first, second, third and fourth heart sound. Each output is given to multichannel CRO display. Thus, different heart sounds are displayed on the CRO and the sound can also be heard using F-M tape recorder. The received signals can be compared with ECG by placing the electrodes on the limbs. These electrodes pick up the electrical activity of the heart and ECG signals can be displayed along with the PCG display as shown.

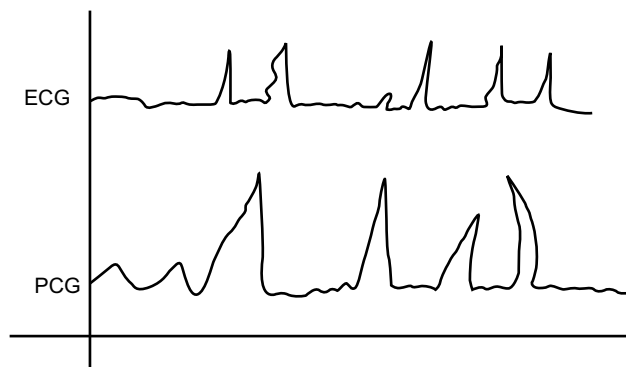


Fig. 17

Applications

1. It is used to discriminate normal and abnormal heart.
2. It can be used to identify the damage in heart valves.
3. It is also used to detect the heart murmur, there by regurgitation can be diagnosed [Regurgitation - backward flow of blood through a defective heart valve].
4. Fetal heart beats and movements can also be monitored using PCG.

2.16 Non-Destructive Testing (NDT) - Detection of Flaw in Metals

Principle

Whenever there is a change in medium, the ultrasonic waves will be reflected. This is the principle used in NDT. Since the flaws can be detected without destroying the materials, it is called non-destructive testing.

Description

It consists of a piezoelectric transducer coupled to the upper surface of specimen (metal) without any air gap between the specimen and the transducer. A frequency generator is connected to the transducer to generate high frequency pulses. The total set up is connected to the amplifier and to a cathode ray oscilloscope as shown in block diagram.

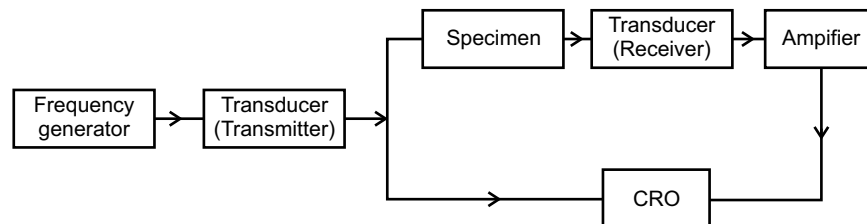


Fig. 18

Working

1. The pulse generator generates a high potential difference and is applied to the piezoelectric transducer.
2. The piezoelectric crystals are resonated to produce ultrasonic waves.
3. The ultrasonic waves (pulse A) are recorded in CRO and is transmitted through the specimen.
4. These waves travel through the specimen (metal) and is reflected back by the other end.
5. The reflected ultrasonic waves (pulse B) are received by the transducer.
6. These reflected signals are amplified and is found to be almost the same as that of the transmitted signals as shown in figure which shows no defect in the specimen.
7. On the other hand, if there is any defect on the specimen like small hole (or) pores, then the ultrasonic waves will be reflected by the holes (i.e.,) defects due to change in medium.

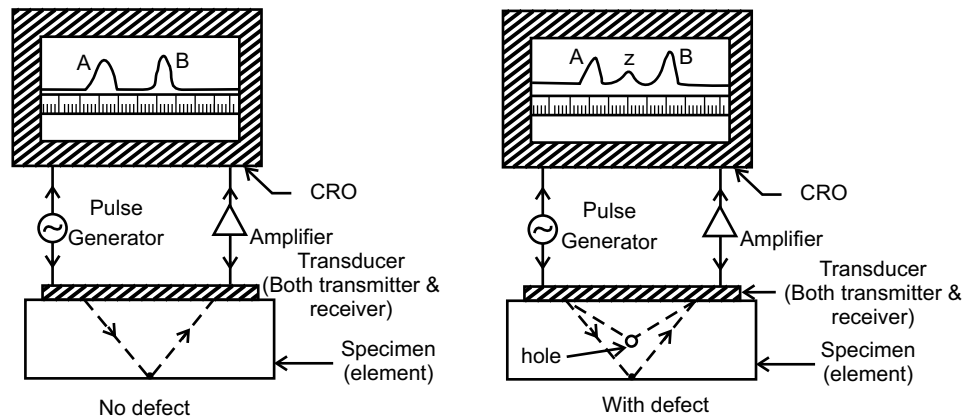


Fig. 19

8. These defects give noise to another signal (pulse z) in between pulse 'A' and 'B'. Similarly if we have many such holes, many z-pulses will be seen over screen of CRO, as shown in figure.
9. From the time delay between the transmitted and received pulses the position of hole can be found.
10. From the height of pulse received the depth of hole or defect can also be determined.

□ Solved Problem 1

Find the sound intensity at 3 m from a speaker that is placed on the side wall of a hall feeding sound evenly into a hemisphere in front of it. The sound power of the speaker is 2 watts.

► Solution

$$\text{Sound intensity} = \frac{\text{Sound power}}{\text{Area spread}}$$

$$\begin{aligned} \text{Area of hemisphere} &= 2\pi r^2 \\ &= 2\pi \times 3^2 \text{ m}^2 = 56.5 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Sound intensity} &= \frac{\text{Power}}{\text{Unit area}} \\ &= \frac{2}{56.6} = 3.54 \times 10^{-2} \text{ Watt/m}^2 \end{aligned}$$

□ **Solved Problem 2**

If the intensity of a sound in watt/m² is doubled, by how much is the dB level changed.

► **Solution**

$$dB = 10 \log_{10} \frac{I}{I_0}$$

Let dB_1 be the intensity level for I_1

and Let dB_2 be the intensity level for $2I_1 = I_2$

$$\begin{aligned} dB_2 - dB_1 &= 10 \log_{10} \frac{2I_1}{I_0} - 10 \log_{10} \frac{I_1}{I_0} \\ &= 10 \log_{10} 2 + 10 \log_{10} \frac{I_1}{I_0} - 10 \log_{10} \frac{I_1}{I_0} \\ &= 10 \log_{10} 2 = 3.01 \end{aligned}$$

□ **Solved Problem 3**

The intensity of sound in a busy street is 8×10^{-5} Watt/m². Calculate the intensity level in dB.

► **Solution**

$$\begin{aligned} \text{Intensity level} &= 10 \log \frac{I}{I_0} \\ &= 10 \log \frac{8 \times 10^{-5}}{10^{-12}} \\ &= 79.031 \text{ dB} \end{aligned}$$

□ **Solved Problem 4**

A hall has a volume of 12500 m³ and reverberation time of 1.5 sec. If 200 cushioned chairs are additionally placed in the hall, what will be the new RT of the hall. Absorption of each chair 1.0 OWU m².

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► **Solution**

$$T = \frac{0.16 V}{\Sigma aS}$$

$$\Sigma aS = \frac{0.16 \times 12500}{1.5} = 1375$$

$$T_2 = \frac{0.165 V}{\Sigma aS + a_1 S_1}$$

$$= \frac{0.165 \times 12500}{1375 + 200} = 1.31 \text{ sec}$$

□ **Solved Problem 5**

A window whose area is 1.4 m^2 opens on a street where the street noise results in an intensity level at the window of 60 dB. How much acoustic power enters the window via the sound waves ?

► **Solution**

$$\text{Intensity level} = 10 \log \frac{I}{I_0} \text{ dB}$$

$$60 = 10 \log \frac{I}{10^{-12}}$$

$$\Rightarrow I = 10^{-6}$$

$$\begin{aligned} \text{Acoustic power} &= \text{Intensity} \times \text{Area} \\ &= 10^{-6} \times 1.4 \\ &= 1.4 \times 10^{-6} \text{ Watt} \end{aligned}$$

□ **Solved Problem 6**

The minimum intensity of audibility of a source is 10^{-12} W/m^2 . If the frequency of the note is 1000 Hz, calculate the amplitude of vibration of air particles. Density of air = 1.293 Kg/m^3 . $v = 340 \text{ m/s}$.

► **Solution**

$$I = 2\pi^2 n^2 a^2 \rho v$$

$$10^{-12} = 2\pi^2 (1000)^2 a^2 \times 1.293 \times 340$$

$$a = 1.073 \times 10^{-11} \text{ m}$$

□ **Solved Problem 7**

Calculate the minimum intensity of audibility in watt/sq.m for a note of 1000 Hz if the amplitude of vibration is 10^{-11} m . Assume the density of air = 1.3 Kg/m^3 and velocity of sound = 340 m/sec .

► **Solution**

$$\begin{aligned}
 I &= 2\pi^2 n^2 a^2 \rho v \\
 &= 2\pi^2 \times 1000^2 \times (10^{-11})^2 \times 1.3 \times 340 \\
 &= 8.176 \times 10^{-13} \text{ W/m}^2
 \end{aligned}$$

□ **Solved Problem 8**

If the intensity is increased by a factor 20, by how many decibel is the sound level increased ?

► **Solution**

Let

$$\begin{aligned}
 L_1 &= 10 \log_{10} \frac{I}{I_0} \\
 L_2 &= 10 \log_{10} \left(\frac{20I}{I_0} \right) \\
 \therefore L_2 - L_1 &= 10 \log_{10} \left(\frac{20I}{I_0} \right) - 10 \log_{10} \left(\frac{I}{I_0} \right) \\
 &= 10 \log_{10} \left(\frac{20I}{I_0} \times \frac{I_0}{I} \right) \\
 &= 10 \log_{10} 20 = 13 \text{ dB}
 \end{aligned}$$

□ **Solved Problem 9**

Two trains are approaching each other with a speed of 36 km per hour. If the engine of the train emits a note of 1000 vibrations, what will be the frequency of the note as heard by a person sitting in the other train?

► **Solution**

Velocity of sound in air = 330 m/sec.

$$n' = n \left(\frac{v - v_0}{v - v_s} \right)$$

Given $v = 330 \text{ m/sec}$

$$v_s = \frac{36 \times 10^3}{60 \times 60} = 10 \text{ m/sec}$$

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$$v_0 = -10 \text{ m/sec} \quad \text{and} \quad n = 1000/\text{sec}$$

$$\begin{aligned} \therefore n' &= 1000 \left(\frac{330 - (-10)}{330 - 10} \right) \\ &= 1000 \times \frac{340}{320} \\ &= 1062.5 / \text{sec} \end{aligned}$$

□ Solved Problem 10

A quartz crystal of thickness of 0.001 meter is vibrating at resonance. Calculate the fundamental frequency. Given Y for quartz is 7.9×10^{10} Newton/m² and ρ for quartz is 2650 kg/m³.

► Solution

$$\begin{aligned} v &= \sqrt{\frac{Y}{\rho}} \\ &= \sqrt{\frac{7.9 \times 10^{10}}{2650}} = 5461 \text{ m/sec} \end{aligned}$$

For fundamental mode of vibrating thickness is $\lambda/2$.

$$\lambda = 2t = 2 \times 0.001 = 0.002 \text{ m}$$

$$\begin{aligned} \text{Now, } v &= n\lambda \Rightarrow n = \frac{v}{\lambda} \\ &= \frac{5461}{0.002} = 2730 \text{ kHz} \end{aligned}$$

Short Questions

1. Define reverberation time.
2. What are the characteristics of musical sound?
3. What is a relation between pitch and frequency?
4. What do you mean by timbre?
5. State weber Feschner's law.
6. Define intensity level and its unit?
7. What do you mean by Doppler effect.
8. Explain the term 'sonic boom'.

9. What are the factors affecting the acoustics of buildings.
10. Define absorption coefficient of a material. Give its unit.
11. What is a decibel.
12. What is meant by sonar? Name two uses of it.
13. What do you mean by acoustics of buildings?
14. What are the requirements for good acoustics?
15. Define one phon and sone.
16. Distinguish between loudness and intensity of sound?
17. What is Echelon effect?
18. Distinguish between reverberation and echo.
19. Give any four applications of Doppler effect.
20. Distinguish between Mach Number and Reynold's number.
21. Why should a hall or auditorium must have optimum reverberation?
22. Differentiate musical sound and noise.
23. What are ultrasonics.
24. List out the properties of ultrasonics.
25. Explain magnetostriction effect.
26. State piezo electric effect and its converse.
27. What do you mean by acoustic grating.
28. Explain why piezo electric generator is called as base tuned circuit.
29. Why a magnetostriction oscillator is called as collector tuned circuit.
30. List out some application of ultrasonic in engineering field.
31. Write down few medical applications of ultrasonics.

Review Questions

1. What is reverberation time? Derive Sabine's formula for reverberation time.
2. Describe with necessary theory a method of measuring of absorption coefficient of a material.
3. Discuss the factors affecting acoustics of building & how these can be rectified?
4. Derive an expression for the Doppler shifted frequency of sound waves there is a relative motion between the source of sound and observes.
5. Discuss the phenomenon of Magnetostriction? How will you produce with the help, high frequency sound waves? Mention the important application of Ultrasonics.
6. What is piezoelectric effect? Explain with a neat circuit, the generator of ultrasonic using a piezoelectric oscillator.
7. What is acoustic grating? Using the principle of acoustic grating how do you determine the velocity of ultrasonic waves in a liquids?

Exercise Problems

1. Find the velocity of the source when the frequency appears to be (a) double (b) Half the original frequency to a stationary observer. **(Ans: $v/2; v$)**
2. A locomotive whistle 256 vib/sec is moving towards you with a velocity $\frac{1}{20}$ th of that sound. What will be the frequency of the notes heard by you before and after the engine passes you? **(Ans: 269.5 vib/sec; 243.8 vib/sec)**
3. A policeman on duty at a crossing challenges a car driver for crossing the speed limit of 100 km/hr by detecting a change of 20 vib in the horn note of frequency 128 as the car passes him. Is he correct? Velocity of sound = 350 m/s. **(Ans: 99 km/hr; wrong)**
4. A train approaching a railway crossing at a speed of 120 km/hr sounds a short whistle at frequency 640 Hz. When it is 300 m away from the crossing. The speed of sound in air is 340 m/s. What will be the frequency heard by a person standing on a road perpendicular to the track through the crossing at a distance of 400 m from the crossing. **(Ans: 680 Hz)**
5. A vibrating tuning fork tied to the end of a string 1.988 m long is whirled round a circle. If it makes two revolutions in a sound, calculate the frequencies of the highest and the lowest notes heard by an observer situated in the plane of the turning fork. Velocity = 350 m/sec. **(Ans: 1.154)**

6. Find the intensity level in dB of a sound of intensity 3.5×10^{-2} watt/m². Zero intensity level is 10^{-12} watt/m². **(Ans: 105.4 dB)**
7. What is the resultant sound level when a 70 dB sound is added to an 85 dB sound. **(Ans: 85.13)**
8. The sound from a drill gives a noise level of 90 decibels at a point a few meter away from it. What is the noise level at this point when four such drills are working at the same distance away? **(Ans: 96.021 dB)**
9. The volume of a hall is 475 m³. The area of wall is 200m², area of floor and ceiling each is 100m². If absorption coefficient of the wall, ceiling and floor are 0.025, 0.02 and 0.55 respectively. Calculate reverberation time. **(Ans: 1.26 sec)**
10. Calculate RT of a hall of 1500m³ having seating capacity for 120 persons,
 - (i) When hall is empty
 - (ii) With full capacity of audience
 - (iii) Audience occupying only cushioned seats.

With following data

	area (m ²)	a(owu)
Plastered walls	112	0.03
Wooden floor	130	0.06
Plastered ceilings	170	0.04
Wooden door	20	0.06
Cushioned chairs	100	1
Audience	120	4.7

(Ans: 2.08 s, 0.362 s, 0.42 s)

11. What should be the total absorption in a hall of volume 10,000 m³ if it is required to have a reverberation time of 1.4 sec. **(Ans: 1192.85 sabine)**
12. A loud speaker emits energy in all directions at the rate of 1.5 J/sec. What is the intensity level at a distance of 20 m. Standard intensity level of sound is 10^{-12} W/m². **(Ans: 84.77 dB)**
13. The velocity of ultrasonics in steel is 500 m/s. An ultrasonic beam is used to determine the thickness of a steel plate. It is noticed that the difference between two adjacent harmonic frequencies is 60 KHz. Determine the thickness of the steel plate. **(Ans: 0.4166 m)**

2.48 *Applied Physics*

14. An ultrasonic interferometer used to measure the velocity in sea water. If the distance between two consecutive antinodes is 0.55 mm. Compute the velocity of the waves in sea water. Frequency of crystal is 1.5 MHz.

(Ans: 1650 m/s)

15. Longitudinal standing waves are set up in a quartz plate with antinodes at opposite faces. The fundamental frequency of vibration is given by $f = \frac{2.87 \times 10^3}{t}$. Calculate Young's modulus and thickness of plate required for a frequency of 1300 KHz ($\rho = 2660 \text{ kg/m}^3$).

(Ans: $8.764 \times 10^{10} \text{ N/m}^2$; $2.207 \times 10^{-3} \text{ m}$)

3.1 Introduction

Laser (Light Amplification by Stimulated Emission of Radiation) are optical phenomena that are used in many fields, such as communication, 3D photography, data processing, medical field etc. Significant feature of a laser is the enormous difference between the character of its light of the light from other sources such as sun, flame or incandescent lamp. Most striking features are

1. Directionality
2. High intensity
3. Extraordinary monochromaticity
4. High degree of coherence.

3.1.1 Directionality

- Conventional light sources emit in all directions (Due to spontaneous emission)
- Lasers, emit only in one direction (Due to stimulated emission)

Directionality of laser beams is usually expressed in terms of *Full angle beam divergence*, which is twice the angle that the outer edge of the beam makes with the axis of the beam. [Outer edge of a beam is a point at which the strength of the beam has dropped to $1/e$ times its value at the centre]

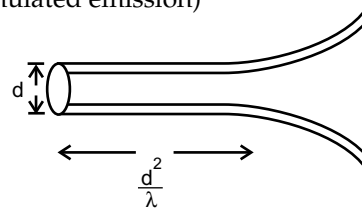


Fig. 1

When a beam with planar wavefront radiates from an aperture of diameter d , the beam propagates as a parallel beam for a distance of about d^2/λ , which is sometimes called as Rayleigh range and begins to spread linearly with distance due to unavoidable effects of diffraction.

$$\text{angular spread } \Delta\theta = \frac{\lambda}{d}$$

3.2 Applied Physics

For laser beam $\Delta\theta = 0.01$ milliradian (i.e) it spreads only 0.01 mm about 1 m for every 1 m of its travel.

3.1.2 Intensity

Laser gives out light into a narrow beam and its energy is concentrated in a small region. This concentration of energy, both spatially & spectrally, accounts for the great intensity of lasers. [Thus even 1 watt laser would appear many thousand times more intense than 1000 watt ordinary lamp]. During focussing of laser radiation, the energy density at the focal spot is enormously increased, resulting in increase in radiation pressure also.

3.1.3 Monochromaticity

The light emitted by a laser is more monochromatic than that of any conventional monochromatic source.

A laser light reveal no spreading at all.

[None including laser, can produce absolute monochromaticity. We can only seek better approximation to ideal]

Degree of non monochromaticity ζ of a wave is defined as its relative bandwidth and is given by $\zeta = \frac{\Delta\nu}{\nu_0}$

For laser $\zeta = 10^{-12}$ and for ordinary light source $\zeta = 10^{-5}$. Thus lasers are strictly monochromatic than conventional light sources.

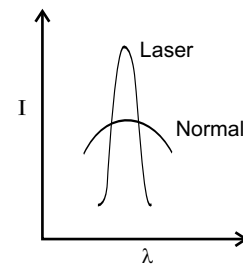


Fig. 2

3.1.4 Coherence

Coherence is expressed in terms of ordering of light field (i.e.,) purity of spectral line. Laser radiation is characterized by high degree of ordering of the light field than the other sources. High coherence of laser emission makes it possible to realize a tremendous spatial concentration of light power, such as $10^{13}W$ in space with linear dimensions of only $1 \mu m$.

3.2 Laser - Basic Principles

When radiation is incident on matter, for interaction to occur, first of all the energy of the interacting photon $h\nu$ must match with energy difference between the two states of the atoms involved in the interaction.

Under this condition, if the radiation interacts with atoms in lower energy state say E_1 , the atoms absorb the energy and get excited to higher energy E_2 by a process called **Induced Absorption** (Fig.3(a)).

Instead if the radiation interacts with atoms already in E_2 then de excitation of those atoms to the lower energy state E_1 occurs with emission of photon of energy $h\nu$. This process is called **Stimulated Emission** (Fig.3(c)).

Another emission process called **Spontaneous Emission** is also possible where in atoms in the excited state drop to tower energy state after they have stayed in excited state for a short elevation of time called their Life Time (Fig.3(b)).

During this process also photon ($E = h\nu$) is emitted.

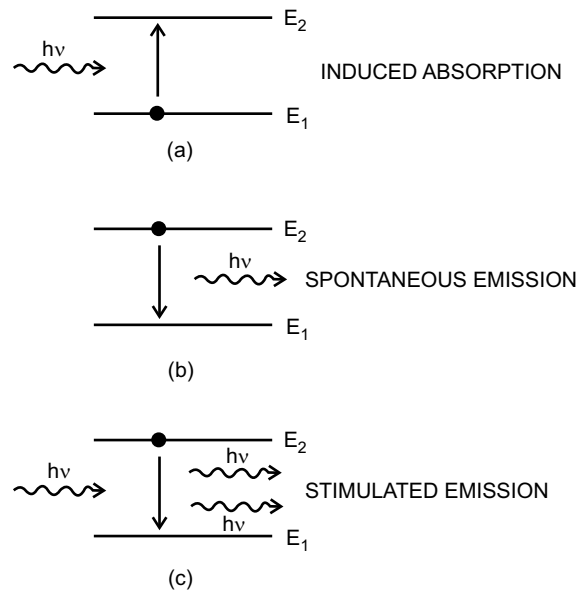


Fig. 3

3.2.1 Einstein coefficients

In a collection of atoms, all the three transition processes occur simultaneously.

Let N_1 be number of atoms / unit volume with energy E_1 and N_2 number of atoms / unit volume with energy E_2 . Let n be number of photons / unit volume at frequency ν such that $h\nu = E_2 - E_1$.

Then energy density of interacting photons $\rho(\nu)$ is given by

$$\rho(\nu) = nh\nu$$

When these photons interact with atoms, both upward (absorption) & downward (emission) transitions occurs.

(a) Upward transition (Absorbption)

When photons are incident, atoms from lower energy state move to higher energy state.

3.4 Applied Physics

Stimulated absorption rate depends on the number of atoms available in the lower energy state for absorption of these photons as well as energy of interacting radiation

$$\begin{aligned} \text{(i.e)} \quad R_{12} &\propto N_1 \\ &\propto \rho(\nu) \\ &= N_1 \rho(\nu) B_{12} \end{aligned} \quad (1)$$

where B_{12} is constant of proportionality.

(b) Downward transition (Emission)

Once the atoms are excited by stimulated absorption, they stay in the excited state for a short duration of time called lifetime of excited state. After this they move to lower level, by emitting photons. So spontaneous emission rate

$$\begin{aligned} R_{21}(Sp) &\propto N_2 \\ &= N_2 A_{21} \end{aligned} \quad (2)$$

where A_{21} is constant of proportionality.

Before de excitation, if the atoms interact with photon, it results in stimulated emission of photons.

\therefore stimulated emission rate

$$\begin{aligned} R_{21}(St) &\propto N_2 \\ &\propto \rho(\nu) \\ &= N_2 \rho(\nu) B_{21} \end{aligned} \quad (3)$$

where B_{21} is constant of proportionality.

During stimulated emission, the interacting photon (stimulating photon) and the stimulated photon, are in phase with each other.

For a system in equilibrium

upward transition rate = downward transition rate

$$\begin{aligned} N_1 \rho(\nu) B_{12} &= N_2 \rho(\nu) B_{21} + N_2 A_{21} \\ \rho(\nu) &= \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} \end{aligned} \quad (4)$$

Dividing numerator and denominator by $N_2 B_{21}$ in Eq.(4), we get

$$\rho(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \quad (5)$$

The population of various energy level of a system in thermal equilibrium is given by Boltzman Distribution law

$$N_i = g_i N_o \exp \left(\frac{-E_i}{kT} \right)$$

where

- N_i - Population density of energy level E_i
- N_o - Ground state population density at Temperature T .
- g_i - degeneracy of i^{th} level
- k - Boltzmann constant ($1.38 \times 10^{-23} \text{ J/k}$)

So

$$N_1 = g_1 N_o \exp \frac{-E_1}{kT}$$

$$N_2 = g_2 N_o \exp \frac{-E_2}{kT}$$

$$\begin{aligned} \text{(or)} \quad \frac{N_1}{N_2} &= \frac{g_1}{g_2} \exp \left(\frac{E_2 - E_1}{kT} \right) \\ &= \frac{g_1}{g_2} \exp \left(\frac{h\nu}{kT} \right) \end{aligned} \quad (\because E_2 - E_1 = h\nu) \quad (6)$$

Substituting (3.6) in (3.5)

$$\rho(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\left(\frac{g_1}{g_2} \frac{B_{12}}{B_{21}} \exp \frac{h\nu}{kT} - 1 \right)} \quad (7)$$

From Plank's Radiation law,

$$\rho(\nu) = \frac{8 \pi h\nu^3}{C^3} \left(\frac{1}{\exp \left(\frac{h\nu}{kT} \right) - 1} \right) \quad (8)$$

Comparing (3.8) & (3.7)

$$\frac{g_1 B_{12}}{g_2 B_{21}} = 1$$

$$\Rightarrow \boxed{g_1 B_{12} = g_2 B_{21}} \quad (9)$$

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{C^3}} \quad (10)$$

Eq.(9) and Eq.(10) are known as Einstein coefficients.

3.6 Applied Physics

Case(i): Ratio of spontaneous emission rate to stimulated emission rate

From Eq.(2) and Eq.(3)

$$\begin{aligned}\frac{R_{21}(sp)}{R_{21}(st)} &= \frac{N_2 A_{21}}{N_2 \rho(\nu) B_{21}} \\ &= \frac{A_{21}}{B_{21}} \times \frac{1}{\rho(\nu)}\end{aligned}$$

Substituting from Eq.(10) and Eq.(8)

$$\begin{aligned}&= \frac{8\pi h\nu^3}{C^3} \times \frac{C^3}{8\pi h\nu^3} \left(\exp \frac{h\nu}{kT} - 1 \right) \\ &= \exp \left(\frac{h\nu}{kT} \right) - 1 \quad \text{which is } \gg 1\end{aligned}$$

$$\text{(i.e.,)} \quad \frac{R_{21}(sp)}{R_{21}(st)} > 1$$

$$\text{(or)} \quad R_{21}(sp) > R_{21}(st) \quad (11)$$

Case(ii): Ratio of stimulated absorption rate & stimulated emission rate

From Eq.(3) and Eq.(1)

$$\begin{aligned}\frac{R_{21}(st)}{R_{12}} &= \frac{N_2 \rho(\nu) B_{21}}{N_1 \rho(\nu) B_{12}} \\ &= \frac{N_2}{N_1} \quad (\because B_{21} = B_{12})\end{aligned}$$

$\therefore N_1 > N_2$ (always)

$$R_{21}(st) < R_{12} \quad (12)$$

From (3.11) and (3.12), when radiation is allowed to interact with matter. Stimulated absorption is predominant and similarly photon is emitted only by spontaneous emission.

For Laser to emit,

$$\begin{aligned}R_{21}(st) &> R_{12} \quad \text{which means} \\ N_2 &> N_1\end{aligned}$$

If this condition is satisfied then, stimulated emission will be predominant than absorption.

\therefore To achieve stimulated emission, population of excited state N_2 should be made larger than population of lower state (N_1) and this condition is called **Population inversion**

Hence for amplifying the beam of light by stimulated emission we must

- Create population inversion
- Increase energy density of interacting radiation.

To achieve this, the following basic requirements are required for laser operation.

- Active Medium - To create population inversion
- Active centers - atoms that actually take part in population inversion
- Population inversion
- Resonance cavity - For optical feedback .
- Threshold inversion density - for emission of in phase photons satisfying characteristics of Laser beam.

Method of increasing the population of higher energy level is known as pumping. To achieve population inversion, various pumping mechanism are involved. Some of them are :

- Optical pumping
- Inelastic atom - atom collision
- Chemical reaction
- Injection luminescence.

3.3 Ruby Laser

In solid substances used in laser devices, the active material is present in concentration less than one percent. The bulk of the material does not participate in the laser action. It only acts as the host. For laser action to be possible, the ions of the active material will have to be excited to proper upper level which is usually accomplished by optical pumping.

Ruby is crystalline Al_2O_3 doped with chromium. The triply ionised chromium ions, which replaces some of Al^{3+} ions, gives the otherwise transparent crystal, a pink red colour depending upon its concentration.

Energy levels of Cr^{3+} ions in Al_2O_3 lattice.

There are 2 main pump bands $4F_1$ and $4F_2$ centered around $0.42\mu m$ $0.55\mu m$ respectively.

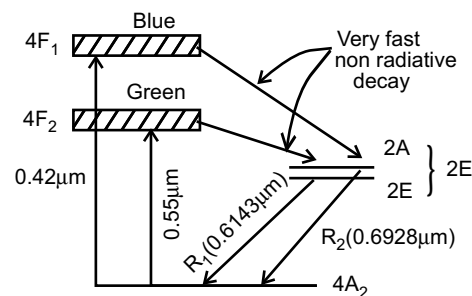


Fig. 4

Construction

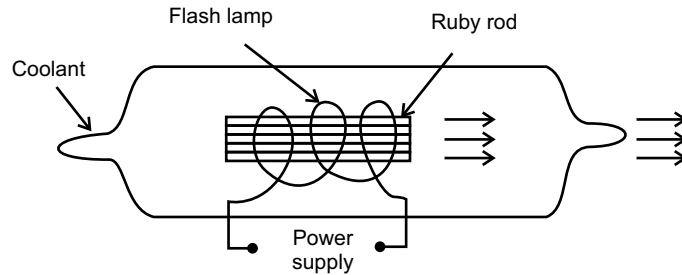


Fig. 5

One centimeter cube of Pink ruby with a concentration of about 0.05% of C_r is used. Ruby is in the form of cylinder 4 cm in length and 0.5 cm in diameter. Its ends are grounded and polished plane and parallel. One end is silvered to give complete reflecting surface with about 10% transmission. A helical photographic flash lamp filled with Xenon, provides white light in the form of a pulse which last for about one thousandth of a second. Above a certain critical light intensity, an inversion of population is achieved and the laser oscillations starts. An intense beam of red light emerges from the end of the rod. The beam divergence is about half a degree and the emerging beam is spatially coherent.

The power during the output pulse is nearly 10 Kilowatts.

Pumping method

The pumping method used here is the optical pumping, where photons are used for excitation. Since solids are characterised by energy bands of small energy separation, photons of near equal frequencies are used for pumping.

Working

When the material is radiated with green light at $\lambda \sim 5500\text{Å}$, chromium ions are excited to $4F_2$ state.

The excited chromium ions quickly lose some of their energy via phonon assisted non radiative transition to a pair of levels associated with $2E$ state. This state slowly decays by emitting a sharp doublet with components.

$$R_1 (6943\text{Å}) \quad \text{and} \quad R_2 (6928\text{Å})$$

Under very intense excitation, the population of metastable state can become greater than that of the ground state. In this situation the initial spontaneously emitted fluorescent photons would travel in all directions, but those travelling parallel to the axis would be reflected to and fro between the reflectors and so would pass

many times through amplifying medium and would stimulate upper state atoms to radiate, thus causing amplification via stimulated emission. The laser action is thus, actually initiated by spontaneous emission.

Actually, laser action can be obtained on both $2A \rightarrow 4A_2$ and $2E \rightarrow 4A_2$ transitions. However, usually it occurs on $2E \rightarrow 4A_2$ transition (R_1 line).

Spiking

Output of a pulsed ruby laser, if examined with a photo cell or oscilloscope is found to consist of a series of pulse of duration of a few microseconds which is called spiking (Fig.6).

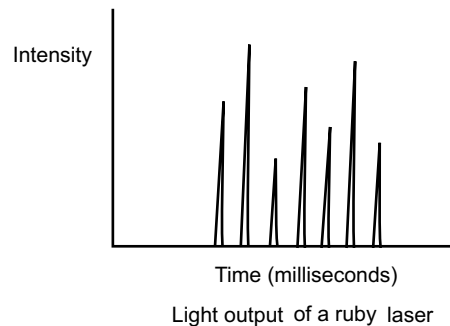


Fig. 6

Duration of exciting flash light is of the order of millisecond & may be sufficiently intense to build up an inverted population very rapidly. As soon as a sufficiently large population has been produced in upper level, laser action starts producing a pulse. This has the effect of depleting the upper level population more rapidly than it can be restored by flash light. Laser oscillation then ceases for a few microseconds. Because the flash lamp is still active, it again builds up population inversion causing another spike, and the sequence is repeated. Peak Power radiated - 10^{12} times as radiated by the sun.

3.4 Energy level diagram of He-Ne laser

The main drawback of ruby laser is that the output beam is not continuous. Hence for continuous beam, gas lasers are used. In gas lasers vapours of metals are employed, as active media. Main Advantages of gas lasers are exceptional monochromaticity, most pure spectrum & high stability of frequency.

Construction

It consists of a fused quartz tube with diameter of about 1.5 cm and 80 cm long. This tube is filled with a mixture of Ne & He under a pressure of $0.1\mu m$ of Hg & 1 mm of Hg respectively. There are majority of He atoms & minority Ne atoms.

At one end of the tube, there is a perfect reflector while on the other end there is a partial reflector.

The active material is excited by means of high frequency generator with a frequency of several tens of MHz and an input of about 50 Watt.

3.10 *Applied Physics*

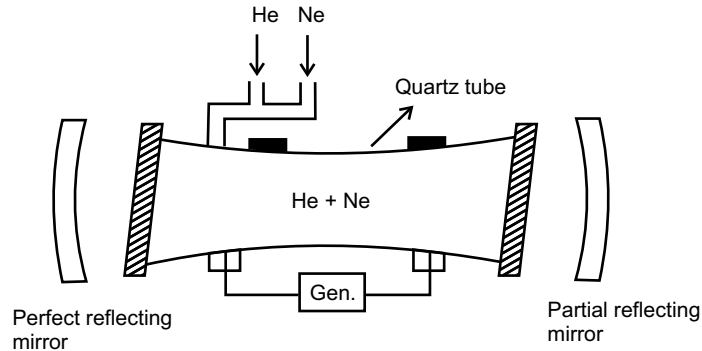


Fig. 7

Pumping method

The pumping method used here is inelastic atom atom collision, where electrons are used for excitation. Since gases are characterised by discrete energies, photons (optical pumping) cannot be used. Here a mixture of two gases having near equal excited energy levels are taken. Readily reactive atoms goes to higher energy level and by resonance transfer of energy, these excited atoms collide with other atoms in ground state and excites them, there by deexciting themselves.

Working

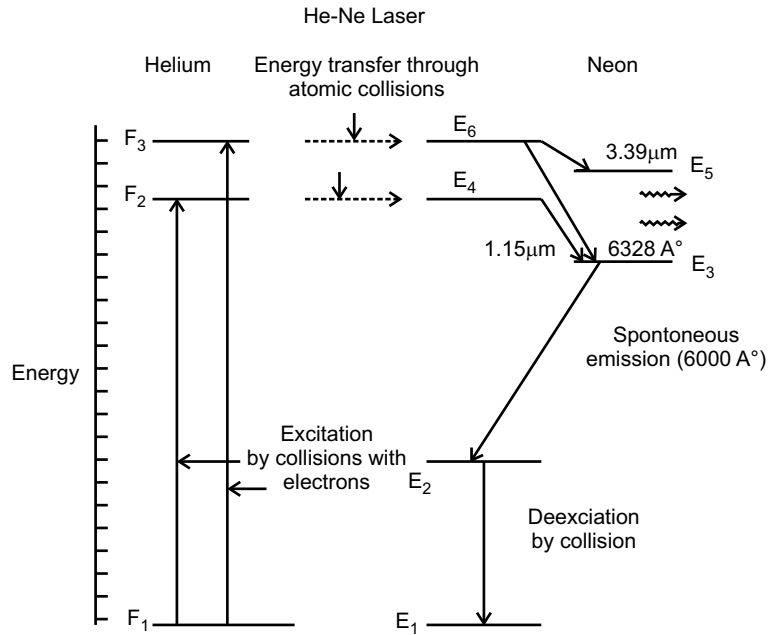


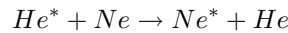
Fig. 8

When a discharge is passed through the gas mixture, *He* atoms are excited to energy levels (E_2, E_3), through collisions with acceleration particles. These levels are meta stable states from which there are no allowed transitions. The excited *He* atoms then collide inelastically with neon atoms still in ground state and transfer the energy to them. Now the *He* atoms return to ground state.

Transition level	Wavelength	Range
$E_6 \rightarrow E_5$	33900 Å°	Far IR
$E_6 \rightarrow E_3$	6328 Å°	Red (visible)
$E_4 \rightarrow E_3$	11500 Å°	IR

The higher neon states E_6 and E_4 are metastable states and have longer lifetime than E_3 .

(ie)



Hence an inversion of population at $E_6, E_4 \times E_3$ results in.

When an excited Ne atom passes spontaneously from metastable states E_6, E_4 to state E_3 , it emits a photon. This photon travels through the gas mixture.

If the photon is moving parallel to the axis of the tube, it is reflected back & forth by the mirror ends & it stimulates an excited Ne atom, resulting in laser transition.

This process continues till a beam of coherent radiation builds up.

Laser Radiation are

To get 632.8 nm output, the laser tube windows are made up of glass or quartz that absorb $1.15\mu m$ & $3.39\mu m$.

There occurs a collection of atoms at metastable E_2 level of Ne, after a fast decay from E_3 levels.

To avoid this, the discharge tube diameter is kept small so that the atom collides with the walls of the tube and comes to ground state. The output is polarised and output power is about 0.5 mW to 100 mW.

3.5 CO_2 Laser

It comes under the category of molecular lasers. It is capable of generating continuously and high power of 10 KW. The active medium is $CO_2 + N_2 + He$. Besides electronic energy level, molecules have various vibrational & rotational levels. The reason is that the constituent atoms of the molecule can vibrate in relation to each other and molecule as a whole can rotate. Due to transitions between vibration - rotation levels of the molecules, molecular laser oscillations are produced.

3.12 Applied Physics

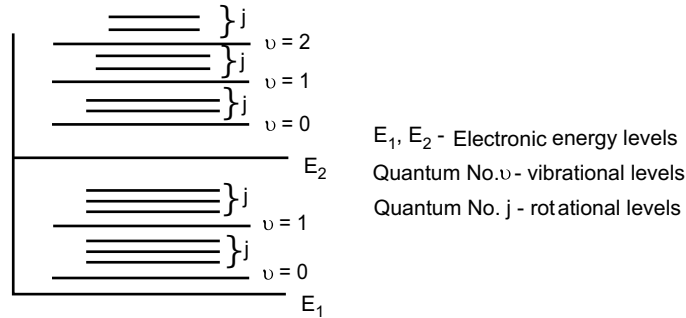


Fig. 9

Types of molecular lasers

There are two types of molecular lasers.

- Type 1 : Transition between vibrational states of same electronic states
 - Type 2 : Transition between the vibrational level of different electronic states.
- CO_2 laser comes in the category of Type 1

3.5.1 Vibrational modes of CO_2

CO_2 is a linear molecule with carbon at the center and oxygen at the ends. There are three different modes of vibrations of CO_2 molecules.

(i) Symmetric stretching mode

The carbon atom is fixed in its position and each oxygen atom vibrate in opposite direction symmetrical to the central carbon atom along a straight line and it is known as symmetric stretching mode and the corresponding frequency is called symmetric stretching frequency

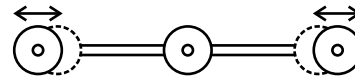


Fig. 10

(iii) Asymmetric stretching mode

All the three atoms will vibrate. The oxygen atoms vibrate in the opposite direction to the vibration direction of carbon atom. i.e two oxygen atom vibrate asymmetrical to the central carbon atom. The corresponding frequency is called asymmetric stretching frequency.

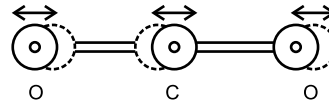


Fig. 11

(ii) Bending mode

The atoms will not be linear. The oxygen atom and the carbon atom may vibrate at right angles to the line passing through the centre of gravity. This is known as bending mode of vibration and the corresponding frequency is called bending frequency.

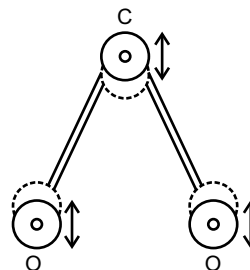


Fig. 12

Design of CO_2 laser

It consists of a discharge tube of 2.5 cm in diameter and 5 cm long. Discharge is produced by means of d.c. excitation (Fig.13).

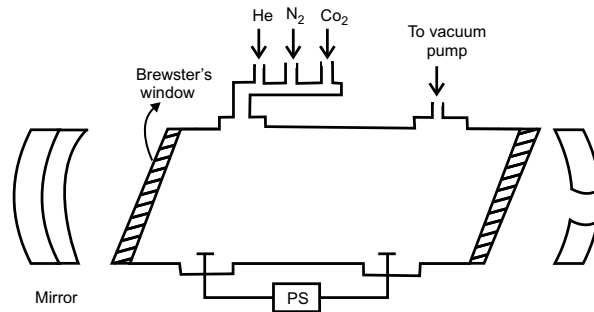


Fig. 13

Brewster windows are used at the ends. Near confocal silicon mirrors coated with aluminium forms resonance cavity. First nitrogen molecules are allowed to enter the area of discharge. Here they become excited by collision with the electron of discharge. These excited nitrogen molecules then flow into the laser volume, where they mix up with unexcited CO_2 molecules. N_2 imparts its energy to CO_2 molecules by resonant energy transfer.

Pumping method

Here also inelastic atom atom collision method is used for pumping. The electrons excite N_2 atoms to their higher energy level. They in turn collide with CO_2 molecules in their ground state, transfer their energies, excite CO_2 to higher level and deexcite themselves to ground state.

Energy level diagram of CO_2 laser

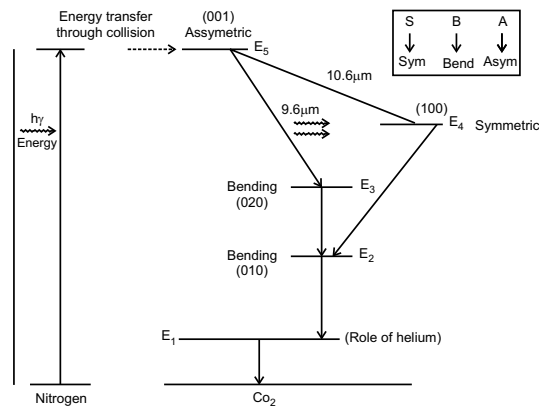


Fig. 14

3.14 Applied Physics

Upper laser level state is (001) and lower laser level are (020) and (100).

Transition between (001) \rightarrow (100) corresponds to wavelength of $10.6 \mu\text{m}$ and the transition (001) \rightarrow (020) corresponds to wavelength of $9.6 \mu\text{m}$.

Function of He

The levels, (100), (020), & (010) being very close, soon reach thermal equilibrium.

Hence the decay from lowest level (010) to ground state should be very fast, otherwise, there would be accumulation of molecules in this level, which would spoil the population inversion.

To avoid this, the temperature of CO_2 must be low. He, is used for this purpose, because it has high thermal conductivity

3.6 Nd: YAG Laser

Nd - Neodymium; YAG - Yttrium Aluminium Garnet

Introduction

Nd-YAG laser is a doped insulator laser (Solid State Laser). It is a four level laser. $\text{Y}_3\text{Al}_5\text{O}_{12}$ commonly called YAG is an optically isotropic crystal and it acts as an active medium. The laser is used to generate high power intense beam of light.

Principle

The term 'Doped insulator laser' refers to the active medium YAG doped with Nd^{+3} . The Neodymium ion has many energy levels. Due to the optical pumping these ions are raised to excited states. The released LASER beam has a wavelength of $1.064 \mu\text{m}$ (IR region)

Construction

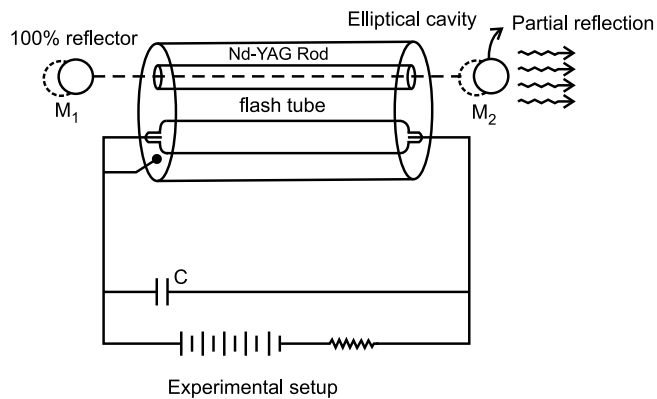


Fig. 15

Fig (15) shows a typical design of the laser. The Nd^{3+} ions normally occupies the Y^{3+} ions in YAG and provides the energy levels for both lasing transition and pumping. This rod is placed inside a highly reflecting elliptical cavity.

Elliptical reflector arrangement

Fig (16) consists of an elliptically cylindrical reflector arrangement. It has laser rod at one focus and krypton flash lamp at other focus. The light emerging from one foci after reflection passes through other foci so that the light energy is completely utilised for the excitation of electrons. An optical source like krypton flash lamp is placed near rod in such a way that most of the radiations from the flash tube passes through the Nd-YAG due to the elliptical cavity. The two ends of the rod are polished and silvered and constitute the optical resonator where one end (M_1) is 100% reflecting and other end (M_2) is partial reflector. It is used to increase the efficiency of the output wave.

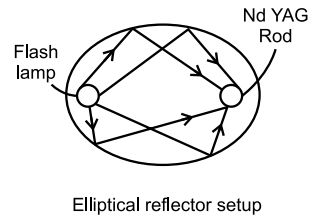


Fig. 16

Working

- i) The Krypton flash lamp is switched on and the light is allowed to fall on Nd-YAG rod
- ii) The intense white light excites the ground state Nd^{3+} ions to higher states
- iii) The excited Nd^{3+} ions quickly make, downward transitions from the upper energy bands. So the metastable state E_3 will be rapidly populated. This state is called upper laser level
- iv) The lower laser level E_2 is far above from the ground state and hence it cannot be populated by Nd^{3+} ions. Therefore, the population inversion is achieved between E_3 and E_2 levels.

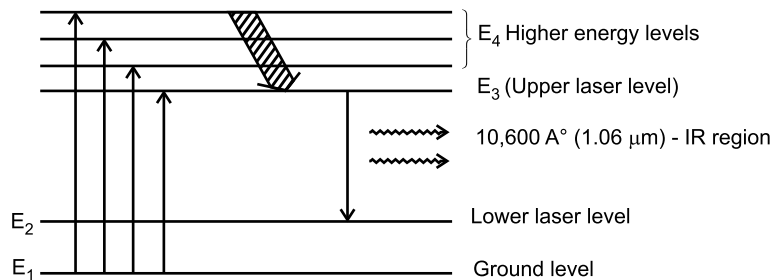


Fig. 17 Energy level diagram

- v) Hence pulsed form of laser beam of wavelength $1.06 \mu\text{m}$ is emitted during transmission.
- vi) The cooling arrangement is made further by blowing air (or) circulating water over the crystal to avoid the heat produced by the lamp

3.16 *Applied Physics*

Characteristics of Nd-YAG Laser

Type	- Doped insulator laser (Solid State)
Active medium	- Yttrium - Aluminium Garnet ($Y_3Al_5O_{12}$)
Active centre	- Neodymium (Nd^{3+} ions)
Pumping method	- Optical pumping
Pumping source	- Xenon flash lamp
Optical resonator	- Ends of the rods polished with silver and two mirrors (one of them is totally reflecting and other is partially reflecting)
Power output	- 2×10^4 watts
Nature of output	- pulsed
Wave length Emitted	- $1.06 \mu m$

3.7 Semi Conductor Laser

Active medium	: P-N junction diode
Active center	: Recombination of electron & holes
Pumping process	: Direct pumping
Optical resonator	: Junction of diodes polished

Introduction

The most compact of all lasers is the semiconductor diode laser.

Laser materials used

There are two types of semiconductors

- (i) Direct band gap
- (ii) Indirect band gap

In direct band gap, there is a huge possibility for direct recombination of holes & electrons which leads to emission of a photon. In indirect band gap semiconductor, no direct recombination is possible, and the recombination does not result in a photon. Hence direct band gap semiconductors are used in the construction of lasers and the wavelength of emitted light depends on the band gap of the material.

Principle

The electron in conduction band combines with a hole in the valence band & hence the recombination of electron and hole produces energy in the form of light. This photon, in turn may induce another electron in conduction band to valence band and thereby stimulate the emission of another photon.

Construction

The active medium is a PN junction diode made from gallium arsenide in which *P* region is doped with *Ge* and *N* region with tellurium. The thickness of PN junction layer is very narrow so that the emitted laser radiation has large divergence. The junction of '*P*' & '*N*' are well polished and are parallel to each other. Since the refractive index of GaAs is high, it acts as optical resonator so that the external mirrors are not needed. The upper and lower electrodes fixed in '*P*' and '*N*' region helps for the flow of current to the diode while biasing.

Working

Population inversion in PN junction is achieved by heavily doping the material. If the junction is forward biased with an applied voltage, direct conduction takes place. Due to high current density, active region is generated near the depletion region. At this junction, if a radiation of frequency ν is made to incident, then photo emission takes place. Further the emitted photons increase the rate of recombination of injected electrons from '*N*' to '*P*' region. Hence emitted photons are in phase and has same frequency & will amplify to get intense beam of laser.

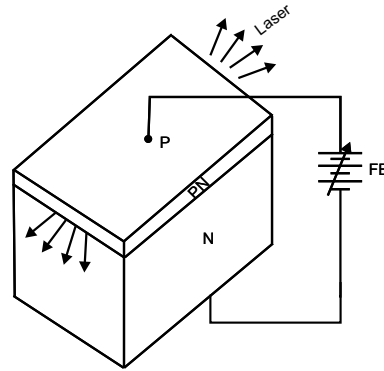


Fig. 18

Output wavelength

Bang gap of GaAs = 1.44 eV

$$E_g = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_g} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.44 \times 1.6 \times 10^{-19}} = 8628 \text{Å} \text{ (IR)}$$

3.8 Applications

1. It is used to drill hole in thick metal sheets.
2. Lasers are used for non destructive testing of metals.
3. They are used in molecular fusion, to accelerate chemical reaction, to cut and weld metals.
4. Semiconductor lasers are used in communication, to produce laser diodes and LED's.
5. Due to its low attenuation, CO_2 laser is used in open air communication.
6. Lasers ($He\ Ne$) are used to produce holograms and to determine size of tiny particles.
7. Lasers are used in LIDAR, remote sensing etc.
8. Lasers are also used in neurosurgery, microsurgery, treatment of liver, lungs and bloodless operations.

9. They are also used in elimination of moles and tumours on the skin surface.
10. They are also used in plastic surgeries, and in ophthalmology etc.

3.8.1 Application of laser in CD - ROM

Optical disks have much higher recording densities than conventional magnetic disks. A compact disc read only memory (CD ROM) is prepared by using a high power laser to burn one million (10^{-6} m) holes in a master disk. From this master a mold is made. This mold is used to make copies on plastic disks. A thin layer of Aluminium is then deposited on the surface, followed by a transparent plastic layer for protection. The holes are called pits and the unburned areas between the pits are called lands. They are distinguishable by their differences in reflectivity. The operation of all optical storage devices depends on laser technology.

A CD-ROM is a rigid plastic platter about 1.2 mm thick and 12 cm across with a centre spindle hole that is 1.5 cm in diameter. Data is written to the disk by burning pits in the recording surface with a carefully focussed laser. Each bit is roughly 0.12 mm micrometre deep and about 0.6 micrometers in diameter. CD-ROM tracks are $1.6\mu\text{m}$ apart for a track density of 16,000 tracks/inch. The floppy disk has a track density of 96 tracks/inch. Data begins at the continuous spiral and it has 2,70,000 data blocks for a total capacity of 553 megabytes. The spiral track is divided into blocks of the same size as shown in fig. A high intensity laser beam is used to store the data, while a low intensity laser beam is used to read the data.

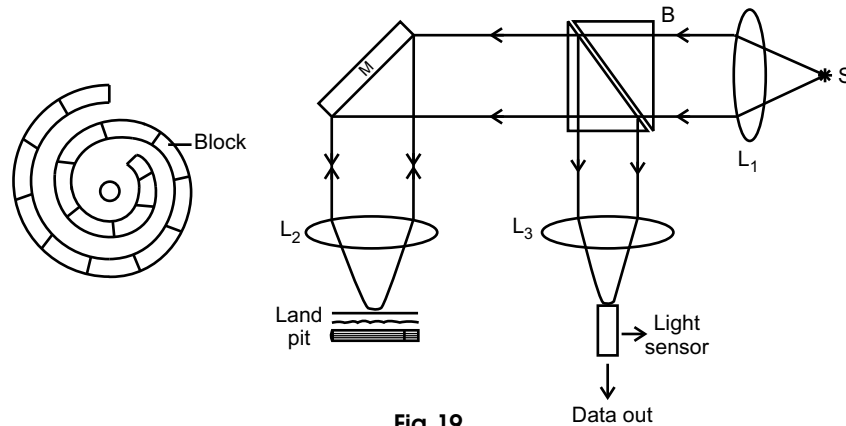


Fig. 19

3.8.2 3D Profiling

Definition

The process of recording and analysing three dimensional character of a particular object using laser scanner and computer software is known as 3D Profiling.

Principle

When an object is illuminated by laser beam, each point on the object acts as a source of secondary waves. These waves spread in all directions and form interference pattern. This interference pattern characterises the object and it is recorded using suitable method. The recorded image is again illuminated by using another coherent laser source. This results in reproduction of the original object in 3 dimension. The recording medium records not only the amplitude distribution but also the phase of the electromagnetic wave scattered from the object.

The set up consists of

- i) Laser source
- ii) Detector array
- iii) Detector scanner
- iv) Computer

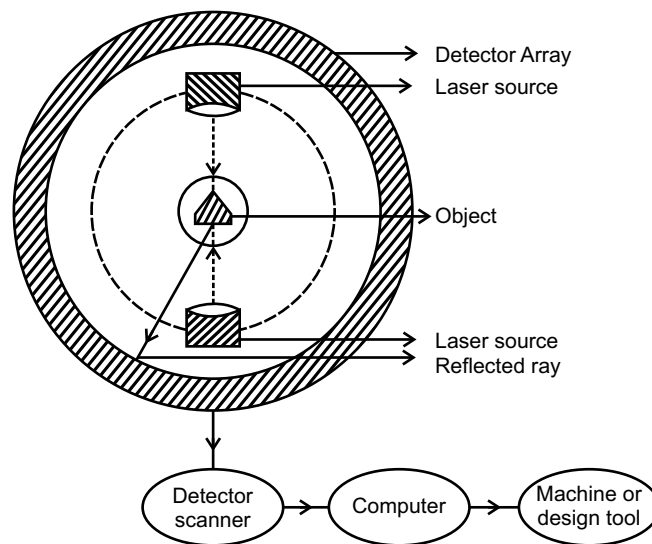


Fig.20 Shows the designing of 3D profile using laser and computer

i) Laser source

A laser source having high monochromaticity, intensity, coherence and directionality is used as a source to get the 3D images of a particular object. The object is placed at the centre of the orbit of the laser source and it is fixed one. The source is movable on circular scale and it can be positioned at any angle over 360° . Thus, the reflected light rays spread in all directions.

ii) Detector array

It is a collection of sensing diodes. These diodes are known as detectors. When the light from object is reflected and falls on these diodes, they sense and produce an equivalent analog pulse.

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iii) Detector scanner

It is going to scan the object in all three planes with the help of received signals from Detector Array. These signals are converted into its equivalent digital from using an analog to digital convertor which is attached to the scanner. The received signals contains both amplitude and phase variation which reflects the nature of the object.

iv) Computer

The scanner signals (digital) are supplied into a computer to stimulated the parallel signals. These signals or data are processed with suitable software and the commands are given to machine by the computer.

Working

The object is placed at centre of the orbit and the laser source is switched on. The laser source will revolve in a circular orbit so that light falls on all parts of the object. The object reflects the light rays and these rays are incident on the detector array. The diodes in the detector array convert the optical signal into electrical signal. These signals are scanned by detector scanner in all the three planes. The digital information about the object from the scanner is given as input to the computer. Now the computer converts these in the form of matrix. Optical signal processors in the computer reconstruct the image of the object in 3-Dimensional mode and the image is stored in the form of digits. In this way we are able to get the 3D image by using laser and the computer. The commands from the computer are given to the respective machine in order to design the given object.

Comparison of types of lasers

Laser	Ruby	He-Ne	CO ₂	Semiconductor (Ga - As)
Type	Solid state	Gas	Molecular gas	Semiconductor
Active medium	Ruby (Al ₂ O ₃ - Cr ₂ O ₃)	He + Ne in ratio 10:1	CO ₂ + N ₂ + He	P - N junction diode
Active center	Chromium	Neon	CO ₂	Recombination of e ⁻ and holes
Pumping method	Optical pumping	Electrical pumping	Electric discharge method	Direct pumping
Optical resonator	Ends of rods polished with silver	Pair of concave mirrors	Mirrors coated with Al	Junction of diodes polished
Power output	10 ⁴ - 10 ⁵ Watts	0.5-50 mW	10KW	1mW
Nature of output	Pulsed	Continuous	Continuous or pulsed	Continuous
Wave length	6943 Å°	6943 Å°	9.6 & 10.6 μm	8400 Å° - 8600 Å°

Fiber optics

Fiber optics deals with light propagation through this glass fibres. Fibre optics plays an important role in the field of communication. The transmission of light via a dielectric wave guide structure was first proposed or investigated at the beginning of twentieth century. A transparent dielectric rod, typically of silica glass with refractive index 1.5, surrounded by air, proved to be an impractical wave guide due to its unsupported structure of excessive losses at any discontinuities of glass air interface. Nevertheless, interest in the application of dielectric optical wave guides in such areas as optical imaging & medical diagnosis led to proposal of clad, surrounding the glass.

Light energy travels in both the core of cladding allowing the associated fields to decay to a negligible value at the cladding air interface. Loss is about 100 dB/km. Tremendous efforts to reduce the attenuation by purification of the materials in which loss was about 4.2 dB/km. Most of this work was focussed on 0.8 to 0.9 μm wavelength band, because the first generation optical sources fabricated from gallium Aluminium Arsenide operates in this region. However, as silica fibers, were investigated at longer wave length (1.1 to 1.6 μm) resulted in lower losses of reduced signal dispersion. Around 1.55 μm , fiber loss = 0.2 dB/km. Recent regions Mid IR (2 to 5 μm) and IR (8 to 12 μm).

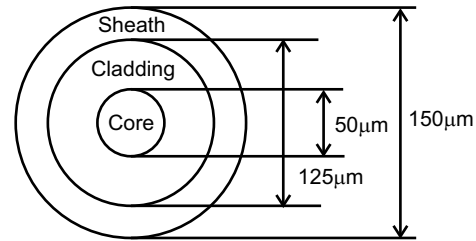


Fig. 21

For transmission mechanism of optical fibers with dimension approximate to those of a human hair, it is necessary to consider it as a cylindrical glass fiber. Such fiber acts as an open optical wave guide, which may be analysed utilizing simple ray theory. Discussion of light propagation in optical fibers using ray theory approach is essential in order to develop some of fundamental parameters associated with optical fiber communication.

3.9 Ray Theory Transmission

3.9.1 Total internal reflection

To consider propagation of light it is necessary to take account of refractive index of dielectric medium. Refractive index is defined as ratio of the velocity of light in vacuum to velocity of length of light in medium. Light travels much slower in denser medium rather than rarer medium.

Consider a ray travelling from a medium of refractive index n_1 to a medium of index n_2 , such that $n_1 > n_2$ Fig.22. Let the angle of incidence be ϕ_1 and angle of refraction be ϕ_2 . As it enters the second medium, it bends away from the normal. If the value of ϕ_1 is increased, the refracted ray still bends away from the normal. For a particular value of ϕ_1 , the angle of refraction will become 90° . For any value above

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this value of ϕ_1 , the ray comes back to the same medium. This mechanism is called total internal reflection.

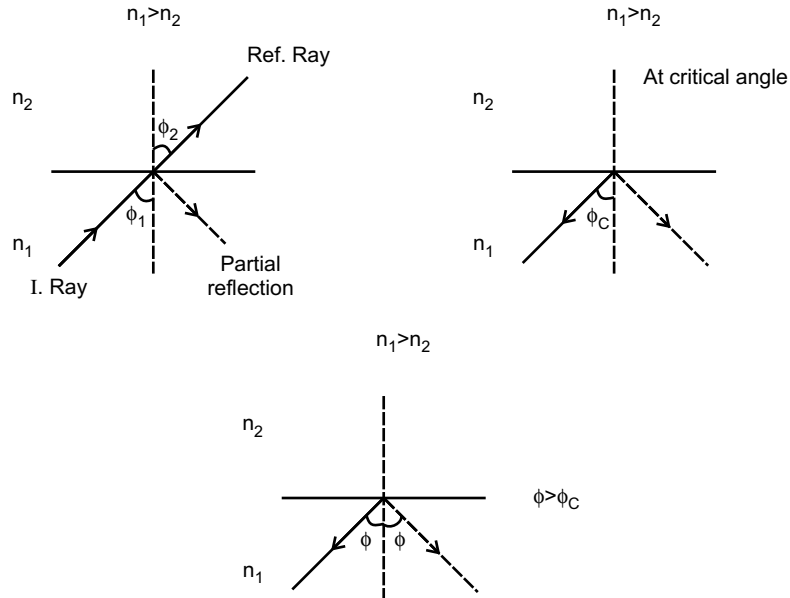


Fig. 22

From Snell's law

$$n_1 \sin \phi_1 = n_2 \sin \phi_2$$

$$\frac{\sin \phi_1}{\sin \phi_2} = \frac{n_2}{n_1}$$

at $\phi_1 = \phi_C; \phi_2 = 90^\circ$

$$\frac{\sin \phi_C}{\sin 90} = \frac{n_2}{n_1}$$

$$\sin \phi_C = \frac{n_2}{n_1}$$

$$\phi_C = \sin^{-1} \frac{n_2}{n_1}$$

(13)

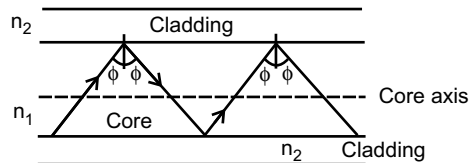


Fig. 23

where ϕ_C is called critical angle defined as the angle of incidence at which the angle of refraction is 90°

This is the mechanism by which light at a sufficiently low angle, may be considered to propagate down an optical fibers with low loss. Transmission of light in optical fibre cable is via series

3.9.2 Acceptance angle

A meridional say A at critical angle ϕ_C , enters the fiber at an angle θ_a , to the fiber axis is total internally reflected at air core interface before transmission core cladding interface at ϕ_c .

Any ray which is incident into fiber core at an angle greater than θ_a will be transmitted to core cladding interface at an angle less than ϕ_C .

Thus for rays to be transmitted by total internal reflection, within the fiber core, must be incident on the fiber core with an acceptance cone defined by conical half angle θ_a .

Hence acceptance angle is the maximum angle to the axis at which light may enter the fiber in order to be propagated, within core region and to suffer total internal reflection at core cladding interface.

Consider a ray entering the fibre at θ_1 and let the fibre be placed in a medium of index n_0 . The core index is n_1 and that of cladding is n_2 (Fig.25). Applying Snell's law for air core interface $n_0 \sin \theta_1 = n_1 \sin \theta_2$

Now; in $\triangle ABC$

$$\phi = \frac{\pi}{2} - \theta_2. \Rightarrow \theta_2 = \frac{\pi}{2} - \phi$$

$$n_0 \sin \theta_1 = n_1 \sin \left(\frac{\pi}{2} - \phi \right) = n_1 \cos \phi \quad (14)$$

$$\text{We know, } \sin^2 \phi + \cos^2 \phi = 1 \Rightarrow \cos \phi = (1 - \sin^2 \phi)^{1/2}$$

$$n_0 \sin \theta_1 = n_1 (1 - \sin^2 \phi)^{\frac{1}{2}} \quad (15)$$

Applying the condition, when $\theta_1 = \theta_a$; $\phi = \phi_C$ to Eq.(15)

$$n_0 \sin \theta_a = n_1 (1 - \sin^2 \phi_C)^{\frac{1}{2}} \quad (16)$$

Substituting from Eq.(13)

$$n_0 \sin \theta_a = n_1 \left(1 - \left(\frac{n_2}{n_1} \right)^2 \right)^{\frac{1}{2}} \quad (17)$$

$$= n_1 \left(\frac{n_1^2 - n_2^2}{n_1^2} \right)^{\frac{1}{2}}$$

$$n_0 \sin \theta_a = n_1 \frac{(n_1^2 - n_2^2)^{\frac{1}{2}}}{n_1}$$

$$\sin \theta_a = \frac{(n_1^2 - n_2^2)^{\frac{1}{2}}}{n_0} \quad (18)$$

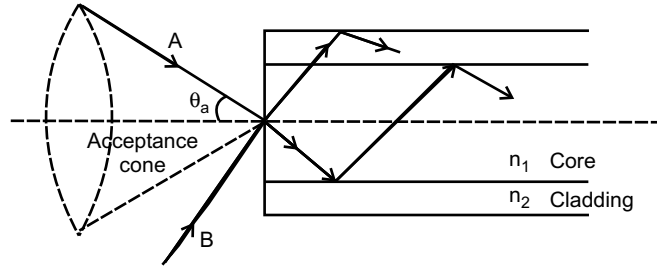


Fig. 24

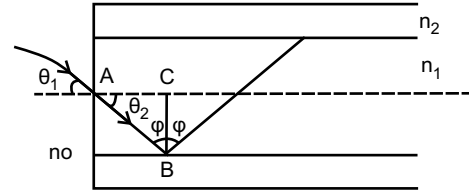


Fig. 25

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$\sin \theta_a$ is called **Numerical Aperture (NA)** which defines the light collecting ability of a fibre and is an inherent property of a fibre.

For air $n_0 = 1$

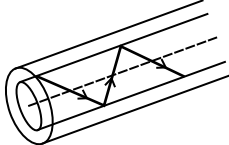
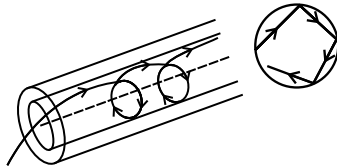
$$NA = \sin \theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}}$$

Fractional refractive index change (Δ)

The fractional difference Δ between the refractive indices of the core and cladding is known as fractional refractive index change.

$$\Delta = (n_1 - n_2)/n_1$$

Types of rays propagating through optical fibres

Meridional ray	Skew ray
<p>Light rays passing through the fibre axis are known as meridional rays. It follows zig zag path They propagate only in step index fibre Small acceptance angle and NA</p>	<p>Rays that are transmitted without passing through the fiber axis. It follows a helical path. They propagate through graded index fibre. Larger acceptance angle and NA</p>
	

3.10 Modes of Propagation

Features	Single mode	Multi mode
Diameter of core	2 to 8 μm	50 mm
No. of modes	1	> 1
Band width	wide	narrow
Output	Faithfully follows input	distorted output
Number of modes	1	$N = 4.9 \left(\frac{dn_1 \sqrt{2\Delta}}{\lambda} \right)^2$
Signal degradation	Less	More
Fabrication	Difficult	Easier

For standard fiber the core is about 50 μm . As this dimension is very large in comparison with wave length of light, there are many possible modes (ways) of electro magnetic field. Thus light will follow a number of paths through the fiber. Optical power is distributed among several modes with different velocity. Two modes of propagation are allowed, single mode and multimode.

3.11 Fiber Materials

Three material systems are usually used.

1. High content silica glass
2. Multi component glass
3. Plastic

Requirements of a good fibre material

1. It must be possible to make long thin flexible fibers from the material
2. Material must be transparent at a particular wavelength to guide light efficiently
3. Physically compatible materials having slightly different refractive indices should be available.

Glass fibers

Basic material used : Silica (SiO_2) at $n = 1.458$ at 850nm

Dopants used: B_2O_3 , GeO_2 or P_2O_5

P_2O_5 or GeO_2 increases the materials index fluorine or B_2O_3 decreases the material refractive index.

Using the above, various core clad materials can be prepared.

e.g.,

1. $\text{GeO}_2 - \text{SiO}_2$ as Core ; SiO_2 as Cladding
2. SiO_2 as Core ' $\text{B}_2\text{O}_3 - \text{SiO}_2$ as Cladding

Fluoride glass can be used which has extremely low transmission losses at 0.2 to 8 m wavelength. Recently Halide glasses can also be used which are mainly made of heavy metal fluoride glasses as main component.

(e.g) ZBLAN $\text{ZrF}_4 - 50\%$; BaF_2 ; 20%; $\text{LaF}_3 - 45\%$; $\text{AlF}_3 - 3.5\%$, $\text{NaF} - 18\%$

Core : ZBLAN Cladding: Z_4F_4 replaced by HaF_4

Plastic fibers

Silica and its polymer can be used in case of plastic fibers.

(e.g)

1. Silica as core and silicone resin as cladding ($n = 1.405$ at 850 nm)
2. Polyester core ($n = 1.60$) and Methyl methacrylate cladding ($n = 1.4$)

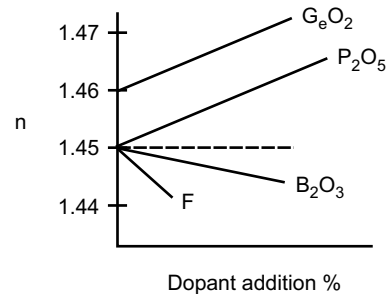
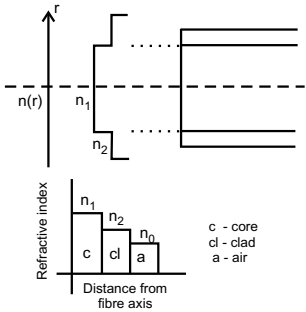
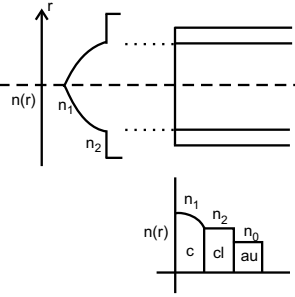


Fig. 26

3.12 Step Index Fiber and Graded Index Fiber

Based on refractive index profile, two types of fibers are available:

Step Index Fibre	Graded Index Fibre
1. Refractive index of core is uniform throughout and undergoes an abrupt or step change at the core cladding interface.	Refractive index of core is made to vary in the parabolic manner (i.e.,) maximum along the center of the core and decreases towards interface and becomes equal to that of cladding at the interface.
2. Diameter of core Multimode : $\approx 50 - 200 \mu m$ Single mode : $\approx 10 \mu m$	Multimode $\approx 50 \mu m$
3. Light travels in the form of meridional ray. (i.e.,) Ray will cross the fibre axis during every reflection at the interface and are propagating in a zig zag manner.	Light travels in the form of skew rays.(i.e.,) Ray will not cross fibre axis at any time but propagate around the fibre axis; propagates in helical or spiral manner.
4. Refractive index profile $n(r) = \begin{cases} n_1 & r < a \\ n_2 & r \geq a \end{cases}$	Refractive index profile $n(r) = \begin{cases} n_1(1 - 2\Delta (\frac{r}{a})^\alpha)^{1/2} & r < a \\ n_1(1 - 2\Delta)^{1/2} & r \geq a \approx n_2 \end{cases}$ where α is refractive index profile
5. Signal distortion is more in multimode step index fibre	Signal distortion is low because of self focussing effect
6. Bandwidth Multimode $\approx 50 MHz$ Single mode $\approx 1000 MHz$	Multimode 200 - 600 MHz
7. Both single and multimode are possible	Only multimode propagation is possible
8. Attenuation more for multimode; very less for singlemode	Attenuation is less
9. NA Multi mode - more Single mode - less	NA is less
10. 	

3.13 Optical Fibers as Wave Guides

A wave guide is one of the components in a communication system and is used for effective transmission of the signals with any attenuation. Thus it guides the signal properly so that the signal can be received at the receiver end without much loss.

A wave guide should have the following properties:

1. It should transmit the signal without any loss and any overlap.
2. It should have the ability to propagate at desired frequency.
3. There should not be any electromagnetic interference, mode conversion, mixing, noise or cross talk.
4. There should not be any leakage and proper security should be ensured.
5. It should have high bandwidth, flexible and light weight.

An optical fiber can satisfy the above properties and hence they can function as optical wave guides.

3.14 Sensors

Optical techniques are widely used for non contact measurements. Optical fiber sensor technology is very important in the field of instrumentation, due to their high reliability, security and immunity to electro magnetic interference.

If the guided light in the fiber gets modulated by the variable to be measured and demodulated, then they are intrinsic or active sensor. If modulation takes place outside the optical fiber and merely transmitted, then they are extrinsic or passive sensors.

3.14.1 Intensity modulated temperature sensor (IMT)

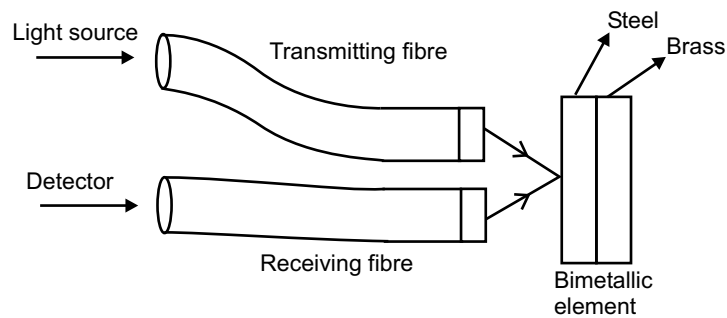


Fig. 27

3.28 Applied Physics

It is based on reflective concept. Here the bimetallic strip acts as the sensing element. It consists of steel and brass welded together in the form of a strip. Since linear expansion of brass is more than steel, the strip bends in the form of an arc. Metal with higher linear expansion will be outside of the arc when heated.

The strip is attached to a bifurcated reflective fibre optic probe. Strip is designed to move continuously and its movement is directly proportional to temperature.

The amount of light reflected is converted into voltage by a photo diode. Fig.24 shows linear response of the sensor. Amount of light reflected decreases with increase in temperature, so that the output of the photodiode decreases with increase in temperature.

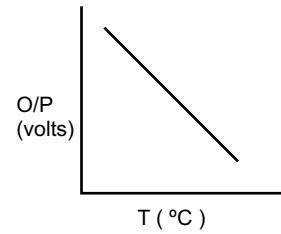


Fig.28

3.14.2 Phase modulated temperature sensor (PMT)

Here, Phase shift produced in the sensing fibre relative to reference fibre is a function of temperature.

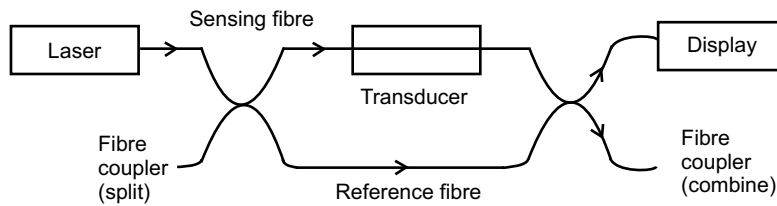


Fig.29

Light source is a semiconductor laser. A coupler splits the beam which travels through sensing and reference fibre. Another coupler combines these two beams and on display, alternate bright and dark bands appear. A change in phase of light in sensing fibre due to change in refractive index with respect to temperature with reference to phase of light in the reference fibre appears as a displacement fringe pattern. A phase change of 2π radians causes a displacement of the fringe pattern. By counting the fringe displacement the magnitude of temperature can be determined.

3.15 Application of Optical Fibre

Endoscope

It is a tubular optical instrument. An endoscope is used by physicians to view the internal parts of the human body. This imaging can help surgeons decide the procedure to be followed for treating the patients.

Construction and working

A typical endoscopy is shown schematically in Fig.30.

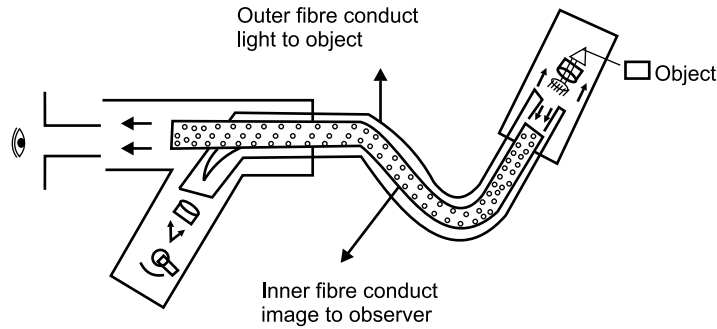


Fig.30

Usually in the endoscope, there are two fibres namely inner fibre and outer fibre. The outer fibre is used to collect the reflected light from that area and using this we can see the inner structure of the object. The inner fibre is used to illuminate the inner structure of the object under study. The ends of the two fibres are provided with suitable optical components to transmit and receive images.

The two fibres together form the endoscopy tube. One end of the endoscopy tube remains outside and the other end is sent into human body.

There is an optical light source at the end of the outer fibre for transmitting light to illuminate the internal part of the body.

The light collecting and image viewing optical arrangements are placed at the respective ends of the inner fibre. So, the other fibre transmits the image of the part of the body to an imaging system coupled outside.

Different types of commonly available endoscopes

	Type	Range of use	Diagnostic problems
(1)	Ophthalmoscope	Eye defects	Retinal detachment
(2)	Gastroscope	Stomach	Gastritis, gastric ulcer, tumours
(3)	Cardioscope	Heart cavities	Spetal defects, valvular defects
(4)	Cystoscope	Urinary bladder	Small stones, inflammation, tumours etc.

□ Solved Problem 1

Prove that laser action is not possible in optical frequencies under thermal equilibrium. ($\lambda = 5000 \text{ \AA}$; $T = 300K$)

3.30 Applied Physics

► Solution

We know

$$\begin{aligned}\frac{R_{sp}}{R_{st}} &= e^{h\nu/kT} - 1 \\ &= \exp\left(\frac{6.625 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10} \times 1.38 \times 10^{-23} \times 300} - 1\right) \\ &= 5.02 \times 10^{41} \\ \Rightarrow R_{sp} &> R_{st}\end{aligned}$$

∴ It is not possible.

□ Solved Problem 2

A CO₂ laser source emits light at a wavelength of 9.6 μm and has an output power of 10 kW. How many photons are emitted in each hour by this laser while operating?

► Solution

$$\begin{aligned}\nu &= \frac{c}{\lambda} = \frac{3 \times 10^8}{9.6 \times 10^{-6}} = 3.125 \times 10^{13} \text{ Hz} \\ E &= h\nu = 6.62 \times 10^{-34} \times 3.125 \times 10^{13} \\ &= 2.07 \times 10^{-20} \text{ J} \\ \text{Energy emitted} &= 10 \text{ kW} = 10 \times 10^3 \text{ W} \\ &= 10^4 \text{ J/s} = 10^4 \times 60 \times 60 \text{ J/hr} \\ \therefore \text{No. of photons} &= \frac{10^4 \times 60 \times 60}{2.07 \times 10^{-20}} = 1.739 \times 10^{27} \text{ ph/hr}\end{aligned}$$

□ Solved Problem 3

The first line of the principal series of sodium D line is at 5890 Å. This corresponds to a transition from the first excited state to the ground state. What is the energy in electron volts of the first excited state?

► Solution

$$\begin{aligned}E &= h\nu = \frac{hc}{\lambda} \\ &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{5890 \times 10^{-10}} \\ &= \frac{3.374 \times 10^{-19}}{1.602 \times 10^{-19}} = 2.106 \text{ eV}\end{aligned}$$

□ Solved Problem 4

Calculate the total number of guided modes propagating in multimode step index fiber having diameter of $60 \mu\text{m}$ and NA of 0.25 operating at a wavelength of $2.7 \mu\text{m}$.

► Solution

$$\begin{aligned} N_{\text{step}} &= 4.9 \left(\frac{d \times NA}{\lambda} \right)^2 \\ &= 4.9 \left(\frac{60 \times 10^{-6} \times 0.25}{2.7 \times 10^{-6}} \right)^2 \\ &\approx 151 \text{ nodes} \end{aligned}$$

□ Solved Problem 5

A step index fibre has a NA of 0.26, a core refractive index of 1.5 and a core diameter of $100 \mu\text{m}$. Calculate

- Cladding refractive index
- Acceptance angle
- Maximum no of modes with a wavelength of $1 \mu\text{m}$ that the fibre can carry.

► Solution

$$\begin{aligned} \text{(a) } NA &= \sqrt{n_1^2 - n_2^2} \\ 0.26 &= \sqrt{1.5^2 - n_2^2} \\ \Rightarrow n_2 &= 1.4772 \end{aligned}$$

(b) Acceptance angle

$$\begin{aligned} \sin \theta_a &= \frac{NA}{n_0} = \frac{0.26}{1} = 0.26 \\ \Rightarrow \theta_a &= 15.07^\circ \end{aligned}$$

(c) No. of modes

$$\begin{aligned} N &= 4.9 \left(\frac{d \times NA}{\lambda} \right)^2 \\ &= 4.9 \left(\frac{100 \times 10^{-6} \times 0.26}{1 \times 10^{-6}} \right)^2 \\ &\approx 3312 \text{ modes.} \end{aligned}$$

Short Questions

1. Write the characteristics of lasers.
2. Distinguish spontaneous and stimulated emission
3. What are Einstein coefficient.
4. State the principle of laser action.
5. Define population inversion.
6. What do you mean by pumping process.
7. Why population inversion is done only by artificial process.
8. What do you mean by active medium, active centre and active material.
9. List out various pumping method.
10. What do you mean by optical pumping.
11. Explain inelastic atom atom collision.
12. Optical pumping cannot be used in the case of gas lasers - Why?
13. Why direct band gap semiconductor are used in manufacturing of laser diodes?
14. List out some application of lasers.
15. Explain the role of He in CO_2 lasers.
16. Explain the role of He in He-Ne lasers.
17. Discuss the three modes of vibration of CO_2 lasers.
18. Give the characteristics features of optical fibers.
19. What is the principle used in light propagation through fibers.
20. What is total internal reflection.
21. Define acceptance angle.
22. Define numerical aperture.
23. What is fractional index change. Write the expression for it.
24. List out of types of optical fibers.
25. State the differences between single mode and multi mode fiber.
26. What is step index fiber.
27. What is graded index fiber.
28. State the differences between step index fiber and graded index fiber.
29. What is extrinsic sensor.
30. What is intrinsic sensor.
31. Write a note on optical fiber materials.

Review Questions

1. Describe the construction and working of Ruby laser with neat diagrams.
2. Describe the construction and working of He-Ne with necessary theory and diagrams.
3. Giving necessary theory, obtain an expression for Einstein coefficients
4. Discuss the various method employed to activate population inversion.
5. With neat diagram explain the different modes of vibration of CO_2 molecule. Explain the construction and working of CO_2 laser and give its application.
6. Explain the construction and working of semiconductor laser and give its application.
7. Using Einstein's A and B coefficients, justify that population inversion and stimulated emission process are required for laser action.
8. Classify the optical fiber on the basis of materials modes of propagation and refractive index differences. Explain them in detail.
9. Explain the propagation of light through optical fiber.
10. Derive an expression for numerical aperture and angle of acceptance of a fiber in terms of refractive indices of the core and cladding.
11. Explain the applications of optical fiber as waveguide and sensors.
12. Explain the working of PMT sensor and IMT sensor.

Exercise Problems

1. Calculate how many photons are emitted in each minute in a He Ne laser source, which emits light at wavelength of 6328 \AA . The output power of this source is 3 mW. (Ans: 5.732×10^{17} ph/min)
2. Examine the possibility of MASER action at thermal equilibrium at wavelength of 10 cm and $T = 300 \text{ K}$, using Einstein's relations.
3. Determine the numerical aperture of a step index fibre when the core refractive index n_1 is 1.5 and cladding refractive index n_2 is 1.48. Find the acceptance angle. (Ans: 14.13°)
4. Estimate NA, when the core index is 1.48 and relative refractive index is 2%. Also calculate the critical angle. (Ans: $0.296, 78.52^\circ$)
5. A fibre has a diameter of $6 \mu\text{m}$ and its core refractive index is 1.47 and for cladding it is 1.43. How many modes can propagate into the fibre if the wavelength of the laser source is $1.5 \mu\text{m}$. (Ans: 9)
6. Calculate the refractive indices of the core and cladding of a fibre from the following data: $NA = 0.27$; $\Delta = 0.015$. (Ans: $1.558, 1.534$)

CHAPTER

4

Modern Physics

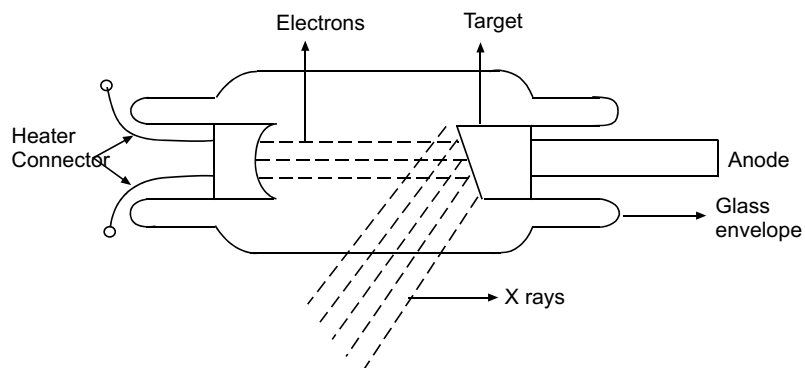
X-ray Instrumentation

In 1895 Conrad Rontgen, a German physicist discovered a previously unknown type of radiation while experimenting with gas discharge tube. He found that this type of radiation could actually penetrate through opaque objects and provide an image of their inner structures. These rays are called X-rays.

Soon after the discovery of X-rays, their importance as a tool for medical diagnosis was recognized. Later it was found that X-rays could also be used for therapeutic purpose. Both applications of X-rays are the domain of the medical speciality known as radiology.

4.1 Generation of X-rays

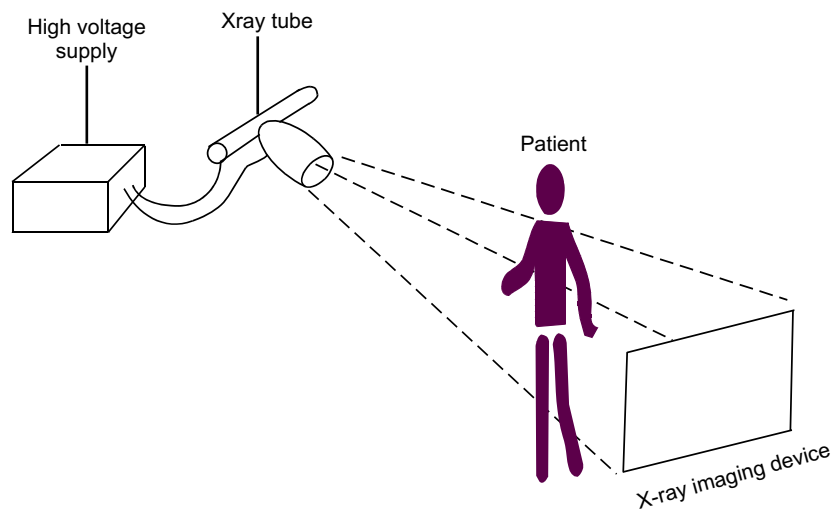
X-rays are generated when fast moving electrons are suddenly decelerated by impinging on a target. An X-ray tube is basically a high vacuum diode with a heated cathode located opposite a target anode. This diode is operated in the saturated mode with a fairly low cathode temperature so that the current through the tube does not depend on the applied anode voltage.



4.2 Applied Physics

The intensity of X-rays depends on the current through the tube. The current can be varied by varying the heater current which in turn controls the cathode temperature. The wave length of the X-rays depends on the target material and the velocity of the electrons hitting the target. It can be varied by varying the target voltage of the tube. X-ray equipment for diagnostic purposes uses target voltages in the range of 30 to 100KV while the current is in the range of several hundred milliamperes. These voltages are obtained from high voltage transformers that are often mounted in oil filled tanks to provide electrical insulation. When an ac voltage is used the X-ray tube conducts only during one half wave and acts as its own rectifier. Other wise high voltage diodes are used as rectifiers. For therapeutic X-ray equipment where even higher radiation energies are required, linear or circular particle accelerators have been used to obtain electrons with sufficiently high energy.

When electrons strike the target only a small part of their energy is converted in to X-rays most of it is dissipated as heat. The target therefore is usually made of tungsten which has a high melting point. It may also be water or air cooled, or it may be in the form of motor driven rotating cone to improve rotation of heat. The electron beam is concentrated to form a small spot on the target. The X-rays emerge in all directions from this spot.



4.2 X-rays

X-rays are electromagnetic waves of shorter wavelength ranging from 10 Å to 0.5 Å. The longer wavelength of the X-ray spectrum is known as soft X-rays and shorter wavelength as hard X-rays.

X-Rays are produced when the fast moving electrons are suddenly stopped by a solid target

Properties

- i) They travel in straight line with the velocity of light.
- ii) X-rays are not deflected by electric and magnetic fields. This indicates that X-rays do not consist of charged particles. This property distinguishes X-rays from cathode rays.
- iii) They affect photographic plate.
- iv) On passing through gas, X-rays ionise the gas.
- v) X-rays produce fluorescence in many substance like barium platinocyanide, cadmium, tungsten etc,
- vi) X-rays are highly penetrating. They can pass through many solids which are opaque to ordinary light like wood, flesh, paper, etc,
- vii) They exhibit the wave nature in a manner similar to light i.e., they undergo reflection, refraction, interference, diffraction and polarisation.
- viii) They produce photo electric effect.

4.3 Bragg's Law

When monochromatic X-rays strike the atoms in a crystal lattice, each atom acts as a source of scattering radiation of the same wavelength. The crystal acts as a series of parallel reflecting planes. The intensity of the reflected beam at certain angle will be maximum when the path difference between two reflected waves from two different planes is an integral multiple of λ .

$$2d \sin \theta = n\lambda$$

Where,

d is inter planar distance, θ is the glancing angle and λ is the wavelength of spectrum.

Derivation

Consider a set of parallel planes in a crystal spaced by d , let a narrow monochromatic X-ray beam of wavelength λ be incident on the first plane at a glancing angle θ . Consider ray 'Q' incident on the second plane and $Q'R'$ is the reflected ray.

To determine path difference between the two rays PQR and $P'Q'R'$, draw normals from the point Q to the lines $P'Q'$ and $Q'R'$. Let the normals be QT and QS .

The path difference between these two rays PQR and $P'Q'R'$ is

$$\text{path difference} = Q'T + Q'S$$

4.4 Applied Physics

In the triangle $QTQ' \Rightarrow \sin \theta = Q'T/QQ'$

$$Q'T = QQ' \sin \theta$$

$$Q'T = d \sin \theta$$

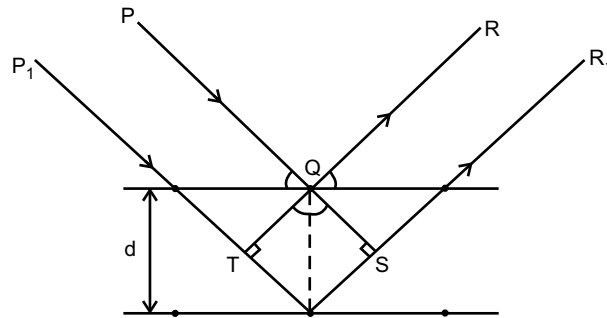
Similarly from triangles $QQ'S$ we can write

$$Q'S = d \sin \theta$$

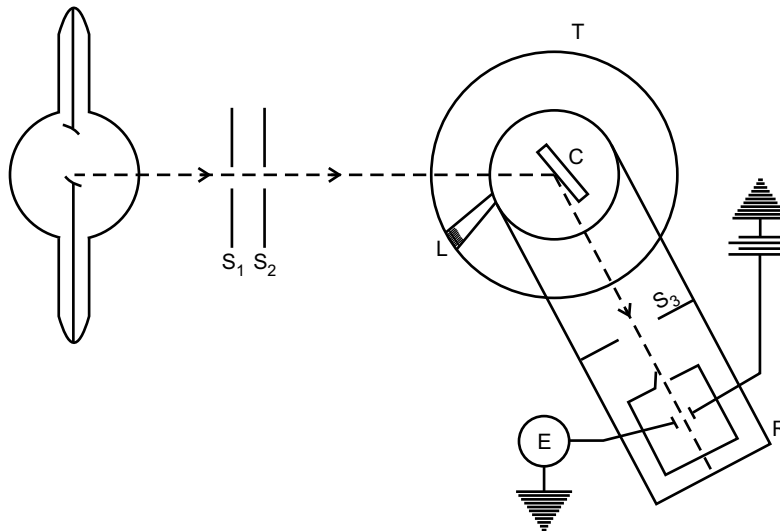
\therefore The path difference is

$$Q'T + Q'S = 2d \sin \theta$$

Hence the rays will reinforce each other and produce maximum intensity when the path difference is an integral multiple of λ



4.3.1 Bragg X-ray spectrometer



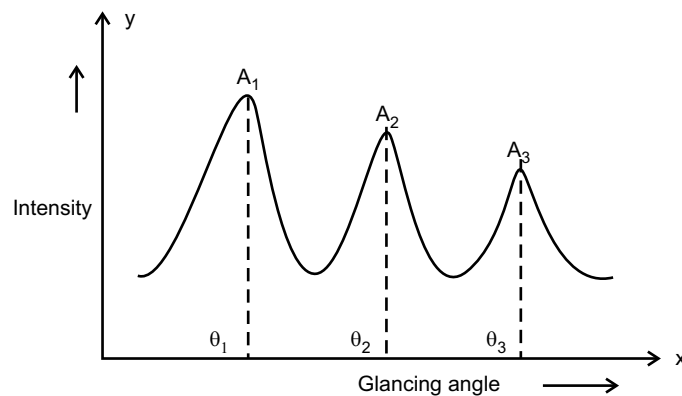
where,

S_1, S_2, S_3	-	Lead slits
R	-	Radial arm
C	-	Crystal
T	-	Turn table
E	-	Electrometer

The construction of Bragg x-ray spectrometer is similar to an optical spectrometer. It consists of

- i) Source of x-rays
- ii) A crystal held on a circular table which is graduated and provided with vernier and
- iii) A detector (ionisation chamber).

X-Rays from X-ray tube, limited by two narrow lead slits S_1 and S_2 are allowed to fall upon the crystal. The crystal is mounted on the circular table T, which can rotate about vertical axis and its position can be determined by vernier V. The table is provided with a radial arm (R) which carries an ionisation chamber. The chamber is connected to an electrometer (E) to measure the ionisation current. We can measure the intensity of diffracted beam of X-ray beam along chamber. In practice, the crystal table is geared to the ionisation chamber so that the chamber turns through 2θ when crystal is turned through an angle θ .



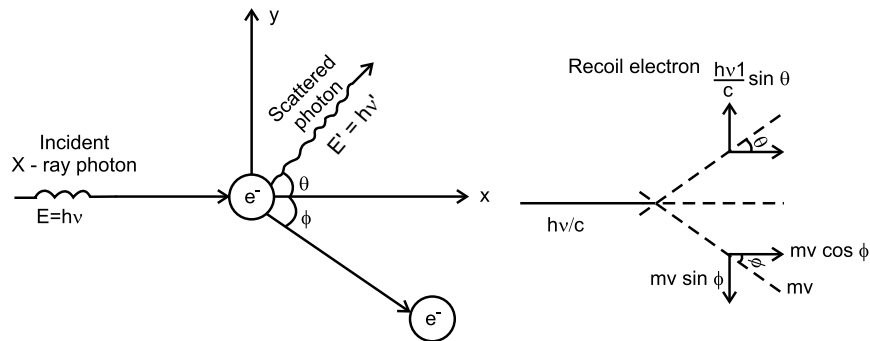
The glancing angle θ for the incident beam is kept very small. The chamber is adjusted to receive the reflected beam till the rate of deflection is maximum. The glancing angle θ and intensity I of the diffracted beam are measured and corresponding ionisation current is noted. The graph is called X-ray spectrum. The prominent peaks A_1, A_2, A_3 refer to X-rays of wavelength λ and the corresponding glancing angle $\theta_1, \theta_2, \theta_3$ for 1, 2, 3rd orders of same wavelength.

The wave length of X-rays is determined by using the equation $2d\sin\theta = n\lambda$. The glancing angle θ is experimentally determined for known order. If d is known, λ can be calculated.

4.4 Compton Effect

When a beam of monochromatic X-rays incident on a material having low atomic number like carbon they suffer a change of frequency on scattering. The scattered beam consists of two components; one has same wavelength as that of incident X-ray and the other one has slightly higher wavelength. The change of wavelength is due to loss of energy of the incident X-ray beam. This effect is called Compton effect.

Compton effect was explained on the basis of quantum theory of radiation. The whole process is treated as a particle collision between x-ray photon and a loosely bound electron of the scatterer. In this process both momentum and energy are conserved. In the photon-electron collision, a portion of the energy of the photon is transferred to the electron. As a result, the X-ray beam proceeds with less than the original energy



The incident photon with an energy $h\nu$ and momentum $h\nu/c$ strikes an electron at rest. The initial momentum of the electron is zero and its initial energy is only the rest mass energy m_0c^2 (i.e before collision). After collision, the scattered photon of energy $h\nu'$ and momentum $h\nu'/c$ moves at an angle θ w.r.t the original direction. The electron acquires a momentum mv and energy mc^2 moves at angle θ with the direction of incident photon

	Before collision	After collision
X-ray photon	Energy $h\nu$ Momentum $\frac{h\nu}{c}$	Energy $h\nu'$ Momentum $\frac{h\nu'}{c}$
Electron	Energy m_0c^2 Momentum 0	Energy mc^2 Momentum mv

According to principle of conservation of energy

$$\begin{aligned} \text{Total energy before collision} &= \text{Total energy after collision} \\ h\nu + m_0c^2 &= h\nu' + mc^2 \end{aligned} \quad (1)$$

Along x-axis

Momentum before collision = Momentum after collision

$$\begin{aligned}\frac{h\nu}{c} + 0 &= \frac{h\nu'}{c} \cos \theta + mv \cos \theta \\ \frac{h\nu}{c} &= \frac{h\nu'}{c} \cos \theta + mv \cos \phi\end{aligned}\quad (2)$$

Along Y-axis

T.M before collision = T.M after collision

$$0 = \frac{h\nu'}{c} \sin \theta - mv \sin \phi \quad (3)$$

From (2)

$$\begin{aligned}h\nu &= h\nu' \cos \theta + mvc \cos \phi \\ mvc \cos \phi &= h(\nu - \nu' \cos \theta)\end{aligned}\quad (4)$$

From (3)

$$\begin{aligned}\frac{h\nu'}{c} \sin \theta &= mv \sin \phi \\ h\nu' \sin \theta &= mvc \sin \phi\end{aligned}\quad (5)$$

Eqs.(4)² + (5)²

$$\begin{aligned}(mvc \cos \phi)^2 + (mvc \sin \phi)^2 &= (h(\nu - \nu' \cos \theta))^2 + (h\nu' \sin \theta)^2 \\ m^2v^2c^2 &= h^2(\nu^2 + \nu'^2 \cos^2 \theta - 2\nu\nu' \cos \theta) + h^2\nu'^2 \sin^2 \theta \\ m^2v^2c^2 &= h^2\nu^2 + h^2\nu'^2 \cos^2 \theta - 2h^2\nu\nu' \cos \theta + h^2\nu'^2 \sin^2 \theta \\ m^2v^2c^2 &= h^2\nu^2 + h^2\nu'^2(1) - 2\nu\nu'h^2 \cos \theta \\ m^2v^2c^2 &= h^2(\nu^2 + \nu'^2 - 2\nu\nu' \cos \theta)\end{aligned}\quad (6)$$

From (1)

$$\begin{aligned}h\nu + m_0c^2 &= h\nu' + mc^2 \\ mc^2 &= h\nu + m_0c^2 - h\nu' \\ mc^2 &= (h(\nu - \nu') + m_0c^2)\end{aligned}$$

Squaring on both sides

$$\begin{aligned}(mc^2)^2 &= (h(\nu - \nu') + m_0c^2)^2 \\ m^2c^4 &= (h(\nu - \nu'))^2 + m_0^2c^4 + 2h(\nu - \nu')m_0c^2 \\ m^2c^4 &= h^2(\nu^2 + \nu'^2 - 2\nu\nu') + m_0^2c^4 + 2h(\nu - \nu')m_0c^2\end{aligned}\quad (7)$$

4.8 Applied Physics

Eqs. (7) – (6)

$$\begin{aligned}
 m^2 c^4 - m^2 v^2 c^2 &= h^2(\nu^2 + \nu'^2 - 2\nu\nu') + m_0^2 c^4 + 2h(\nu - \nu')m_0 c^2 \\
 &\quad h^2(\nu^2 + \nu'^2 - 2\nu\nu' \cos \theta) \\
 m^2 c^2 (c^2 - v^2) &= -h^2 \cdot 2\nu\nu' + m_0^2 c^4 + 2h(\nu - \nu')m_0^2 c^2 + 2\nu\nu' h^2 \cos \theta \\
 m^2 c^2 (c^2 - v^2) &= 2h(\nu - \nu') m_0 c^2 - 2h^2 \nu\nu' + 2h^2 \nu\nu' \cos \theta + m_0^2 c^4 \\
 m^2 c^2 (c^2 - v^2) &= -2h^2 \nu\nu' (1 - \cos \theta) + m_0^2 c^4 + 2h(\nu - \nu') m_0^2 c^2 \quad (8)
 \end{aligned}$$

From Einstein's mass-relativistic equation, we can write

$$\begin{aligned}
 m &= \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (9) \\
 m^2 \left(1 - \frac{v^2}{c^2}\right) &= m_0^2 \\
 m^2 (c^2 - v^2) &= m_0^2 c^2
 \end{aligned}$$

Similarly by c^2 on both sides,

$$m^2 c^2 (c^2 - v^2) = m_0^2 c^4 \quad (10)$$

Use (10) in (8),

$$\begin{aligned}
 (8) \implies m_0^2 c^4 &= 2h^2 \nu\nu' (1 - \cos \theta) + m_0^2 c^4 + 2h(\nu - \nu') m_0 c^2 \\
 2h^2 \nu\nu' (1 - \cos \theta) &= 2h(\nu - \nu') m_0 c^2 \\
 \frac{h(1 - \cos \theta)}{m_0 c^2} &= \frac{\nu - \nu'}{\nu\nu'} \\
 \frac{h(1 - \cos \theta)}{m_0 c^2} &= \frac{1}{\nu'} - \frac{1}{\nu}
 \end{aligned}$$

Multiply by ' c ' on both sides

$$\begin{aligned}
 \frac{c}{\nu'} - \frac{c}{\nu} &= \frac{h(1 - \cos \theta)}{m_0 c} \\
 \lambda' - \lambda &= \frac{h(1 - \cos \theta)}{m_0 c} \\
 d\lambda &= \frac{h}{m_0 c} (1 - \cos \theta) \quad (11)
 \end{aligned}$$

The above equation gives the change in wavelength of the scattered x-ray photon.

Case 1: when $\theta = 0$;

$d\lambda = 0$. This means that there is no change in wavelength

Case 2: when $\theta = 90^\circ$;

$$\begin{aligned}
 d\lambda &= \frac{h}{m_0 c} = \frac{6.634 \times 10^{-34}}{(9.1 \times 10^{-31}) \times (3 \times 10^8)} \\
 d\lambda &= 0.024 \text{ \AA}
 \end{aligned}$$

This is known as Compton wavelength

Case 3: when $\theta = 180^\circ$; $\cos 180 = -1$

$$d\lambda = \frac{2h}{m_0c}$$

$$d\lambda = 0.048\text{\AA}$$

The above equation gives the maximum value of change in wavelength.

4.5 Introduction to Nucleus

The atomic nucleus was discovered by Rutherford in 1911. He showed that the atom consists of a very small nucleus. ($\approx 10^{-14}$ m diameter).

All atomic nuclei are made up of elementary particles called protons and neutrons. A proton has a positive charge of the same magnitude as that of an electron. A neutron is electrically neutral. The proton and the neutron are considered to be two different charge states of the same particles which is called a nucleon.

A species of nucleus, known as nuclide, is represented schematically by ${}_zX^A$ where z , the atomic number indicates the number of protons, A mass number, total number of protons plus neutrons and X is chemical Symbol of the Species.

$$N = \text{number of Neutrons} = A - Z$$

Isotopes

Isotopes are nuclei with the same atomic number z but different mass numbers A . The nuclei Si^{28} , Si^{29} , Si^{30} and Si^{32} are all of isotopes of silicon. All isotopes of an element have identical chemical behaviour and differ, physically only in mass.

Isobars

Those nuclei, with the same mass number A , but different atomic number Z , are called Isobars, (Example) ${}_8\text{O}^{16}$, ${}_7\text{N}^{16}$

Isotones

Nuclei with an equal number of neutrons, that is, with the same N , are called Isotones. (Example) ${}_6\text{C}^{14}$, ${}_7\text{N}^{15}$, ${}_8\text{O}^{16}$

Isomers

There are atoms, which have the same Z and A , but differ from one another in their nuclear energy states and exhibit differences in their internal Structure. These nuclei are distinguished by their different life times. Such nuclei are called Isomers.

General Properties of Nucleus

Nuclear size

Rutherford's α -scattering experiment showed that the mean radius of an atomic nucleus is of the order of 10^{-14} to 10^{-15} m while that of the atom is about 10^{-10} m. Thus the nucleus is about 10000 times smaller in radius than the atom.

The empirical formula for the nuclear radius

$$R = r_0 A^{1/3}$$

Where A mass number,

$$r_0 = 1.3 \times 10^{-15} \text{ m} \quad m = 1.3F$$

(1 Fermi = 10^{-15} m)

Nuclear mass

Nucleus consists of protons and neutron. Then assumed nuclear mass = $Zm_p + Nm_n$ where m_p and m_n are the respective proton and neutron masses and N is the neutron number. It is measured accurately by mass spectrometers.

Real nuclear mass < $Zm_p + Nm_n$

Difference in masses

$Zm_p + Nm_n$ real mass = Δm is called mass defect.

Nuclear density

The nuclear density ρ_N can be calculated

$$\rho_N = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{Am_N}{\frac{4}{3}\pi R^3} = \frac{m_N}{\frac{4}{3}\pi r_0^3} = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi(1.3 \times 10^{-15})^3}$$

(When $m_N = 1.67 \times 10^{-27}$ kg, $R = r_0 A^{1/3}$)

$$= 1.816 \times 10^{17} \text{ kgm}^3$$

This high value shows that the nuclear matter is an extremely compressed state.

Nuclear charge

The charge of the nucleus is due to the protons in it. Each protons have a positive charge = 1.6×10^{-19} C.

\therefore nuclear charge is ze ,

where z - atomic number.

Atomic Mass Unit (AMU)

1 amu is taken as one - twelfth (1/12) of the mass of carbon atom ${}^6C^{12}$. Roughly it is the mass of a proton or a neutron. They are expressed in amu carbon of atomic weight 12 and atomic number 6 has a mass equal to 12 amu.

$$\text{The mass of proton} = 1.007277 \text{ amu}$$

$$\text{The mass of neutron} = 1.008665 \text{ amu}$$

$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg.}$$

Energy can be calculated in electron volt (*ev*), when a mass equal to 1 amu is converted into energy

$$E = mc^2$$

$$m = 1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$E = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 \text{ J}$$

$$\text{But } 1 \text{ ev of energy} = 1.602 \times 10^{-19} \text{ J}$$

$$E = \frac{1.66 \times 10^{-27} \times (3 \times 10^8)^2}{1.602 \times 10^{-19}} \text{ J}$$

$$= 931 \times 10^6 \text{ ev}$$

$$E = 931 \text{ Mev}$$

Binding Energy

The difference between the sum of the masses of the individual nucleons and the mass of the nucleus is called mass defect. In the process of nuclear formation energy is given out, therefore the actual mass of the nucleons reduces. The disappeared mass is converted to energy. The minimum energy needed to break the binding force that exists between the nucleons is called binding energy. The energy equivalent of the mass defect is called the binding energy of the nucleus.

$$B.E = [zm_p + (A - z)m_n - {}_z M^A] c^2$$

Where,

z - no. of protons

$(A - z)$ - no. of neutrons

m_p - mass of protons

m_n - mass of neutron and

${}_z M^A$ - Experimental mass of final nucleus.

Nuclear forces

The nucleus consist of positively charged protons and neutrons. According to Coulomb law, protons must repel each other with a very large force, when clustered together and hence the nucleus must be broken. But this does not happen because some other force which overcome the electrostatic repulsion between protons and binds the protons inside the nucleus. This force is called nuclear force.

Properties

1. The nuclear force is attractive in nature and is the strongest of all the basic forces.
2. Nuclear force is charge independent. It is the same for all the 3 types of pairs of nucleons (n-n), (p-p) and (n-p).
3. Nuclear force is a short range force.
4. Nuclear forces have saturation property.

4.6 Model of the Nucleus

Liquid Drop Model

This model was proposed by Niels Bohr. He suggested that some of the properties of the nucleus is analogous in certain aspects to the charged liquid drop. The following analogies hold between a small liquid drop and the nucleus.

1. The liquid drop is spherical in shape due to surface tension. Nucleus is also assumed to be spherical in shape due to nuclear forces.
2. The density of the nucleus is independent of its volume ($2.305 \times 10^{17} \text{ Kg/m}^3$) and the density of the liquid is also independent of the volume.
3. The molecules of the liquid drop interact over short ranges. Similarly, nucleon in the nucleus also interact only with their immediate neighbouring nucleons.
4. The molecules of the drop leave the drop during evaporation when the temperature is raised and it gains energy in this process. Similarly, when a nucleon in the nucleus gains energy, it will leave the nucleus.
5. When a drop of water is allowed to oscillate, it breaks up into two smaller drops of nearly equal size. The process of nuclear fission is similar to thus and the nucleus breaks up into two smaller nuclei.

The process of nuclear fission has been successfully explained on the basis of the liquid drop model.

Shell Model and Magic Numbers

The shell model explains the behaviour of a nucleon in the nucleus. According to this model, the protons and neutrons are grouped in shells in the nucleus similar to

extranuclear electrons in various orbits outside the nucleus. The shells are regarded as “filled” when they contain a specific number of protons or neutrons or both. Inert gases have their shells filled with maximum possible electrons and are stable. Similarly, certain nuclei have been found to be light.

In the extranuclear shells only one type of particle (i.e., electrons) is to be arranged in different orbits and Pauli’s exclusion principle is applied. In the case of nucleus, there are two types of particles (protons and neutrons) and the shell arrangement is only empirical and it is based on the study of the stability and interactions of the nucleides which are known.

A nuclei is stable if it has definite number of either protons or neutrons of the combination of both. These numbers are known as Magic numbers. If a nucleus has protons and neutrons other than the magic number, it is less stable.

The magic numbers are 2, 8, 20, 50, 82, 126 and semi-magic numbers are 6, 14, 28, 40. The nuclei having 2, 8, 20, 50, or 82 protons or neutrons are stable. All other nuclei having protons or neutrons other than magic or semi-magic number are comparatively less stable.

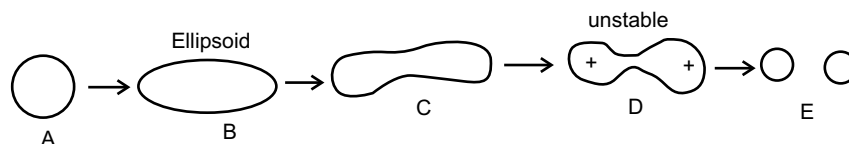
For example tin ($z = 50$) having 50 protons, have stable isotopes. This agrees with shell model because 50 is magic number. Stable nucleides are abundant in nature.

It is found that nuclei having a number of neutrons equal to magic number, cannot capture a neutron because shells are closed and they cannot contain an extra neutron.

4.7 Bohr Wheeler theory

Bohr and Wheeler successfully explained the phenomenon of nuclear fission using liquid drop model. According to this theory, when an external force is applied on liquid drop, its original spherical shape due to symmetrical surface tension may change into an ellipsoid. If the external force is sufficiently large, the ellipsoid may change into a dumb bell shape and may even break at the narrow end into two portions.

The analogy may be extended to a nucleus which behaves like a liquid drop. When a nucleus absorbs neutron, it forms a compound nucleus which is highly energetic. The extra energy possessed by it comes usually from the binding energy of the neutron which is absorbed by it. The extra energy will initiate a series of oscillations in the spherical nucleus shown as ‘A’ in the diagram.



As a result of these oscillations, the shape of the nucleus may change into ellipsoid as in ‘B’. If the applied extra energy is large, oscillations may be violent and that stage ‘C’ and ultimately stage ‘D’ may be approached. The nucleus is now dumb-bell

4.14 Applied Physics

shaped (stage D). Each bell of the dumb-bell is accumulated with a positive charge and repulsion takes place. This results in fission as shown in stage 'E'.

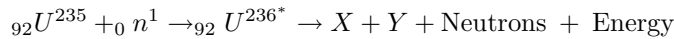
The nuclei that results from fission are called fission fragments. Usually fission fragments are of different size. A heavy nucleus undergoes fission when it acquires enough excitation energy to oscillate violently. Certain nuclei like ${}_{92}\text{U}^{235}$ are sufficiently excited by the mere absorption of a neutron.

4.8 Nuclear Fission

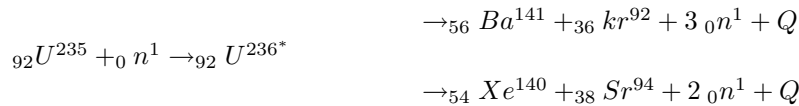
The process of breaking up of a single heavy nuclei atom into two (more or less equal) fragments with the release of large amount of energy is known as Fission.

Example

When uranium is bombarded with neutrons, the uranium nucleus captures a slow neutron forming an unstable compound nucleus. The compound nucleus splits into two nearly equal parts. Some neutrons and energy are released in this process. The fission equation is as follows:-

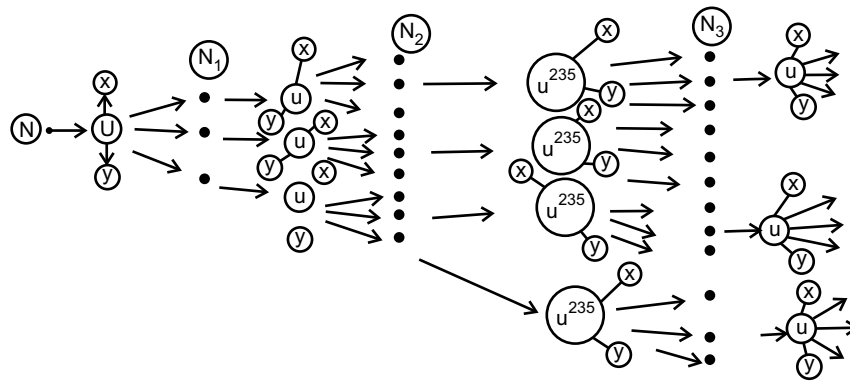


Here ${}_{92}\text{U}^{236*}$ is highly unstable and x and y are the fission fragments. Typical fission reactions are



The fission products X and Y have different mass numbers and atomic numbers.

[${}_{92}\text{U}^{236*} \rightarrow$ Highly unstable]



N_1 - 3 neutrons

N_2 - 9 neutrons

N_3 - 27 neutrons

The chain reaction proceeds as above.

Multiplication factor (k)

It is defined as the ratio of the secondary neutrons produced to the original neutrons.

$$K = \frac{\text{Number of Neutrons produced in one generation}}{\text{Number of Neutrons produced in preceeding generation}}$$

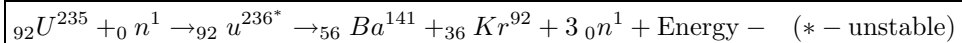
- If $k = 1$, The chain reaction is steady, controlled (or) critical.
 If $k > 1$, The chain reaction is 'supercritical' and uncontrolled.
 If $k < 1$, The chain reaction will slow down and fission stops. It is 'subcritical' state.

4.8.1 Energy released per fission

The process in which a heavy nucleus splits up into two or smaller nuclei of nearly comparable masses is called Nuclear Fission. More energy is released during this process, which is equivalent to the mass defect.

Example

Uranium, when bombarded with a neutron, is converted into unstable nucleus which splits up into two nuclei of atomic masses, Barium ($z = 56$) and krypton ($z = 36$). The reaction is as follows.



Mass of ${}_{92}\text{U}^{235}$	=	235.045733 amu
Mass of ${}_0n^1$	=	1.008665 amu
<hr/>		
* Total Initial mass	=	236.054398 amu
<hr/>		
Mass of Barium	=	140.9177 amu
Mass of krypton	=	91.8854 amu
Mass of 3 neutrons	=	3.025995 amu
<hr/>		
* Total Initial mass	=	235.829095 amu
<hr/>		
Difference	=	0.2253 amu
Since 1 amu	=	931 Mev
Energy released	=	0.2253×931 Mev
	=	200.4443 Mev

Thus, in the process of fission of one nucleus of uranium, about 200 Mev is released.

Energy released by 1 gram of uranium

$$\begin{aligned} \text{Number of atoms in 1 mole of uranium} &= 6.023 \times 10^{23} \\ \text{Energy produced by 1 gram of uranium during fission} &= \frac{6.023 \times 10^{23}}{235} \times 200 \text{ Mev} \end{aligned}$$

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$$\therefore E = 5.128 \times 10^{23} \text{ Mev}$$

But we know $1 \text{ Mev} = 1.602 \times 10^{-19} \text{ J}$

$$\therefore E = 5.128 \times 10^{23} \times 1.602 \times 10^{-19} \text{ joules}$$

As $1 \text{ kilowatt hour} = 10^3 \times 3600 \text{ Joules}$

$$\therefore E = \frac{5.128 \times 10^{23} \times 1.602 \times 10^{-19}}{10^3 \times 3600}$$

$$E = 2.26 \times 10^4 \text{ kilowatt hour}$$

Thus, the disintegration of 1 gm of uranium produces energy = $2.26 \times 10^4 \text{ kwh}$. Due to this reason, atomic energy is being used for the generation of electricity.

4.9 Chain Reaction

It is a self-propagating process in which number of neutrons goes on multiplying rapidly in geometrical progression during fission till the whole fissile material is completely disintegrated. For example, consider the chain reaction process in U^{235} .

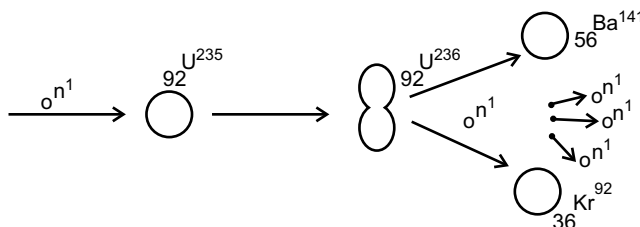
When U^{235} is bombarded with a slow neutron, fission takes place and 3 neutrons are produced along with release of energy (200 Mev). These neutrons further cause fission by reacting with the same element (U^{235}) of different mass. This process continues till the entire U^{235} nuclei is completely disintegrated. This self-propagating process is called Chain Reaction. If this process is allowed to proceed in an uncontrolled manner disintegration takes place in no time and enormous magnitude of energy is released.

Types of chain reaction

Chain reaction can be classified into two types

1. Controlled chain reaction and
2. Uncontrolled chain reaction.

A chain reaction becomes self propagating only if the number of fission producing neutrons are kept constant. This reaction is known as "controlled chain reaction". It is used in nuclear reactors.



If the number of neutrons is allowed to multiply indefinitely and the entire energy is released within a very short period, it is known as uncontrolled chain reaction. Ex: Atom bomb.

These conditions can be conveniently expressed in terms of multiplication factor (or) reproduction of the system.

The following figure shows the bombardment of neutron with uranium and the fragments with the release of neutrons.

The fission products fall into two groups:

1. Heavy Group, with mass number between 130 and 155 and
2. Light Group, with mass number between 80 and 110.

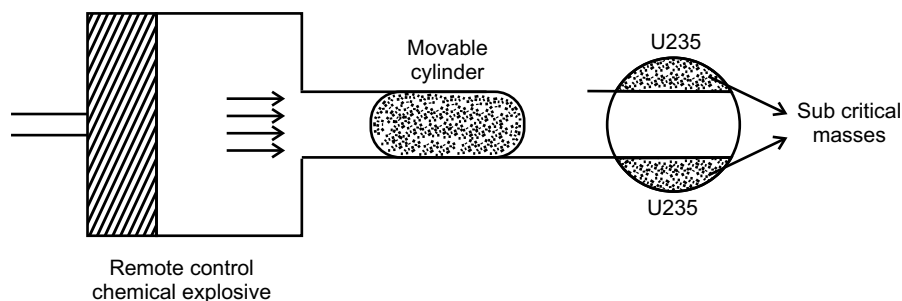
Critical size

It is defined as the minimum size for which the numbers of neutrons produced in fission process, just balances those lost by leakage and non-fission capture.

The mass of the fissionable material at this stage is called critical mass. It is the size less than which chain reaction is not possible.

4.10 Atom Bomb

It is based on the principle of nuclear fission. Here uncontrolled chain reaction takes place and a enormous amount of energy is released in a short time. Natural uranium is not suitable for sudden release of energy because U^{238} is fissionable by fast neutrons (high energy) only. It is essential in a bomb that U^{235} or Pu^{239} should be used with slow (low energy) neutrons.



An atom bomb consists of two pieces of U^{235} or Pu^{239} , each smaller than the critical size and a source of neutrons. The two subcritical masses of U^{235} in the form of hemispheres are kept apart as shown in the figure.

When the bomb has to be exploded, a third well fitting cylinder of U^{235} is propelled so that it will fit in or fuse together with the other two pieces. Now, the total quantity of U^{235} is greater than the critical mass. Hence an uncontrolled chain reaction takes place resulting in a terrific explosion.

The explosion of an atom bomb releases tremendous amount of energy in the form of heat, light and radiation. The temperature of the order of 10^9 C and a pres-

sure of millions of atmospheres are produced thereby all the objects and living creatures within a region of several miles are completely destroyed.

4.11 Nuclear Reactor

These are two types of nuclear reactors

1. **Homogeneous Reactors:-** In this reactor, heavy water (D_2O) is the moderator. Uranium can be in the form of a solution of Urany sulphate a very small particles of uranium can be suspended in D_2O .
2. **Heterogenous Reactors:-** Here graphite is the moderator. Rods of Uranium are distributed through graphite in a regular way, forming lattice.

A nuclear reactor is a system where nuclear fission is produced in a controlled, self sustaining chain reaction. The following are the essential features of nuclear reactors.

- | | |
|--------------------------------------|-------------------------------|
| (1) Fissionable material called Fuel | (4) Cooling system and |
| (2) Moderator | |
| (3) Neutron Reflector | (5) Safety and control system |

(1) **Fissionable Substance:-** The commonly used fissionable materials are

- (a) Uranium isotopes U^{233} , U^{235}
- (b) Thorium isotopes Th^{232} , and
- (c) Plutonium isotopes Pu^{239} , Pu^{240} and Pu^{241} .
- (d) Natural uranium, containing 99.28% of U^{238} and 0.72 % of U^{235}

(2) **Moderators:-**

- (a) The function of moderator is to slow down the high energetic neutrons produced in the process of U^{235} to thermal energies.
- (b) Ideally moderators have low atomic weight and low absorption cross section for neutrons.
- (c) Heavy water (D_2O), Graphite, beryllium oxide or beryllium, water etc., are used as moderators.

(3) **Neutron Reflector:-**

- (a) By using reflectors on the surface of reactors, leakage of neutrons can be reduced very much and the neutron flux in the interior can be increased.
- (b) Materials having high scattering cross section and low absorption cross section are good reflectors.
- (c) Since moderator has good reflecting properties, it also acts as a reflector.
- (d) Reflectors are made of Nickel, Thorium or other materials.

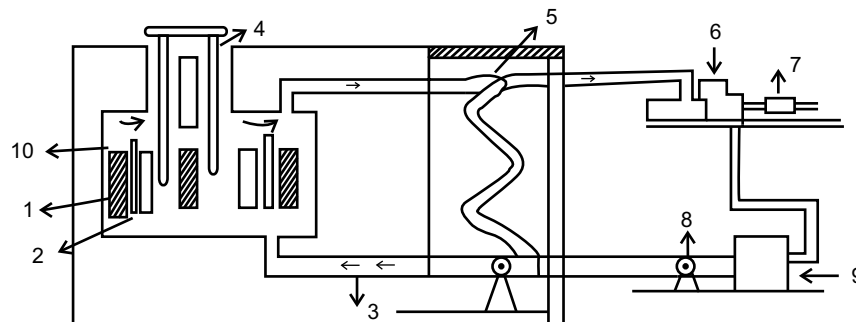
(4) **Cooling system:-**

- (a) The cooling system removes the heat evolved in the reactor core.
 - (b) The heat is evolved from K.E. of the fission fragments when they are slowed down in the fissionable substance and moderator.
 - (c) The coolant or heat transfer agent is pumped through the reactor core.
 - (d) Then through a heat exchanger, the coolant transfers heat to the secondary thermal system of the reactor.
 - (e) Water, heavy water, air, CO_2 , He, liquid metal such as sodium are commonly used coolants.
- (5) **Safety and Control system:-** The chain reaction and energy output are controlled by control rods. This is done by pushing control rods into reactor core. These rods are of material like Boron or Cd, having a large neutron - absorption cross section. These rods absorb neutrons and hence reduce the reaction. By pushing the rods in, the operation of the reactor can be stopped. The safety system protect the space surrounding the reactor against intensive neutron flux and gamma rays existing in the reactor core. This is achieved by surrounding the reactor with massive walls of concrete and lead which would absorb neutrons and gamma Rays.

4.12 Power Reactor

The heat generated in a nuclear reactor is used for producing power in a nuclear power plant. The figure shows the essential components.

- | | |
|--|-----------------------|
| 1. Fuel Elements (Uranium) | 6. Turbines |
| 2. Moderator (graphite block) | 7. Generator |
| 3. Coolant (water, gas or liquid alloys) | 8. Pump |
| 4. Control rods (Boron, steel or Cd) | 9. Condenser |
| 5. Heat exchanger | 10. Reactor Reflector |



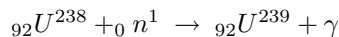
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- (i) A quantity of enriched uranium in the form of pure metal (or) solution of a soluble salt in water constitutes the center of the heat energy source.
- (ii) More heat is produced during fission process.
- (iii) The cadmium rods regulate the temperature to a pre determined value.
- (iv) If it is desired to bring down the temperature, the rods are pushed down further to absorb more neutrons.
- (v) If the temperature is to be raised, the rods are pulled up a little.
- (vi) A fluid is circulated through the shielded reactor and heat exchanger.
- (vii) The hot fluid, while passing through heat exchanger, converts water into steam.
- (viii) The steam produced runs the conventional turbines to produce electricity.

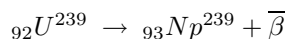
4.13 Breeder Reactor

The breeder reactor is important because it uses U^{238} which is a easily obtainable fuel.

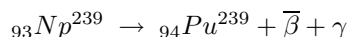
If a thermal reactor core with U^{235} fuel is surrounded by a blanket of a fissile material like U^{238} , U^{238} can be converted into fissile fuel. Reactors of this type are called Fuel Producing Reactors. The reaction is as follows:



This is followed by



${}_{93}Np^{239}$ is also radioactive. It distinguishes and the reaction is given as below.



This process of producing one type of fissionable material (Pu^{239}) from a non-fissionable material (U^{238}) is called 'Breeding' and the reactor used for this purpose is called "Breeder Reactor".

4.14 Nuclear Fusion

Nuclear fusion is the process in which two lighter nuclei combine together to form a heavier and stable nuclei.

The fusion process will result in the liberation of energy. Since the reacting nuclei are positively charged particles, a repulsive force acts between them. To overcome the electrostatic repulsion between them during fusion, a high amount of energy is required. This energy is given in the form of heat. Therefore, before fusing the nuclei, the mass of the single nucleus formed is always less than the sum of the masses of the

individual lighter nuclei. The difference in mass is converted into energy according to Einstein's relation $E = mc^2$.

Energy released in fusion process

Based on the mass defect method, the amount of energy released during a nuclear fusion process is calculated by *actual mass before fusion*.

The mass of two deuteron atoms = $2 \times 2.01478 \text{ amu} = 4.02956 \text{ amu}$

Actual mass after the fusion

Mass of He atom	=	4.00388	amu
Mass of deuterium atoms	=	4.02956	amu
Decrease in mass	=	<u>0.02568</u>	<u>amu</u>

\therefore The energy released = $0.02568 \times 931 \text{ MeV} = 24 \text{ MeV}$

Thus, the energy released during in fusion is 24 MeV which is much less than the energy produced in the fission of U^{235} which is about 200 MeV. But energy released per unit mass during fusion of the lighter nuclei is much greater than the fission of U^{235} .

4.15 Thermonuclear Reactions as a Source of Stellar energy

It is established that the stellar energy is due to the process of fusion of light nuclei to form a heavy nucleus. The temperature of the sun is very high ($3 \times 10^7^\circ\text{C}$) and it radiates a large amount of energy in the order of $3.6 \times 10^6 \text{ J/s}$. The origin of such tremendous amount of energy is due to the nuclear fusion reaction.

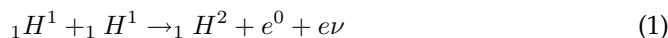
Actually sun's matter mainly consists of hydrogen and helium. Nuclear fusion reactions are continuously taking place in the sun. Thus, the energy released from the sun and other stars are mainly due to fusion of the hydrogen nuclei into helium nucleus.

Bethe suggested that the fusion process takes place under the following two different cycles.

- (i) Proton - Proton cycle (P-P chain)
- (ii) Carbon - Nitrogen cycle (C-N chain)

(i) P-P cycle

This P-P cycle starts with the combination of two protons and hence it is known as P-P cycle. When 2 protons fuse together, the resultant products are deuteron, positron and neutrino.

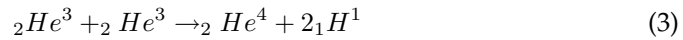


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Deuteron then combines with another proton to yield He^3 .

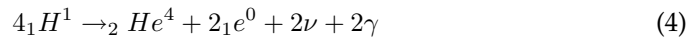


The two ${}_2He^3$ are fused to produce He^4 .



In the P-P cycle, four protons are fused together to produce one Helium atom.

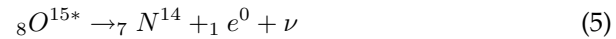
Therefore, the net effect of the reaction is



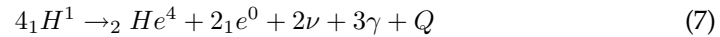
The total energy is released in the above reaction is 26 MeV (approx).

(ii) C - N cycle

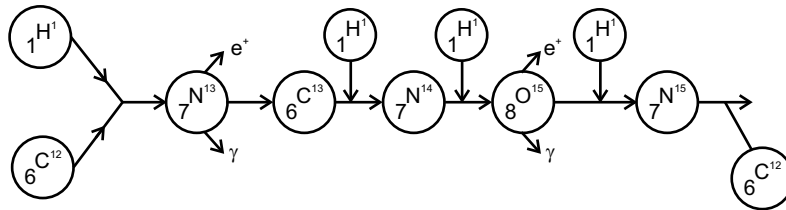
In this cycle, carbon acts as catalyst. In the first step, hydrogen atom interacts with C^{12} nucleus with the release of fusion energy as,



The reaction cycle is essentially the following reaction,



The C-N cycle is illustrated in figure.



The loss in mass is calculated as

$$4{}_1H^1 = 4.031300 \text{ amu}$$

$$2He^4 = 4.002603 \text{ amu}$$

$$1{}_1e^0 = 0.001098 \text{ amu}$$

∴ Loss in mass = 0.02756 amu

∴ Energy released = 0.02756 × 931 = 26 MeV

It is found that in one million years the sun loses about 10^{-7} of its mass by the above process. Taking mass at the sun as 2×10^{30} kg and its present age as 10^{10} years, it is estimated that C-N cycle may keep going for another 30 billion years.

4.16 Hydrogen Bomb

In a hydrogen bomb fusion of lighter nuclei takes place and tremendous amount of energy is released. In hydrogen bomb fission - fusion process is applied.

In this bomb hydrogen is the core. The fission bomb produces a very high temperature at which thermonuclear reactions start resulting in the fusion of hydrogen to form helium. Greater energy per unit mass is obtained from a hydrogen bomb than from a nuclear fission bomb (atom bomb).

There is no limit to the size of the bomb because there is no necessity for critical mass in this process.

Differences between nuclear fission and fusion

Nuclear fission	Nuclear fusion
1. Heavy nucleus splits into two nuclei.	Two lighter nuclei are fused together to form a single nucleus.
2. This reaction is initiated by neutron.	This reaction is initiated by proton.
3. This process occurs even at room temperature.	This process occurs only at very high temperature in the order at 10^7 to 10^9 k.
4. The energy released during fission is 200 MeV. But, the energy released per nucleon is 0.85 MeV	The energy released during fission is 26 MeV. But, the energy released per nuclear is 6.75 MeV.
5. Fission equation: ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + Q$	Fusion equation: ${}_1H^1 + {}_1H^1 \rightarrow {}_2He^4 + Q$

Short Questions and Answers

1. What are X rays?

X rays are a type of radiation which could actually penetrate through opaque objects and provide an image of their inner structures.

They were discovered by a German physicist Conrad Rontgen in 1895.

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2. **What is Radiology?**

The medical speciality where X rays are used for diagnosis and therapeutic purpose is known as radiology.

3. **How are X ray generated?**

When fast moving electrons are suddenly decelerated by impinging on a target X rays are produced.

4. **How are X rays used as diagnostic tool?**

The use of X ray as a diagnostic tool is based on the fact that various components of the body have different densities for the rays. When X ray from a point source penetrate a body section the internal structure of the body absorbs varying amounts of the radiation. The radiation that leaves the body, therefore has a spatial intensity variation that is an image of the internal structure of the body.

5. **What factors determines the efficiency of the X rays?**

The anode voltage and the target material determine the efficiency of the X rays.

6. **Mention the use of Aluminium filters in X ray machine?**

The emitted X rays contain a broad range of frequencies. Aluminium filters absorb lower X rays frequencies and hence the intensity of low frequency X rays incident on the patient is reduced.

7. **What is the use of collimator?**

Collimator which is placed between the patient and aluminium filter restricts the beam falling on the patient.

8. **Mention the use of bucky grid?**

Some of the X rays entering the body of a patient are actually scattered and causes a blurring of the image. This effect can be reduced using buckey grid.

9. **What is an image intensifier?**

Some of the X rays are lost by means of a Bucky grid and hence the density of the image in the film is reduced and the image resolution is also reduced therefore to improve the density and resolution of the image the image intensifiers are user.

10. **What is flouoroscopy?**

In fluoroscopy the patients condition is viewed on a fluorescent screen which convert X rays into visible light by scintillations.

11. Discuss some differences between radiography and fluoroscopy?

Radiography	Flouroscopy
i. X ray image is developed on a photo sensitive film	Viewed directly on a Flourescent screen.
ii. Patient dose is low	Patient dose is high.
iii. Efficiency is more	Efficiency is
iv. High resolution	Low resolution.

12. How is a contrast media used in X rays?

When some organs and soft issues show very little difference in X ray absorption, it is necessary to fill them with a contrast media in order to make their outline visible on the X ray image.

13. Define one Rontgen?

One Rontgen is the unit of exposure of ionizing radiation and is defined as the quantity of gamma rays or X rays required to produce 1.61×10^{12} pairs of ions in one gram of dry air at standard condition of temperature and pressure.

14. Give four properties of X rays?

- i. Travels with speed of light
- ii. Produce ionization in gases
- iii. Affects photographic films
- iv. Penetrates through material

15. Define REM?

REM is Rontgen equivalence man which is defined as the unit of dose of any ionizing radiation which produces the same biological effect as a unit of absorbed dose of ordinary X rays.

Review Questions

1. Explain the generation of X rays using a neat diagram?
2. Explain the instrumentation for diagnostic X rays?
3. Explain with neat diagram the working of image intensifiers?

CHAPTER

5

Material Science

5.1 Magnetic Materials

Introduction

The materials which gets magnetised when placed in external magnetic field are known as magnetic materials. These materials play a vital role in modern technology. They are widely used in electrical machines, computers, television tube, transducers, audio devices, meters etc. These materials actually play an important role in storage devices.

Among the many types of magnetic materials, diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic and ferrimagnetic are the most important, from the point of view of principal applications. In order to understand the magnetic behaviour of different materials mentioned above, a thorough knowledge of such material is very essential.

Let us first consider a few terms and definitions used in magnetism.

5.1.1 Terms and Definitions in magnetism

1. Magnetic poles

Two opposite ends of a bar magnet are known as magnetic poles. They are called as north pole and south pole. The north and south poles are separated by a distance l form a magnetic dipole.

2. Magnetic moment (M)

Magnetic moment of a bar magnet is defined as the product of its pole strength (m) and length of the magnet ($2l$).

$$M = m \times 2l \quad (1)$$

unit of M is $wb - m$.

3. Magnetic field

The space (or) region around the magnet where the influence of magnetism is felt is known as magnetic field.

4. Magnetic lines of force

It is a line along which a unit north pole (compass needle) is free to move. The direction of magnetic field at any point is given by the tangent drawn at that point. The existence of a unit north pole is imaginary.

A magnetic field is represented in picture by drawing number of lines of force using a compass box. These lines are known as the lines of magnetic induction.

For a strong magnetic field, the lines of the force are close together and for a weak magnetic field, the lines of forces are far apart.

For a magnet, the lines start from North pole and terminate or enter at the south pole.

5. Magnetic flux (ϕ)

A collection of magnetic lines of force (or) lines of magnetic induction is known as magnetic flux and is represented by ϕ .

Unit of magnetic flux (ϕ) is weber (or) wb.

6. Magnetic induction (or) Magnetic flux density (B)

It is defined as the number of magnetic lines of force (flux) passing through unit area of a material (or) medium kept perpendicular to the direction of lines of force.

Magnetic flux density (or)

$$\text{Magnetic induction (B)} = \frac{\text{Number of magnetic lines of force}}{\text{Area of crosssection}}$$

$$\boxed{B = \frac{\phi}{A}} \quad (2)$$

unit of B is wb/m^2 (or) Tesla (T)

$$\text{Hence Magnetic flux} \quad \boxed{\phi = BA} \quad (3)$$

7. Magnetic field intensity (H)

Magnetic field intensity at a point in a magnetic field (produced by a magnetic material) is the force experienced by a unit north pole placed at that point. It is represented by H.

Unit of H is Ampere turns per meter (or) AT/m.

But the existence of unit north pole is imaginary.

$$H = \frac{F}{m} = \frac{\text{Force experienced by unit north pole in magnetic field}}{\text{pole strength}}$$

(or) $F = mH$ (4)

8. Magnetic permeability (μ)

It is a measure of the amount of magnetic lines penetrating through the material (or) it is the easiness of a material (or) medium to allow the magnetic lines through it.

It is found that the magnetic flux density (B) is directly proportional to the field strength (H) that is

$$B \propto H$$

$$\boxed{B = \mu H} \quad (5)$$

$$\boxed{\mu = \frac{B}{H}} \quad (6)$$

where μ is a constant of proportionality and is known as the permeability (or) absolute permeability of the medium.

Magnetic permeability (or) absolute permeability (μ) of a medium (or) a material is defined as the ratio of magnetic induction (B) in the sample to the applied field (H).

Unit of magnetic permeability (μ) is Henry/metre.

9. Magnetic permeability of free space (or) air (μ_0)

If the flux density (or) magnetic induction is established in air (or) in a non-magnetic material by keeping it in a magnetizing field of strength H then

$$B_0 = \mu_0 H \quad (7)$$

$$\boxed{\mu_0 = \frac{B_0}{H}} \quad (8)$$

where B_0 = magnetic induction (or) flux density in air (or) vacuum.
 μ_0 = permeability of free space (or) air.

magnetic permeability of free space (μ_0) is defined as the ratio of the magnetic induction (B_0) in air (or) free space to the magnetic field (H).

Unit of μ_0 is Henry/metre.

5.4 Applied Physics

10. Magnetic Relative permeability (μ_r)

It is defined as the ratio of the absolute permeability of a medium (μ) and the permeability of free space (μ_0) that is

$$\boxed{\mu_r = \frac{\mu}{\mu_0}} \quad (9)$$

μ_r is a dimensionless quantity. It is only a ratio and has no unit.

Also

$$\boxed{\mu_r = \frac{\mu}{\mu_0} = \frac{BH}{B_0H} = \frac{B}{B_0}} \quad (10)$$

Hence relative permeability of a medium (μ_m) can also be defined as the ratio of magnetic flux density B established in the medium to the magnetic flux density B_0 established in air.

11. Intensity of magnetisation (I)

The term magnetisation represents the process of converting a non magnetic material into a magnetic material. The intensity of magnetisation of a magnetic material is a measure (or) degree of its magnetisation.

Intensity of magnetisation of a material is defined as the magnetic moment (induced magnetisation) per unit volume. It is represented by I ,

$$I = \frac{\text{magnetic moment}}{\text{volume of the material}}$$

$$\boxed{I = \frac{M}{V}} \quad (11)$$

Unit of I is wb/m (or) wbm^{-1} .

If A is the area of cross section $2l$ is the length and m is the poles length of a magnet (magnetic material) then

$$I = \frac{M}{V} = \frac{m \times 2l}{A \times 2l} \quad \text{sin } M = m \times 2l$$

$$\boxed{I = \frac{m}{A}} \quad (12)$$

Hence the intensity of magnetisation of a material can also be defined as the ratio between its pole strength and area of cross section (or) pole strengths per unit area kept right angles to the direction of magnetisation.

Significance: when the same amount of magnetic field is applied to different magnetic substances like iron, steel alloys etc, they get magnetised to different extent. Hence the intensity of magnetisation is a measure (or) degree of magnetisation of a specimen.

12. Magnetic susceptibility (χ)

This term represents the easiness with which a material gets magnetised. Actually it represents the magnetic response of a magnetic material to get magnetised.

Ex: If χ is +ve, the material can be magnetised. If χ is -ve, the material cannot be magnetised.

Definition: Magnetic susceptibility of a material is defined as the ratio of intensity of magnetisation (I) to the magnetic field strength (H). It is represented by χ

$$\chi = \frac{I}{H} \quad (13)$$

χ is a dimension less quantity. It has no unit (since I and H are having same units).

Relation between μ_r and χ is

$$\boxed{\mu_r = 1 + \chi} \quad (14)$$

5.2 Different types of magnetic materials and their properties

Classification of magnetic materials

5.2.1 Introduction

All materials made up of atoms. In an atom, the electrons are revolving round the nucleus in various shells. Moving electron (moving charges) produces electric current, which in turn produces a magnetic field. Electron has both orbital motion around the nucleus and spin motion about its own axis. Orbital motion of electron produces orbital magnetic moment. Similarly spin motion of electron produces spin magnetic moment. Hence an atom of a material can be considered as atomic magnet whose magnetic moment is given by the vector sum of orbital and spin magnetic moments. The magnetic moment (or) magnetic dipole of the atoms in a material will decide the nature of magnetism in that material.

5.2.2 Classification

Magnetic materials are broadly classified into two categories (types)

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1. Magnetic materials not having any permanent magnetic moment.
Example: Diamagnetic materials.
2. Magnetic materials having permanent magnetic moment.
Examples:
 - (a) Paramagnetic materials.
 - (b) Ferromagnetic materials.
 - (c) Antiferromagnetic materials.
 - (d) Ferrimagnetic materials.

Let us now discuss briefly the five types of magnetic materials mentioned above.

Diamagnetic materials

These materials when placed in an external magnetic field get feebly magnetised in a direction opposite to the external field. Because of this, when a diamagnetic material is suspended in a uniform magnetic field, it at once turns in a direction perpendicular to the direction of external field (Resultant of external and internal fields is in perpendicular direction).

Explanation

An electron moving round the nucleus produces a magnetic moment. Due to different orientations of various orbits in an atom, the net magnetic moment of a diamagnetic material is zero. When an external magnetic field is applied on these materials, there is a change produced in the motion (speed of electrons) in the orbits which produces an induced magnetic moment in a direction opposite to the external field. That is why a suspended diamagnetic material turns perpendicular to external field.

Properties

1. Direction of alignment: (turning)

In a uniform external magnetic field: when a diamagnetic material is suspended in a uniform magnetic field, it at once turns perpendicular to the direction of external field.

In a non uniform field: it turns from a stronger field region to a weaker field region.

2. Effect on the magnetic lines of force

Diamagnetic materials repel the magnetic lines of force. Hence when they are brought near an electromagnet, they are repelled.

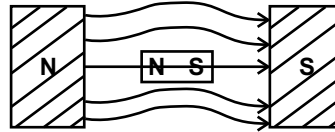


Fig. 1 Magnetic lines are passing away from the diamagnetic material

3. Magnetic moment

There are no permanent dipoles. Hence the magnetic effects are small.

4. Magnetisation

Magnetisation in a diamagnetic material is directed opposite to the direction of external magnetic field.

5. Susceptibility (χ)

Diamagnetic materials have negative susceptibility. The absolute value of susceptibility is of the order of 10^{-6} .

Eg: For Al_2O_3

$$\chi = 5 \times 10^{-5}$$

6. Dependence of susceptibility on temperature

The magnetic susceptibility of diamagnetic materials is independent of temperature.

7. Dependence of susceptibility on applied field

The susceptibility of diamagnetic materials is independent of strength of the applied field. (Susceptibility value is a constant for a particular diamagnetic material and will not vary with temperature and strength of the applied field).

8. Relative permeability (μ_r)

Relative permeability of diamagnetic materials is less than unity ($\mu_r < 1$).

a. The magnetisation in weak magnetic field

The magnetisation of diamagnetic material is a linear function of the magnetic field, when the field is not so strong.

Examples of diamagnetic substances

Sb, Bi, Zn, Ag, Au, Pb, Al_2O_3

a. Paramagnetic materials

These materials when placed in a uniform external magnetic field, they get magnetised in the direction of the external field. Because of this, when a paramagnetic material is suspended in an external field, they at once turns in the same direction of the applied field.

Explanation:

Paramagnetic materials possess permanent magnetic moments. This magnetic moments in a single atom of the material is due to (a) orbital motion of electrons around the nucleus producing orbital magnetic moment, and (b) spin motion of electrons producing spin magnetic moment.

The orbital magnetic moments disappear due to the electric field of neighbouring charges but spin magnetic moments remain unaltered due to this field.

In the absence of external field, because of random orientation of magnetic moments under thermal fluctuations, the net magnetic moment in the material is zero. When the external field is applied, the magnetic moments tend to line up with the external field. Since the magnetisation produced in the materials is in the same direction of external field, they set themselves in the direction of applied field.

Properties

1. Direction of alignment in external magnetic field

When a paramagnetic substance is suspended in a uniform magnetic field, it at once turns along the direction of the external field.

In a non uniform field, it turns from a weaker field region to a stronger field region.

2. Effect on the magnetic lines of force

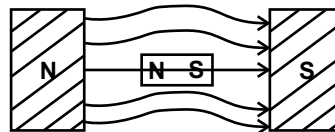


Fig.2 Magnetic lines are pulled towards the paramagnetic material

The diamagnetic materials attracts the magnetic lines of force. Hence when they brought near an electromagnet, the lines of forces due to electromagnet are pulled towards the paramagnetic materials.

3. Magnetic moment

Paramagnetic materials possess permanent magnetic dipole moment.

4. Magnetisation

Magnetisation in a paramagnetic material is directed along the direction of external magnetic field.

5. Susceptibility (χ)

Paramagnetic materials have a positive magnetic susceptibility (very low). The susceptibility is of the order of 10^{-6} .

Examples: χ for Al = 2×10^{-5} .

6. Dependence of susceptibility on temperature

Susceptibility paramagnetic substance greatly depends on temperature. As the temperature increases, susceptibility decreases. according to *Curie law*

$$\chi \propto \frac{1}{T}$$

$$\chi = \frac{C}{T}$$

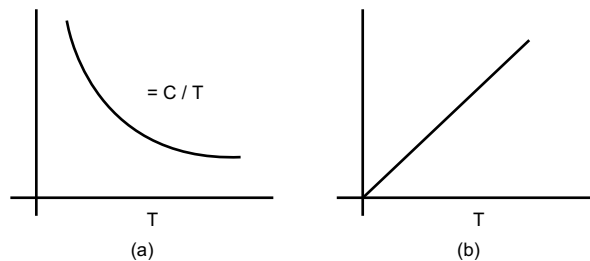


Fig. 3

where C is Curie's constant.

T = temperature in kelvin.

At low temp, *Curie-Weiss law* can be used.

The relation is

$$\chi = \frac{C}{T - \theta}$$

where θ = A constant known as paramagnetic Curie temperature.

Graph gives the relation between χ and T as per Curie law.

Graphs between χ & T , $\frac{1}{\chi}$ and T the shown in Figs.3.

7. Dependence of susceptibility on applied field

Paramagnetic susceptibility is independent of applied magnetic field and it will not vary with applied field.

8. Relative permeability (μ_r)

Relative permeability of paramagnetic materials is slightly greater than unity ($\mu_r > 1$) lines of forces due to external field are attracted towards the material and penetrate through it.

Eg: μ_r for Al = 1.00002.

Since $\mu_r = 1 + \chi$ where χ has a value $0.00002 = 2 \times 10^{-5}$.

9. Variation of magnetisation under weak external magnetic field

The magnetisation is a linear function of the external field (H), when the field is not so strong.

Examples: Pt, Al, Cr, Mn, CuSO_4 , liquid oxygen and solutions of salt of iron and nickel.

b. Ferromagnetic materials

These materials when placed in a uniform external magnetic field, they get strongly magnetised in the direction of the applied field. Because of this, when a ferromagnetic material is suspended in an external field they quickly turns along the direction of the applied field.

Explanation

A ferromagnetic material has a spontaneous magnetic moment naturally even in the absence of external field that is the net magnetic moments due to the atoms in the material is not zero. This net intrinsic magnetic dipole moment which is primarily due to the spin of the electrons. There is a strong interaction between the neighbouring atomic magnetic dipoles. It is known as *spin exchange interaction* and this exchange interaction align the neighbouring magnetic dipoles parallel to each other and this spread over a very small region (1 - 0.1 mm across) called *domain*. The domains are oriented in different directions and there is a net magnetic moment in the material which the vector sum of the magnetic moment of domains.

In the absence of external field, the magnetic moment of domains are not aligned. When the external field is applied, the domains that are aligned in the direction of field, increase in size at the expense of other. In a very strong magnetic field, all the domains are lined up providing highly observed magnetism. Since the magnetisation produced inside the material is in the direction of external field, the material quickly turns along the direction of applied field.

Properties

1. Direction of alignment in external magnetic field

When a ferromagnetic material is suspended in a uniform magnetic field, it quickly turns along the direction of the external field.

In a non-uniform field, it turns from a weaker field region to a stronger field region quickly.

2. Effect on the magnetic lines of force

Ferromagnetic material strongly attracts the magnetic lines of force. The field lines are crowded in to the material, as shown in Fig.4.

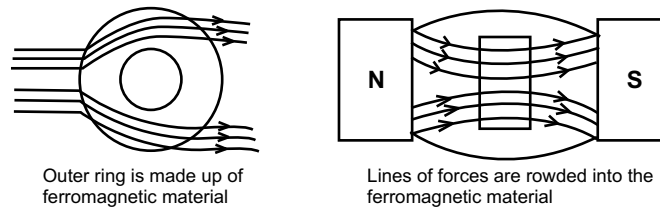


Fig.4

3. Magnetic moment

Ferromagnetic materials have permanent dipole moment. Hence they attract field strongly.

4. Magnetisation

They exhibit magnetisation even in the absence of external magnetic field. This property is known as *spontaneous magnetisation*.

In the external field, the magnetisation produced is directed along the applied field.

5. Susceptibility (χ)

Ferromagnetic materials have positive magnetic susceptibility. The value of susceptibility is very high and is of the order of 10^6 .

Eg: χ for pure Fe = 2×10^5 .

6. Dependence of susceptibility on temperature

Ferromagnetic susceptibility greatly depends on temperature. Each ferromagnetic material has a characteristic temperature called ferromagnetic *curie temperature*

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(θ_C). When $T < \theta_C$, they show ferromagnetic property. They obey Curie law, $\chi = \frac{C}{T - \theta}$. When $T > \theta_C$, they show paramagnetic property.

Graph between χ and T is shown in Fig.5.

$$\chi = \frac{C}{T - \theta} \quad (\chi \text{ is very large})$$

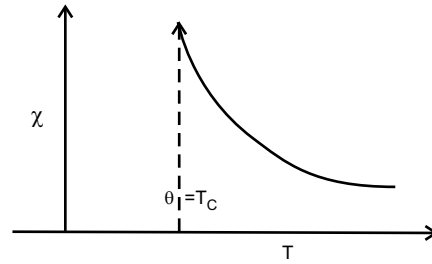


Fig.5

7. Dependence of susceptibility on applied field

Because of non linear relationship between B (magnetic induction) and the applied field, susceptibility of a ferromagnetic material does not have a constant value. The graph between χ and H is similar to the graph between μ_r and H .

8. Relative permeability (μ_r)

Relative permeability of ferromagnetic materials is very high (positive value) and is of the order of 10^6 .

Eg: μ_r for Fe = 2×10^5 .

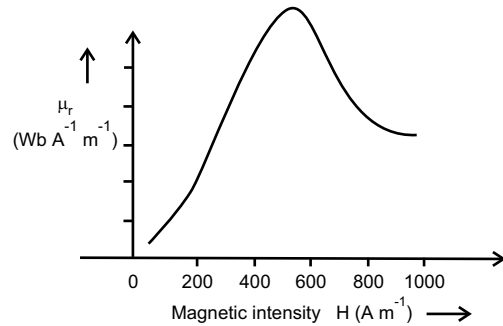


Fig.6

Because of non linear relation between magnetic induction (B) and applied field (H). The permeability of a ferromagnetic material does not have a constant value.

9. Variation of magnetisation with applied field

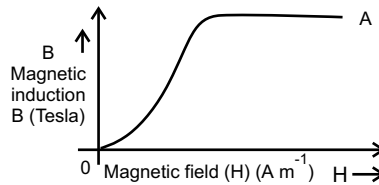


Fig.7 (OA represents the initial magnetisation curve)

Magnetisation of a ferromagnetic material does not vary linearly with the applied field H . It has a very complex function with the applied field.

10. Hysteresis:

Ferromagnetic material exhibits hysteresis. Internal magnetisation (B) lags behind the external field. Because of this property, there is a residual magnetisation left after the external field is switched off.

Eg: Fe, Co, Ni.

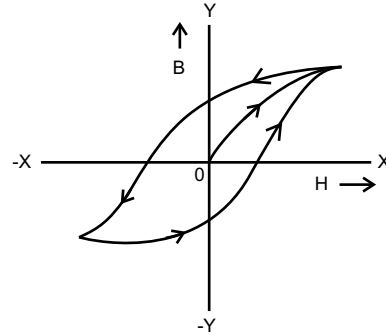


Fig. 8 Hysteresis loop

c. Antiferromagnetic materials

These materials are magnetised parallel to the direction of external magnetic field. When a antiferromagnetic material is suspended in a uniform (strong) magnetic field, it turns slowly along the direction of applied field.

Explanation

Antiferromagnetic materials are crystalline solids, which exhibit a small positive susceptibility of the order of 10^{-3} to 10^{-5} . Since the distance between interacting atoms is small in

an antiferromagnetic material, the magnetic interaction between any two dipoles align themselves *anti-parallel* to each other and all the dipoles have *equal magnetic moment* in the absence of external field. Hence the resultant magnetisation is zero. But when an external field is applied, the dipoles align themselves in the direction of the external field, (since the magnetic induction produced in the material is in the same direction of the external field).

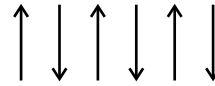


Fig. 9 Dipole magnetic moments are equal

Properties

1. Direction alignment is external magnetic field

When a antiferromagnetic material is suspended in a uniform magnet field, it turns along the direction of applied field.

2. Magnetic moment

Antiferromagnetic materials have permanent magnetic moment.

3. Alignment of dipoles

Since the distance between interacting atoms is small, the dipoles align in opposite directions. This is due to an *exchange interaction*. Actually electron spin of neigh-

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bouring atoms are aligned antiparallel. But magnitudes of magnetic moments of dipoles are equal.

4. Magnetisation

Magnetisation in antiferromagnetic material is directed along the direction of the external field.

5. Susceptibility (χ)

Susceptibility (χ) of antiferromagnetic materials is very small and is positive. It is of the order of 10^{-3} to 10^{-5} .

6. Dependence of susceptibility on temperature

The susceptibility of antiferromagnetic materials increases with temperature and reaches a maximum at a temperature called *Neel temperature* T_N . With the further increase in temperature susceptibility decreases showing paramagnetic properties.

Relation between χ and T is

$$\chi = \frac{C}{T + \theta}$$

C = curie constant

θ = paramagnetic curie temperature

The variation of susceptibility (χ) of a antiferromagnetic material with temperature (T) is shown in Fig.10.

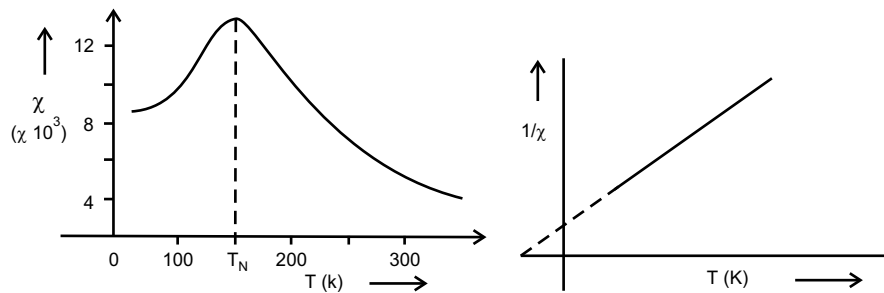


Fig.10

Variation of $\frac{1}{\chi}$ with T is a straight line.

7. Permeability (μ_r)

Permeability (μ_r) of antiferromagnetic materials is slightly greater than 1

since $\mu_r = 1 + \chi$

Examples: Salts of transition elements.

d. Ferrimagnetic materials

These materials are magnetised parallel to the direction of external magnetic field. When a ferrimagnetic material is suspended in a uniform magnetic field, it turns quickly along the direction of the applied field.

Explanation

In a ferrimagnetic material the magnetic interaction between any two dipoles is such that they align themselves *antiparallel* to each other.

But the magnitudes of any two adjacent *dipoles moments are not equal*. Hence the material possess a net magnetic moment. Hence if we apply a small external magnetic field, it will produce a large value of magnetisation of specimen along the direction of external field. Hence the material turns along the direction of external field. But above a *curie temp* T_C , similar to *Neel temperature* in antiferromagnetic materials the inherent magnetic moment disappears due to random arrangement of dipoles under thermal agitation.

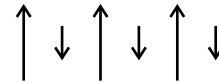


Fig. 11 Dipole magnetic moments are not equal

Properties

1. Direction of alignment in external magnetic field

When a ferrimagnetic material is suspended in a uniform magnetic field, it turns quickly along the direction of the applied field.

2. Magnetic moment

Ferrimagnet materials have permanent magnetic moment.

3. Alignment of dipoles

Since the distance between neighbouring atoms is very small, the dipoles align in opposite directions. This is *due to exchange interaction*. But the *magnitudes of magnetic moments of dipoles are not equal*. Hence the material possess a *net magnetic moment*. That is why ferrimagnetic materials are referred as *uncompensated magnetic materials*.

4. Magnetisation

Magnetisation in a ferrimagnetic material is parallel to the direction of external magnetic field.

5. Susceptibility (χ)

Susceptibility (χ) of ferrimagnetic materials is very large and is positive but not so high as ferromagnetic materials.

6. Dependence of susceptibility on temperature

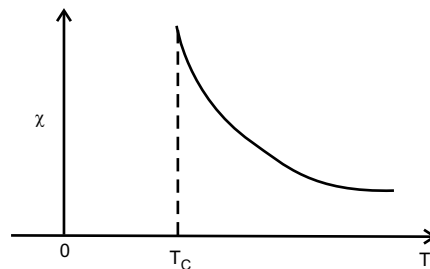


Fig. 12

The susceptibility of magnetic materials decreases externally with the increase of temperature (T) as shown in Fig.12.

Relation between χ and T is

$$\chi = \frac{C}{T + \theta}$$

7. Relative permeability (μ_r)

Relative permeability of ferrimagnetic materials is very large and is positive.

Relation between μ_r and χ is, $\mu_r = 1 + \chi$.

Examples: Ferrous ferrite, nickel ferrite, Ni - Zn composite ferrite.

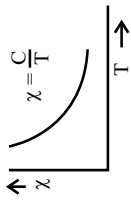
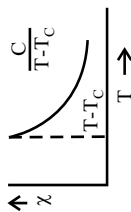
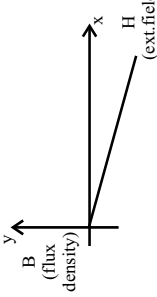
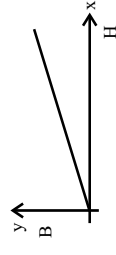
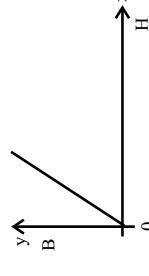
Advantages:

Ferrimagnetic materials are widely used in high frequency applications (where ferromagnetic materials cannot be used).

5.2.3 Comparison of dia, para and ferro magnetism

<i>Sl.No</i>	<i>Particulars</i>	<i>Diamagnetism</i>	<i>Paramagnetism</i>	<i>Ferromagnetism</i>
1.	When the material is suspended in a uniform magnetic field	Turns perpendicular to the direction of external field	Turns along the direction of external field	Turns along the direction of external field
2.	When the material is suspended in a non uniform magnetic field	Turns from the region of strong field to weak field	Turns from the region of weak field to strong field	Turns from the region of weak field to strong field
3.	Direction of internal field (magnetisation produced)	Opposite to the direction of external field	Along the direction of external field	Along the direction of external field
4.	Effect on magnetic lines of force	They repel the magnetic lines (of force) of external field	They attract the magnetic lines of external field	They attract the magnetic lines of external field
5.	Magnetic moment	Materials not having permanent magnetic moment	Materials have permanent magnetic moment	Materials have permanent magnetic moment
6.	Nature of magnetic susceptibility (χ)	χ is negative and is very low	χ is positive and is very low	χ is positive and is very large of the order of 10^5
7.	Dependence of χ on temperature	Independent of temperature	χ varies inversely with temperature that is $\chi = \frac{C}{T}$ at room temp $\chi = \frac{C}{T-\theta}$ at low temp	χ varies inversely with temperature that is $\chi = \frac{C}{T-\theta}$

θ = Paramagnetic curie temperature C = Curie constant

Sl.No	Particulars	Diamagnetism	Paramagnetism	Ferromagnetism
8.	Nature of graph between χ and T	—		
9.	Dependence on magnetisation with external field. (When the field is not so strong)	Varies linearly in -ve direction	Varies linearly having small +ve slope	Varies linearly having larger +ve slope
10.	Magnetic Relative permeability (μ_r)			
11.	Hysteresis (lagging of internal magnetisation with the external field)	Does not exhibit Hysteresis	Does not exhibit Hysteresis	Exhibit Hysteresis (There is a residual magnetism in the material after the external field is switched off.)
12.	Presence of Domains (A group of atomic dipoles oriented in a particular direction)	—	—	Presence of large number of domains
13.	Examples:	Sb, Bi, Zn, Ag, Au, Pb	Pt, Al, Cv, Mn, CuSoY liquid oxygen	Fe, Co, Ni

5.2.4 Domains theory of ferro magnetism

Weiss proposed the concept of domains in order to explain the properties of ferromagnetic materials and their hysteresis effect.

Ferromagnetic materials are very strongly magnetic. A ferromagnet has a spontaneous magnetic moment that is a magnetic moment even in zero applied field. The atoms (molecules) of ferromagnetic materials have a *net intrinsic* (internal) *magnetic moment* which is mainly *due to the spin of electrons*. The spin motion of electrons produces a current and in turn has a magnetic moment. Hence atoms of the materials are considered as atomic magnetic poles. The interaction between the neighbouring atomic magnetic dipoles is very strong. This interaction is known as spin *exchange interaction* and is present even in the absence of an external magnetic field. This exchange interaction aligns (makes) the neighbouring magnetic dipole moments parallel to one another and this spread over a small region [(1 – 0.1mm across) some times 10^{-6} m size (or) larger] of the bulk material. This small volume of the bulk is known as *domains*. In each individual domain, the magnetic moments of the atoms are aligned in the same direction. Thus domain is a region of the ferromagnetic material in which all the magnetic moments of the atoms (magnetic dipoles) are aligned to produce a net magnetic moment in one direction only. Hence each domain acts as a tiny magnet having a magnetic moment & axis (points in a direction). A bulk material consists of many domains. The direction of spontaneous magnetisation (Net magnetic moment) vary from domain to domain. The total magnetic moment of the material is given by the vector sum of magnetic moment of all the domains present in it. Since the domains are randomly oriented, the resultant magnetisation of material may be zero or nearly zero. This happens in the *absence of external field*.

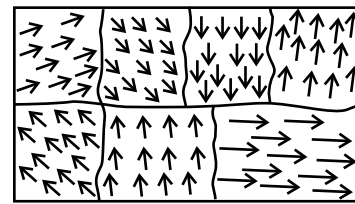
The boundaries separating the domains are called *domain walls* (or) *Bloch walls*. The domain walls are similar to grain boundaries in a polycrystalline material but domain walls are *thicker than grain boundaries*. Like grain growth, domain size also grow due to the movement of domain walls.

In an *unmagnetised piece* of Ferro magnetic material, the magnetic moment of the domains themselves are not aligned in a particular direction.

When an *external field is applied* there are *two ways* of alignment of a random domain as given below.

a) By motion of domain walls

The volume of the domains that are oriented (magnetised) in the direc-



Domains in a ferromagnetic material

Fig. 13

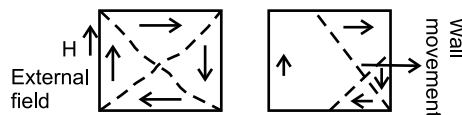


Fig. 14

tion of external field increases in size at the expense of the others by the *movement the walls* of domains towards unfavourably oriented with respect to external field.

b) By rotation of domains

As the magnetic field is increased to a larger value (near saturation) further *domain growth becomes impossible*. Therefore most favorably oriented and fully growth *domains tend to rotate* so as to be in complete alignment in the direction of external field.

A typical magnetisation curve (B-H curve) of a pure ferromagnetic material is shown in Fig.15.

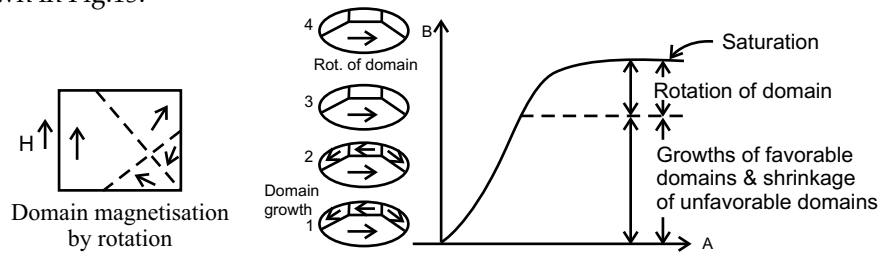


Fig.15

c) Effect of temperature

If a ferromagnetic material is heated to a high temperature, the thermal vibrations become so strong enough to offset the alignment of magnetic moments of dipoles within the domain. At a particular temperature, the material loses its ferromagnetic property and behaves like a paramagnetic material.

The temperature above which a ferromagnetic material becomes paramagnet is called as *Curie temperature* given by the relation $\chi = \frac{C}{T-T_C}$. Where χ is the magnetic susceptibility of the material variation of χ with respect to temperature (T) is shown in Fig.16.

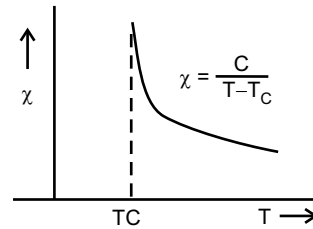


Fig. 16

Heisenberg’s quantum theory of ferromagnetism (or) Heisenberg’s criteria on internal field

Ferromagnetic materials have a large internal magnetic field even in the absence of an external magnetic field. The first theoretical explanation of the large internal field (Weiss field) in ferromagnetic materials was proposed by Heisenberg, in 1928. He pointed out that the magnetisation of ferromagnetic materials arises due to electron spin rather than due to orbital motion. It has been proved that it is impossible to explain an internal field of strength lying in the range of 10^6 to 10^7 oersted by classical dipole interaction. Hence Heisenberg interpreted (suggested) that such high value of inter-

nal fields can be explained in terms of *quantum exchange interactions* between the electrons.

An exchange force is essentially caused by a static coulombic interaction which is purely electrostatic in nature and its magnitude can be much larger than the magnetic interaction. This exchange force appears in the form of *spin-spin interaction* because orbital states of electrons naturally impose a *limitation* on the spin states according to *pauli's exclusion principle*.

The strength of interaction depends on the inter atomic separation and it may change its sign as the separation is varied. It can be predicted that as the two unpaired electrons (spins in the same direction $\uparrow\uparrow$) approach each other, the electron spins of unpaired electrons in each atom assume *parallel orientation*.

According to Heisenberg, if the spins of the electrons are parallel, the exchange interaction between the electrons in different quantum states leads to a lower energy. If the interatomic separation decreases more and more, the exchange force decreases until they pass through a zero value and an *antiparallel spin* is favoured. ($\uparrow\downarrow$)

Let us consider two electrons in a magnetic field, r_{ab} is the interatomic separation and r_0 is the orbital radius of electron of any material, and if $\frac{r_{ab}}{r_0} > 3$, then the *exchange energy is negative* and hence it is minimum for parallel interaction. Such a material exhibits ferromagnetic properties. For example ferromagnetic materials like Fe, Co, Ni and Gd have $\frac{r_{ab}}{r_0}$ ratio 3.26, 3.64, 3.94, and 3.12 respectively.

If the ratio $\frac{r_{ab}}{r_0} < 3$, then the materials are found to be non ferromagnetic. For example, non ferromagnetic materials like Cr and Mn have $\frac{r_{ab}}{r_0}$ ratio 2.60 and 2.94. Thus Heisenberg explained the large value of the intermolecular field successfully and gave *theories for ferro, antiferro and ferrimagnetic materials* on the basis of exchange interaction between spins.

Conclusion: Hence it is possible to combine two materials with unfavourable ratio of $\frac{r_{ab}}{r_0}$ (< 3 and > 3) to form a new compound whose ratio of $\frac{r_{ab}}{r_0} > 3$ and thus make it ferromagnetic. This is the usefulness of Heisenberg's interpretation discussed above. With the help of this theory, one can fabricate a new material with desired properties required for electromagnetic applications.

5.2.5 Hysteresis

The *lagging of flux density* (B) (magnetisation) behind the *magnetising field* H (external magnetic field) in the case of a *ferromagnetic material* is known as *Hysteresis*. The hysteresis graph represents the variation of flux density B with the change in the external magnetic field strength (H) in a ferromagnetic material.

Consider an unmagnetised specimen of ferromagnetic material subjected to an external magnetic field. When the magnetic field strength (H) is gradually increased from zero (point 0), the flux density (B) also increases rapidly upto a point A in the graph. It is observed that if the magnetic field strength (H) is increased further, the

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rate of increase of flux density (slope $\frac{dB}{dH}$) is reduced and finally becomes zero (slope) and the value of flux density reaches a saturation shown by the curve PC.

When the magnetic field strength (H) is reversed and slowly decreases to zero then the flux density (B) decreases not along CO, but along CPD and the flux density (B) not becomes zero when the external field strength (H) is zero. A certain amount of flux density represented by OD in the graph is still left with the material even when the external field strength is zero and is known as *residual magnetism* (or) *retentivity* and its represented in the graph as B_r . Thus retentivity of a specimen is defined as the magnetisation (or) flux density retained by the specimen when the magnetising field is reduced from saturation value to zero. Its magnitude is represented as B_r in the graph.

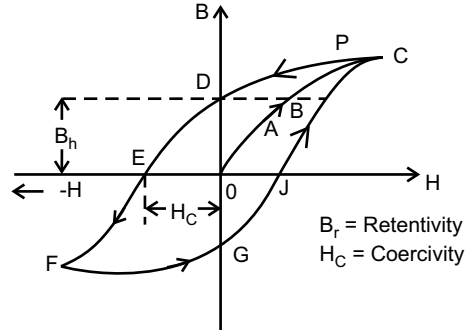


Fig. 17

If the external magnetic field (H) is decreased (or) increased in the negative direction, the flux density varies along DE and at E, the magnitude of flux density becomes zero. Hence there is no residual magnetism in the specimen at this stage. Hence in order to make the flux density left with the material to zero, a certain amount of external magnetic field represented by OE is applied in reverse direction. This negative value of the magnetic field causes demagnetisation of the specimen and it is called as the *coercive force* (or) *coercivity*. Hence *coercivity* of a specimen is defined as the amount of reverse external magnetic field required to make the residual magnetism zero (flux density $B=0$) (or) complete demagnetisation. Its magnitude is represented as H_c in the graph.

It is observed that the field strength (H) is decreased further (increased in negative direction) the flux density B, increases rapidly along the curve EF, until a negative saturation is reached at F. If the field is increased towards positive direction, we get a curve FGJC and the cycle is completed, the enclosed area of the curve CPDE-FGJC is called as hysteresis loop. It may be noted from the figure, that the *flux density (B)* always *lags behind the magnetic field strength (H)* and this phenomenon is known as *magnetic hysteresis*.

The work done on the ferromagnetic specimen gives an energy loss known as *Hysteresis loss*. The *hysteresis loss* is defined as the loss of energy in taking a ferromagnetic material through a complete cycle of magnetisation and the *magnitude of loss* is represented by the *area enclosed by the hysteresis loop*. Actually *domain structure changes* during the hysteresis.

5.2.6 Hard & Soft materials

Based on hysteresis loss, the magnetic materials can be classified into two types namely hard and soft materials. **Hard magnetic** materials have *large hysteresis loss* and hence the area of hysteresis loop (loop of B-H curve) is large. Eg: *carbon, steel, tungsten, steel chromium steel and Al-Ni-Co alloys (Alnico). Copper nickel iron alloys (cunife) and Copper nickel cobalt alloys (cunico).*

Soft materials have *low* hysteresis loss and hence the area of hysteresis loop is small. Eg: *Iron-silicon alloys, ferrous nickel alloys, Iron cobalt-alloys (Ferroites) and Garnets.* **Hard materials** are used for the *production of permanent magnets used in magnetic detectors, microphones, flux metres, voltage regulators & damping devices and magnetic separators.* **Soft materials** are used in *electromagnetic machinery* and transformer cores. They are also used in *switching circuits, microwave isolators and matrix storage of computers.*

5.2.7 Explanation of hysteresis on the basis of domain theory

The phenomenon of hysteresis can be successfully explained on the basis of domain theory. The *increase in value of the resultant magnetic moment* (magnetisation) of the specimen under the action of the applied field can be due to (1) *the motion of domain walls* and (2) *the rotation of domains.*

When a *weak magnetic field* is applied the domains which are aligned parallel to the external field and in the easy direction of magnetisation (inclined to the direction of magnetisation) grow in size at the expense of less favourably oriented domains by expanding the walls of domains. When the weak field is removed, the domains reverse back to their original state and size. This *reversibility* of wall movement is possible in the region OA of B-H curve.

When the *external field becomes stronger*, the movement of **Block** wall continues and the movement becomes irreversible. This behaviour is indicated by the portion of the curve AB of the B-H graph. The phenomenon of hysteresis (lagging of B behind H) is only due to this irreversibility under stronger magnetic field and at the point B, all domains have magnetised along their easy direction of magnetisation (still not along the direction of external field).

Application of still higher fields, the process of rotation of domains takes place and now all the domains are made to align in the direction of external field, which may be away from their easy direction thereby storing anisotropy energy. When once the domain rotation is complete *the specimen is saturated.* This process is represented by the part ep of the B-H curve. Now *all the domains align in the direction of external field.*

On removal of the field the specimen try to attain the original configuration by the movement of block walls. But this movement is related to impurities lattice imperfections etc. Hence more energy is required to reduce the magnetisation of specimen to

zero. The amount of energy spent in this process (reverse back to original condition) is known as **Hysteresis loss** and this loss enclosed by the represented by the **area of hysteresis loop of B-H curve**.

Bohr magneton is the magnetic moment produced by one unpaired electron in an atom. It is the basic (quantum) unit of atomic magnetism, represented by μ_B

$$1\mu_B = 9.27 \times 10^{-24} \text{ Am}^{-2}$$

5.2.8 Energy product of a magnetic material

For a permanent magnet, we require a hard material with a high saturation magnetisation, high residual magnetism (**remanence**) and **high coercivity**. For such a material, the area of hysteresis loop is also large (B-H curve). The area of the hysteresis loop between B_r and H_C represents the energy required to demagnetise a permanent magnet. The maximum value of this area, that is the **product** $B_r \times H_C$ is called as the **energy product**. The area of the loop of B-H curve second quadrant is often used to measure the effectiveness of the material as a permanent magnet. The **area must be as large as possible for permanent magnets**.

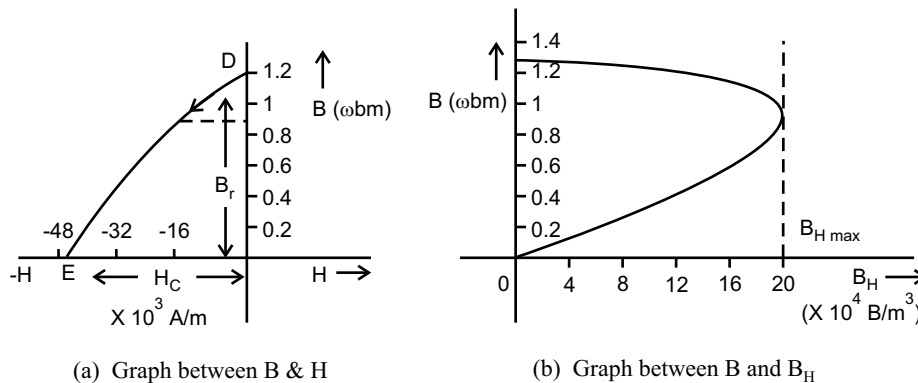


Fig. 18

Discussion

The second and fourth quadrant of B-H curve, the loop represents the demagnetising curves of the material and is related to the energy required to demagnetise the magnet. It is conventional to represent the second quadrant of B-H curve as the demagnetising curve. The energy product BH versus magnetic induction B curve for a hard material is shown in figure.

It is seen from the graph that the energy product is zero at the points D and E and it attains maximum at an intermediate point. Known as the maximum energy product BH_{max} . Actually the maximum **energy product of a hard material** is given by the area of the **largest rectangle that can be inscribed in a demagnetising curve**.

Application

The energy product should be as large as possible for permanent magnets. Hence the materials with higher values of energy product can be used as a magnetic recording materials and these materials can be selected by drawing graph between B and BH (product of B and H).

5.2.9 Ferrites and their application

Ferrites belong to soft magnetic materials and they are also called as **ferrimagnetic materials**. They are complex compounds of different metals and oxygen and they exhibit the property of ferromagnetism.

The general chemical formula of ferrite molecule's MFe_2O_4 (or) MFe_2O_4 (in ionic form $\text{M}^{+2}\text{Fe}_2^{+3}\text{O}_4^{-2}$). It is a mixture of MO and Fe_2O_3 , where M represents a divalent metals (having two electrons in the outermost shell) like Fe, Co, Mn, Zn, Ca and Mg.

Since the oxide contains ions, the magnetic properties can be predicted with some good approximation each Fe^{+2} ion corresponds to 4 Bohr magneton (unit of measuring atomic magnetism) and each Fe^{+3} corresponds to 5 Bohr magneton. A material such as $\text{Fe}^{+2}\text{Fe}_2^{+3}\text{O}_4^{-2}$ exhibits a saturation magnetisation which amounts to 4 Bohr magneton. If all the ions are lined up in parallel, we can expect $4 + (2 \times 5) = 14$ magneton per molecule ($4\mu_B$ for Fe^{+2} , $10\mu_B$ for 2Fe^{+3}). This difference in the estimation of atomic magnetism can be accounted on the basis of arrangement of atoms in ferrites.

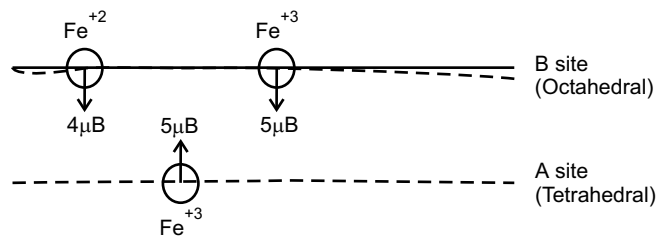


Fig. 19 Schematic arrangement of Fe^{+2} and Fe^{+3} ions magnetite.

The oxygen ions in a ferrite form a close packed face centred cubic structure (FCC). It is found that for every four O^{-2} ions, there are two octahedral holes surrounded by six O^{-2} ions and 1 tetrahedral hole surrounded by four O^{-2} ions. The metal ions are distributed over tetrahedral sites (A site) and octahedral sites (B site). Thus octahedral sublattice has twice as many sites as in tetrahedral one. There are two Fe^{+3} ions and one Fe^{+2} ion per molecule of Fe_3O_4 , Fe^{+3} magnetic moments just cancel each other ($5 - 5 = 0$) and hence the net magnetic moment per molecules of ferrite is $4\mu_B$. This is in good agreement with experiment. The behaviour of other ferrites may be explained in a similar manner.

Applications

1. Ferrite rods used in radio receiver to increase the sensitivity and selectivity of the receiver.
2. Ferrites are used as cores in audio and T.V. transformers.
3. Ferrites are used in digital computers, data processing circuits and digital data storage devices.
4. Ferrites are used to produce low frequency ultrasonic waves in magnetostriction method.
5. Ferrites are widely used in non reciprocal microwave devices like Gyrotator, circulator and isolator.
6. Ferrites are used in power limiting and harmonic generation devices.
7. They are used in the design of ferromagnetic amplifiers of microwave signals.
8. Hard ferrites are used to make permanent magnets which are used in galvanometers, ammeters, voltmeters, speed meters, wattmeters and recorder.
9. Ferrite core can be used as bistable element.
10. Rectangular ferrite cores can be used as a magnetic shift register.

5.3 Dielectrics

5.3.1 Introduction

Dielectrics are essentially *insulating materials*. Dielectric materials are used to store electric energy. The basic function of an insulating material is to resist the flow of electric current and at the same time the dielectric material stores electric energy. For a material to be a good dielectric, it must be an insulator. Hence any insulator is a dielectric.

They are *non metallic materials* having *high specific resistance* and *negative temperature coefficient*. In a dielectric, all the electrons are bound to their parent molecules and there are no free charges, electrons are not released under normal voltage applied or thermal energy supplied.

5.3.2 Description of dielectric in terms of band theory of solids

The property of dielectric can be explained in terms of band theory (structure) of solids. The energy band diagram of an insulating materials (dielectric) is shown in Fig.20. There is a wide energy gap which is of the order of 3eV more between conduction band and valence band.

This energy gap is so wide that at ordinary temperature, electrons cannot jump from valence band to conduction band. As a result of this, no conduction takes place at ordinary temperature. But as the temperature increases, the resistance of dielectric decreases.

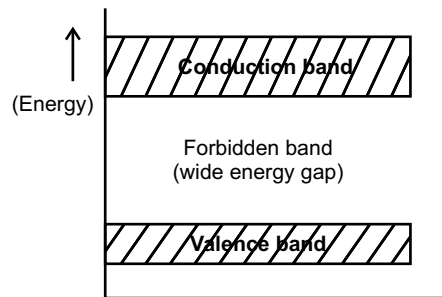


Fig. 20

The common examples of dielectric are rubber, bakelite, mica, glass etc.

5.3.3 Application

1. Dielectrics are widely used as *insulating materials*. Some examples are given below.
 - (a) Copper (or) Aluminium wires are used as *conductors in electrical circuits* but they are insulated with a outer jacket of plastic (or) rubber (dielectric).
 - (b) In *heater coils* (filament) dielectric like *ceramics beads* are used to avoid short circuiting.
 - (c) In *electric iron box*, mica (or) *asbestos* are used to prevent the flow of electric current to outer body.
 - (d) In transformer and generator winding, *varnished cotton* is used as insulator to prevent short circuit.
 - (e) In electrical transmission lines *porcelain structures* are used as insulators between points of different potential.
2. Another important application of dielectric materials is their use as *storage capacitors*.

Depending up of the type of dielectric used and application, they are classified in to the following groups.

(a) Capacitors with air (or) vacuum, or inert gas as dielectric (gas dielectric)

When the values of these capacitors are not so large, the dielectric losses are very less. They are used in radio frequency circuits and low frequency measuring circuits.

(b) Capacitors with mineral oil as dielectric (liquid dielectric) and electrolytic solution as dielectric

These capacitors have a very large value of capacitance. But dielectric losses are not so small. They are used in high voltage applications (power pack and filtering circuits).

(c) Capacitors with solid dielectric

In these capacitors, the dielectric materials like mica, paper glass etc can be used. Also dielectric constant does not vary with temperature. They have high value of dielectric constant and high insulating resistance.

They are used as standard capacitors in the laboratories and in electrical circuits.

(d) Capacitors with both solid and liquid dielectric

In this type of capacitors combination of solid dielectrics like paper, glass, mica, thin films of synthetic materials etc and liquids like mineral oil, silicon oil and castor oil etc are used. These capacitance have high value of capacitance.

They are used in electrical power distribution systems to provide power factor correction.

5.3.4 Fundamental definitions

1. Electric field and electric lines of force

The space round a charge, where the influence of electricity is felt (or) in which it exerts a force on another charge. The electric field is assumed to consist of electric lines of force. These lines of force start from positive charge and end on the negative charge. It is a line along which a unit positive charge is free to move.

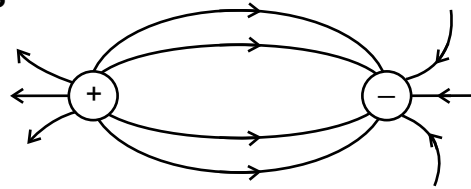


Fig. 21

The tangent at any point on the electric line of force gives the direction of electric at that point.

2. Electric field strength (or) electric field intensity (E)

Electric field strength at a point in an electric field is the force experienced by a unit positive charge placed at that point. It is represented by the letter E.

If q is the magnitude of charge in coulomb and F is the force experienced by the charge in Newton, then

Electric field strength $E = \frac{F}{q}$

Also $E = \frac{V}{d}$

V = Voltage applied

d = Distance between the plates

unit E is N/C (or) V/m.

3. Electric flux (ψ)

It represents the total number of electric lines of force starting from a positive charge. It is numerically equal to the charge. It has the same unit as charge.

Unit of ψ is coulomb.

4. Electric flux density (or) electric displacement vector (D)

Electric flux density at a point is defined as the flux (ψ) (or) electric lines of force passing normally through the unit area of cross section at that point. It is represented by the letter D.

If ψ is the total flux in coulombs

$$A = \text{Area of cross section in metre}^2$$

then Electric flux density

$$D = \frac{\psi}{A}$$

unit of D is C/m^2 .

5. Permittivity (ϵ)

It represents the effectiveness (or) easiness of a medium (or) substance to allow the electric flux through it. It is defined as the ratio of electric displacement (D) in a dielectric medium to the applied field strength (E). Permittivity is represented by the latter ϵ ,

$$\text{Permittivity } \epsilon = \frac{D}{E}$$

Permittivity can also be defined as the degree up to which a medium can restrict the flow of charge through it and it is always greater than 1.

If ϵ_0 = absolute permittivity of free space

$$= 8.854 \times 10^{-12} F/m \text{ and}$$

ϵ_r = Relative permittivity (or) dielectric constant of a medium

then permittivity ϵ of a medium is given by

$$\epsilon = \epsilon_0 \times \epsilon_r$$

unit of ϵ is Farad/metre (F/m).

6. Dielectric constant (ϵ_r)

The dielectric characteristic of a material can be determined by the dielectric constant (or) relative dielectric constant (ϵ_r) of the material.

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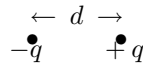
It is defined as the ratio of the permittivity of the medium (ϵ) to the permittivity of free space (ϵ_0) and is represented by ϵ_r

Dielectric constant (or) relative permittivity $\epsilon_r = \frac{\epsilon}{\epsilon_0}$.

Since it is ratio, it has no unit. It also gives the measure of polarisation (alignment of charges) in the dielectric material under external electric field.

7. Dipole

Two charges of equal magnitude but of opposite polarity separated by a small distance form a dipole. It is represented in the following figure



8. Dipolemoment (μ)

The product of either charge (one of the charge) on the dipole and distance between the charges is defined as the dipole moment represented by μ and it is directed from $-q$ to $+q$.

The dipolemoment $\mu = q \times d$

unit of dipolemoment is coulomb – metre ($C - m$).

9. Electric polarisation (\vec{p}) (or) polarisation vector

The dipolemoment (μ) per unit volume of dielectric material is known as polarisation. It is represented by the letter P

$$P = \frac{\mu}{V} \quad \text{where } V = \text{volume of dielectric in } m^3.$$

If $\vec{\mu}$ is the average dipole moment per molecule and N is the number of molecular per unit volume then

$$\vec{P} = N\vec{\mu}$$

Explanation

Consider an atom of dielectric material placed inside an electric field. The centre of positive charge is displaced along the direction of electric field (+ve polarity to -ve polarity) and the centre of negative charge is displaced in opposite direction. Thus a **dipole is produced**.

Hence when a dielectric material is kept in an electric field, dipoles are created in

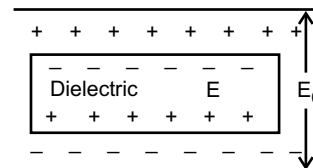


Fig.22

all the atoms of the material. The process of producing electric dipoles in an electric field which are oriented along the field direction is known as **polarisation** in dielectrics. But the field produced in dielectric (E) is opposite to the applied field (E_0).

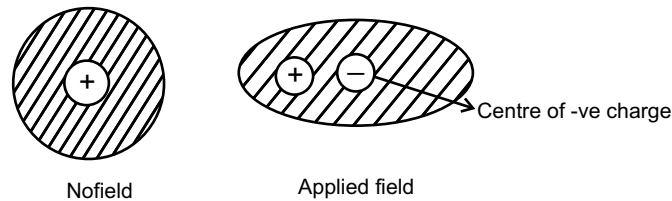


Fig.23

10. Polarizability (α)

When a dielectric material is kept in an electric field and if the strength of electric field is increased the strength of induced dipoles also increases. Thus the induced dipole moment (μ) is directly proportional to the strength of the applied field (E).

that is $\mu \propto E$

$$\mu = \alpha E$$

Where α is a constant of proportionality known as **polarizability**

$$\alpha = \frac{\mu}{E} = \frac{\text{Induced dipole moment}}{\text{Strength of applied field}}$$

Unit of α is F/m^2

Relation between P , ϵ_0 , ϵ_r and E is

$$P = \epsilon_0(\epsilon_r - 1)E$$

11. Electric flux (D) in terms of field strength

Since the electric flux density (D) of a dielectric material is directly proportional to the field strength (E), then D can be written as

$$D = \epsilon_0 \epsilon_r E \tag{15}$$

Since polarisation measures the additional field density in the presence of material as compared to the vacuum, D can be written as

$$D = \epsilon_0 E + P$$

Sub for D from (15), we have

$$\begin{aligned} \epsilon_0 \epsilon_r E &= \epsilon_0 E + P \\ \therefore P &= \epsilon_0 \epsilon_r E - \epsilon_0 E \end{aligned}$$

$$P = \epsilon_0(\epsilon_r - 1)E$$

Hence flux density (D) is similar to the magnetic induction (B).

12. Electric susceptibility (χ_e)

Since polarisation vector \vec{p} is proportional to the applied field E and is in the direction of E , the polarisation vector \vec{p} can be written as

$$p = \epsilon_0 \chi_e E$$

$$\therefore \chi_e = \frac{p}{\epsilon_0 E}$$

Sub for p

$$\chi_e = \frac{\epsilon_0(\epsilon_r - 1)E}{\epsilon_0 E}$$

$$\chi_e = \epsilon_r - 1$$

It is a measure of a substance to get electrified.

5.3.5 Measurement of dielectric constant (or) relative permittivity (ϵ_r)

The polarisation (or) dielectric constant (ϵ_r) of a dielectric can be measured by using a parallel plate capacitor.

First the parallel plates of a capacitor are separated by vacuum and if a voltage V is applied to the capacitor, a charge is developed on each plate (equal magnitude). Measure the capacitance C using a L-C-R meter and C can be written as

$$C = \frac{\epsilon_0 A}{d} \quad (16)$$

where A = area of one of the plates.

Now if a dielectric material is introduced between the plates, it will increase the charges on the plates due to polarisation in the material. Again capacitance C_1 is measured by using L-C-R metre then

$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d} \quad (17)$$

$$\therefore \frac{C_1}{C} = \epsilon_r \quad (18)$$

The ratio $\frac{C_1}{C}$ gives the value of dielectric constant of dielectric material.

5.3.6 Various polarisation mechanism (processes) in dielectric

We know that the net charge within a *neutral atom* is zero. Further more, the centre of negative charge of electrons coincides with centre of positive charge of nucleus and hence the net dipole of an atom is zero.

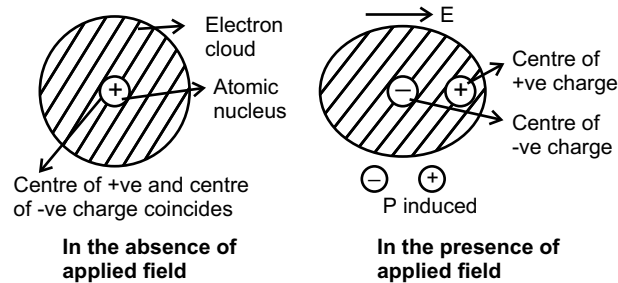


Fig. 24

When an atom is placed in an external field, an induced dipole moment is developed. The electrons, being much lighter than the positive nucleus, are easily displaced by the field, which results in the separation of centre of negative charge and centre of positive charge. Hence a *dipole* is created.

The separation of negative and positive charges and the resulting dipole moment are termed as *polarisation*. An atom is said to be polarised, if it has *effective dipole moment*.

Polarisation occurs due to several mechanisms. When a dielectric material is placed inside d.c. electric field, the polarisation takes place due to four types of processes mentioned below.

- (a) Electronic polarisation
- (b) Ionic polarisation
- (c) Orientation polarisation
- (d) Space charge polarisation.

(a) Electronic polarisation (or) induced polarisation (p_e)

This polarisation is due to the displacement of positively charged nucleus and the negatively charged electrons (electronic cloud) in the opposite direction under the external electric field. As a result a dipole moment is created within each atom. The extent of this shift (displacement) is proportional to the field strength.

The dipole moment is given by the product of the charge and shift distance ($q \times d$). But induced dipole moment is proportional to the strength of external field E .

$$\therefore \mu_e \propto E$$

$$\boxed{\mu_e = \alpha_e E} \quad (19)$$

where $\alpha_e = \text{electronic polarizability}$.

But the dipole moment per unit volume is known as *electronic polarisation* represented by p_e

$$p_e = N \mu_e \quad (20)$$

$$\boxed{p_e = N \alpha_e E} \quad (21)$$

where $N = \text{Number atoms}/m^3$.

This electronic polarisation occurs in all dielectrics for any state of arrangement. It sets over a very short period of time, of the order of 10^{-14} to 10^{-15} sec (fermi second). It is independent of temperature. The process of polarisation is already represented in Fig.

Calculation of electronic polarizability (α_e)

A simple classical model of an atom is shown in Fig.25. Here a nucleus of charge Ze is surrounded by an electronic cloud of charge $-Ze$. This electronic charge distributed over a sphere of radius R .

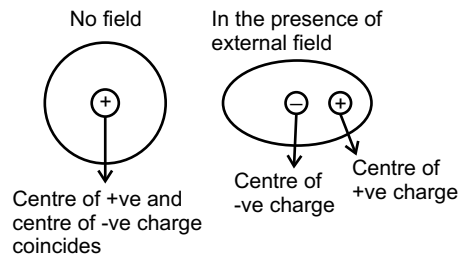


Fig.25

Then the charge density of electron cloud is

$$P = \frac{\text{charge}}{\text{volume of space (atom)}}$$

$$P = \frac{-Ze}{\frac{4}{3}\pi R^3} = \frac{-3Ze}{4\pi R^3} \quad (22)$$

where an external field of intensity E is applied, the nucleus (with the charge Ze) and electron (with the charge $-Ze$) experiences a Lorentz force of magnitude,

$$F = E q \quad \text{where } q = Ze$$

$$\boxed{F = Ze E} \quad (23)$$

The Lorentz forces on nucleus and electron act in opposite direction. Hence nucleus and electron are pulled a part (A dipole is created).

When they are separated, there is a coulombic force of attraction developed between them. This force opposes the displacement produced by Lorentz force (which

separates them) when these forces are equal and opposite, an equilibrium is reached and x be the displacement between centre of positive charge (nucleus) and centre of electronic charge (electron cloud). Since nucleus is much heavier (1836 times of electron) than electron cloud, it is assumed that the electron cloud is displaced under external field.

$$\begin{aligned} \therefore \text{ Lorentz force} &= Eq = -E(Ze) \quad q = -Ze \text{ for electron} \\ &= -Ze E \\ \text{Coulombic force} &= \frac{-(Ze) \times \text{charge enclosed in a sphere of radius}}{4\pi \epsilon_0 x^2} \quad (24) \\ \text{charge enclosed} &= \frac{4}{3}\pi x^3 P \end{aligned}$$

Sub. for P from (22)

$$\begin{aligned} \text{charged enclosed } (Q) &= \frac{4}{3}\pi x^3 \left(\frac{-3Ze}{4\pi R^3} \right) \\ Q &= \frac{-Ze x^3}{R^3} \end{aligned}$$

Sub for Q in (24)

$$\begin{aligned} \therefore \text{ coulombic force} &= \frac{Ze}{4\pi \epsilon_0 x^2} \left(\frac{-Ze x^3}{R^3} \right) \\ \text{coulombic force} &= \frac{-Z^2 e^2 x}{4\pi \epsilon_0 R^3} \quad (25) \end{aligned}$$

In the equilibrium position

Lorentz force = coulombic force

$$\begin{aligned} -Ze E &= \frac{-Z^2 e^2 x}{4\pi \epsilon_0 R^3} \\ \boxed{x = \frac{4\pi \epsilon_0 R^3 E}{Ze}} \quad (26) \end{aligned}$$

The displacement of electron cloud is proportional to the strength of applied field E . The two electric charges Ze and $-Ze$ separated by a distance x under the action of external field form an **induced electric dipole** producing **induced dipole moment**.

Induced dipole moment $\mu_e =$ one of the charge \times dist. between them

$$\text{Induced dipole moment } \mu_e = (Ze) \times x \quad (27)$$

Sub. for x

$$\begin{aligned} \text{Induced Dipole moment} \quad \mu_e &= Ze \frac{(4\pi \epsilon_0 R^3 E)}{Ze} \\ \boxed{\mu_e = 4\pi \epsilon_0 R^3 E} \quad (28) \end{aligned}$$

$$\boxed{\mu_e = \alpha_e E} \quad (29)$$

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where $\alpha_e = 4\pi \epsilon_0 R^3 E$ is known as **electronic polarizability**.

The dipole moment per unit volume is called as electronic polarisation represented by P_e

$$\therefore P_e = N \mu_e \quad (30)$$

Sub for μ_e electronic polarisation is given by

$$\boxed{P_e = N \alpha_e E} \quad (31)$$

But $P_e = E \epsilon_0 (\epsilon_r - 1) = N \alpha_e E$

Eq.(31) represents

$$\text{electronic polarizability } \alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N} \quad (32)$$

(b) Ionic polarisation (P_i)

Ionic polarisation is due to the displacement of cations and anions in opposite directions and takes place in an ionic solid, suppose an electric field is applied in the positive x direction. The positive ions move towards right by x_1 , and negative ions move towards left by x_2 . Assuming that each unit cell contain one cation and one anion, the resultant dipole moment per unit cell due to ionic displacement is given by

$$\mu_i = ex_1 + ex_2$$

where e = magnitude of charge of each ion.

$$\mu_i = e(x_1 + x_2) \quad (33)$$

Let β_1, β_2 represent the restoring force constants and F is the force due to the applied (external) field in Newton. Since the displacement of ions is proportional to the applied field, then force is given by

$$F = \beta_1 x_1 = \beta_2 x_2 \quad (34)$$

$$x_1 = \frac{F}{\beta_1} \quad (35)$$

$$x_2 = \frac{F}{\beta_2} \quad (36)$$

The restoring force constants (β_1, β_2) depends on (1) mass of the ions and (2) angular frequency of molecule (ω_0) in which ions are present, then

Lorentz force = centripetal force

$$Ee = m \omega_0^2 x_1$$

$$\therefore x_1 = \frac{Ee}{m \omega_0^2} \quad (37)$$

similarly $x_2 = \frac{Ee}{M\omega_0^2}$

where $m = \text{mass of cation}$
 $m = \text{mass of negative ion}$

\therefore Resultant displacement $= x_1 + x_2 = \frac{eE}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$ (38)

Resultant dipolemoment $\mu = e(x_1 + x_2)$

Sub for $x_1 + x_2$ from (38)

$\mu_i = \frac{e^2 E}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$ (39)

But $\mu_i \propto E$

$\mu_i = \alpha_i E$ (40)

where $\alpha_i = \text{Ionic polarizability}$

$\alpha_i = \frac{\mu_i}{E} = \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$
 $\alpha_i = \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)$ (41)

where $\left(\frac{1}{M} + \frac{1}{m} \right)^{-1} = \frac{Mm}{M+m}$ is called as **reduced mass of ionic molecule**.

Here **ionic polarizability** is (a) **directly proportional to the square of the charges** of ions and (b) inversely proportional the square of the **natural frequency** of the ionic molecule. The process of ionic polarization is given in Fig.26.

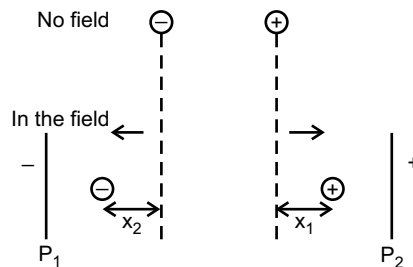


Fig. 26 Electric field is applied between plates P_1 & P_2

Ionic polarisation (P_i) is given by

$P_i = N \alpha_i E$ (42)

(c) Orientation polarisation (P_0)

The orientation polarisation is due to the existence of a permanent dipole moment in the electric medium. For example in methane (CH_4) molecule, the centre of positive charges coincides so that there is no permanent dipole moment on the other hand, in CH_3Cl , the centre of positive charge and the centre of negative charge do not coincide. Since the electronegativity (tendency to pull the electrons towards it self in a hetero nucleus molecule) of chlorine atom is more than that of hydrogen, the chlorine atom pulls the bonding electrons nears itself more strongly than hydrogen. Hence even in *the absence of an electric field*, this molecule *carries a dipole moment*.

When an electric field is applied, the molecules having permanent displacement align themselves in the direction of the applied field. The polarisation due to such alignment in the presence of applied field is known as orientation polarisation and it depends on temperature. As the temperature increases, the orientation polarisation (P_0) decreases. Since thermal energy tends to produce random arrangement there by disturbing the alignment. This *orientation polarisation* occurs when the frequency of applied field is the *range of audio frequencies of the order* of 20 KHz.

The orientation polarisation P_0 is given by

$$P_0 = N \vec{\mu} \quad (43)$$

$$\text{where } \vec{\mu} = \frac{\mu^2 E}{3KT} \quad (44)$$

$$P_0 = \frac{N\mu^2 E}{3KT} \quad (45)$$

$$\text{But } P_0 = N \alpha_0 E \quad (46)$$

where $\alpha_0 = \text{orientation polarizability}$

$$\therefore \alpha_0 = \frac{P_0}{NE} \quad (\text{or) comparing equation 45 \& 46} \quad (47)$$

$$\text{we get } \alpha_0 = \frac{\mu^2}{3KT} \quad (48)$$

The α_0 is inversely proportional to the absolute temperature. The process is as shown in Fig.27.

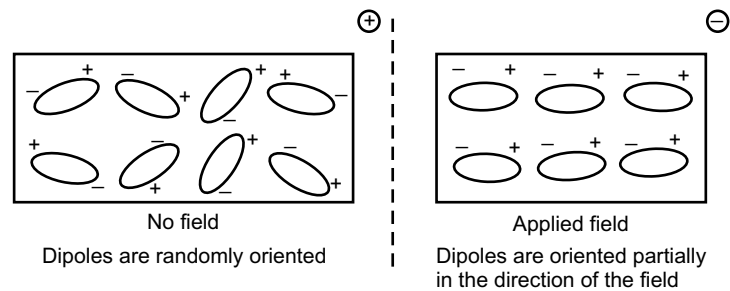


Fig.27

(d) Space charge polarisation (P_s)

This polarisation occurs due to the accumulation of charges at the electrodes or interfaces in multi phase material. The ions diffuse over appreciable distances in the presence of the applied field, giving rise to a redistribution of charges in the dielectric medium.

The *space charge polarisation* (P_s) occurs, when the frequency of applied field is in the range of *power frequencies* (about 50-60 Hz). It depends on temperature. It increases with the increase in temperature. It also *depends* on the *applied field strength* (E) also.

The space charge polarisation is given by

$$P_s = N \alpha_s E$$

Where α_s =space charge polarizability.

Since α_s is very small when compared with other polarizabilities, and it can be neglected. The process of polarisation is shown in Fig.28.

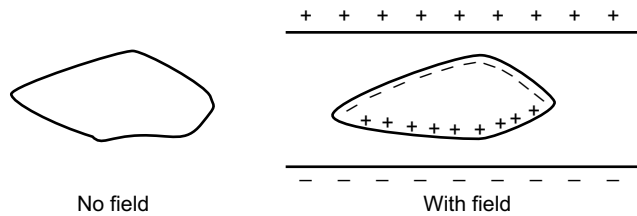


Fig.28

Total polarisation (P)

The total polarisation (P) of a material under the influence of applied field is given by the sum of the contribution of the electronic, Ionic, orientational and space charge polarisation discussed above.

$$\therefore P_{total} = P_e + P_i + P_o + P_s \tag{49}$$

Since *space charge polarizability* α_s is very small when compared with other types of polarizabilities, the term $P_s = N \alpha_s E$ can be neglected.

\therefore Total polarisation is given by

$$P_{total} = P_e + P_i + P_o \tag{50}$$

Total polarizability is given by

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

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Sub for α_e , α_i , & α_o , we get

$$\alpha = 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{\mu^2}{3KT} \quad (51)$$

Hence total polarisation of a material is given by

$$P_{total} = N \alpha E$$

Sub for α

$$P_T = N E \left\{ 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{\mu^2}{3KT} \right\} \quad (52)$$

Equation (52) is known as *Langevin-Debye equation*.

5.3.7 Effects of frequency and temperature on polarisation and dielectric constant

1. Effect of frequency of the applied field on polarisation and dielectric constant

The *polarisation* and consequently dielectric constant *depends* on the *frequency* of the *applied field*. This phenomenon is called *dielectric dispersion*. On the application of an electric field on a dielectric material, a polarisation takes place as a function of time. The polarisation $P(t)$ as a function of time is given by

$$P(t) = P \left(1 - e^{-\frac{t}{T_r}} \right)$$

where P = The maximum polarisation (polarisation amplitude) attained in a time t .

T_r = The relaxation time for a particular type of polarisation process.

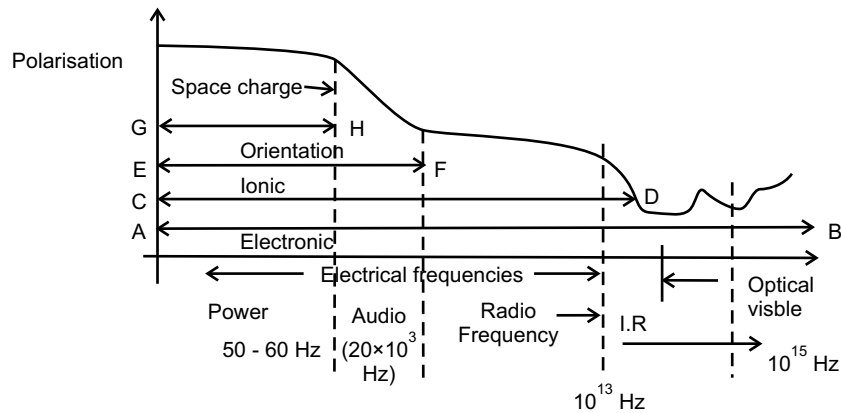


Fig. 29

The relaxation time is measure of the *time scale* of a polarisation process.

Relaxation time (t_r) is defined as the time taken for a *polarisation* process to *reach 0.63 of maximum value*. It is a constant for a particular process and is different for different polarisation process.

The variation of polarisation with the frequency of the applied field is shown in Fig.29.

Discussion

(a) Electronic polarisation

The *electronic polarisation* is extremely rapid (fast) and is essentially complete at the instant the voltage is applied. It occurs at all frequencies. It occurs even when the frequency of the applied voltage is very high of the order of optical frequencies ($\sim 10^{15}$ Hz) and it occurs during every cycle of applied voltage (Range AB in graph).

(b) Ionic polarisation

Ionic polarisation is slower than electronic polarisation, Ionic polarisation is due to the displacement of ions through a distance in the external field. Since ions are heavier than electron cloud, the time taken for the displacement is larger and hence it is a slower process than electronic polarisation. The frequency with which ions are displaced is of the same order of the lattice vibrational frequency (10^{13} Hz). Hence these ions do not respond to optical frequency that is, the time for required for lattice vibration is 100 times more than the period applied voltage at optical frequency. *If the frequency of the applied voltage is less than 10^{13} Hz, ionic polarisation will take place.* At 10^{13} Hz, both electronic and ionic polarisation take place. (Range CD in graph).

(c) Orientation polarisation

It is even *slower than ionic polarisation*. The relaxation time for orientation polarisation in a liquid dielectric is less than in a solid dielectric. For example for liquid propyl alcohol, the relaxation time is 10^{-10} s where as for solid ice it is 3×10^{-6} s. Orientation polarisation takes place when frequency of the applied voltage is in the range of audio frequencies (about 20 KHz) (Range EF in graph).

(d) Space charge polarisation

It is the slowest *polarisation process* among the four *polarisation process*. Since it involves the diffusion of ions over several interatomic distances, it takes a longer relaxation time than other process. Its relaxation time is related to the successful jump of ions under the influence of the applied field of frequency 100 Hz. Hence space polarisation takes place at machine (power) frequencies (about 50 - 60 Hz). (Range GH in the graph).

Conclusion

1. Hence we can conclude that the electronic polarisation occurs at *optical frequencies* (10^{15} Hz). Ionic polarisation at 10^{13} Hz along with electronic polarisation. For frequencies 10^6 to 10^{10} Hz, orientation polarisation takes place along with ionic and electronic polarisation. At 100 Hz (power frequencies) space charge polarisation takes place.
2. It has been observed from the graph as the frequency increases, dielectric constant of material decrease. (That is polarisation decreases as frequency as increases). Thus at low frequencies, the value of polarisation (coordinate in the graph) is very high and at high frequencies its value small.

2. Effect of temperature on polarisation and dielectric constant

It has been observed that

- (a) The electronic polarisation is independent of temperature.
- (b) The ionic polarisation is also independent of temperature.
- (c) *The orientation polarisation decreases with the increase of temperature.* It is because of the fact that the thermal energy tends to randomize (disturb) the alignment of dipole moment. Hence as the temperature increases, *dielectric constant of materials decreases* (since polarisation decreases).
- (d) The space *charge polarisation increases with the increases of temperature.* It is because of the fact that the *thermal energy* may aid *in overcoming* the *activation* barrier for the *orientation* (alignment) of polar molecules in the direction of the applied field. Hence *dielectric constant increases with the increase of temperature.* (since polarisation increases with the increase of temperature)

5.3.8 Expression for Internal field (Lorentz method) and Clausius - Mosotti equation

Consider a dielectric material (a slab) placed between the plates of a parallel plate capacitor. Let an external electric field is applied across the plates of the capacitor.

Let us imagine a spherical cavity is formed around the atom A inside the dielectric slab. It is also assumed that the radius (size) of the cavity is large when compared to the radius (size) of the atom.

The internal field at the atom site A can be considered to be made up of four components of electric field intensities namely E_1 , E_2 , E_3 and E_4 which are discussed as follows:

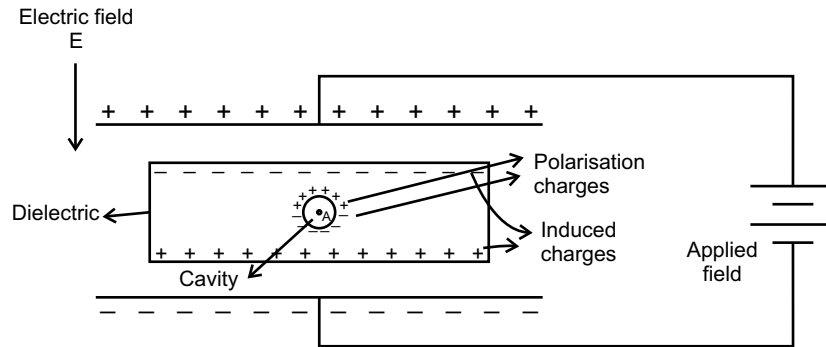


Fig.30

(a) Electric field intensity E_1

This *field intensity* E_1 at A is due to the *charge density on the plates of capacitor* (across which an external field is applied).

From theory we can write

$$D = \epsilon_0 E_1 \tag{53}$$

where D = Electric flux density in vacuum (due applied field)

ϵ_0 = permittivity of air (or) vacuum,

$$\therefore E_1 = \frac{D}{\epsilon_0} \tag{54}$$

$$\text{But } D = D + \epsilon_0 E \tag{55}$$

When the polarisation takes place additional flux density (P) arises due to the presence of dielectric material along with air or free space.

Hence P = Flux density due to dielectric material present.

$\epsilon_0 E$ = Flux density in vacuum (or) air.

Sub for D in (54)

$$E_1 = \frac{P + \epsilon_0 E}{\epsilon_0} = \frac{P}{\epsilon_0} + E$$

$$E_1 = E + \frac{P}{\epsilon_0} \tag{56}$$

(b) Electric field intensity E_2

This *field intensity* E_2 at A is due to the *charge density induced on two sides of the dielectric material* (kept between plates of a charged capacitor).

$$\text{Hence } E_2 = -\frac{P}{\epsilon_0} \quad (P = \text{flux density in the slab})$$

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Negative sign shows that this field is opposite to external field (E) [acts up] in Figure.

(c) Electric field intensity E_3

This *field intensity* at A is due to the other *atoms present in the cavity*. If we assume a cubic structure, $E_3 = 0$ due to the symmetric structure (fields due to atoms surrounding the considering atom cancel each other).

(d) Electric field intensity E_4

This *field intensity* E_4 at A is due to *the polarisation charges on the surface of cavity* around the atom A. This can be calculated by Lorentz method as given below.

In the Fig.31 if dA is the surface area of the sphere of radius r lying between θ and $\theta + d\theta$. Where θ is the angle made by the element of sphere with reference to the applied force (field E).

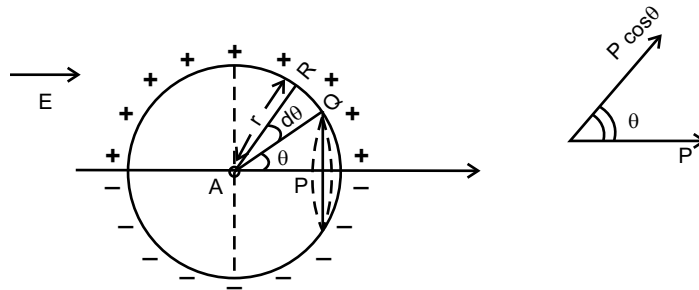


Fig.31 Enlarged view of cavity around the atoms

Circumference of element of sphere (in the form of circle of radius PQ).

$$= 2\pi (PQ)$$

Surface area of the element = circumference \times thickness

$$dA = 2\pi(PQ) \times (QR) \quad (57)$$

In fig $\sin \theta = \frac{PQ}{r}$

$$\boxed{PQ = r \sin \theta} \quad (58)$$

$$d\theta = \frac{QR}{r}$$

$$\boxed{QR = r d\theta} \quad (59)$$

Sub in (57)

$$dA = 2\pi \times r \sin \theta \times r d\theta$$

$$\boxed{dA = 2\pi r^2 \sin \theta d\theta} \quad (60)$$

If dq is the charge on the element and is equal to the product of the normal component of the **polarisation** (P) by the surface area.

$$\boxed{dq = P \cos \theta \times dA} \quad (61)$$

Sub for in (61) from Eq.(60)

$$dq = P \cos \theta \times 2\pi r^2 \sin \theta d\theta$$

$$\boxed{dq = P \times 2\pi r^2 \sin \theta \cos \theta d\theta} \quad (62)$$

The field due to this at A is denoted by dE_4 in the direction $\theta = 0$ is given by

By coulomb's law

$$dE_4 = \frac{q_1 \times q_2}{4\pi \epsilon_0 r^2}$$

where $q_1 = dq \cos \theta$

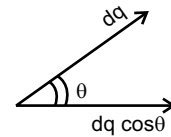
$$q_2 = 1$$

$$dE_4 = \frac{dq \cos \theta \times 1}{4\pi \epsilon_0 r^2} \quad (63)$$

Sub for dq

$$dE_4 = \frac{P \times 2\pi r^2 \sin \theta \cos \theta d\theta \times \cos \theta}{4\pi \epsilon_0 r^2}$$

$$dE_4 = \frac{P \times 2\pi r^2 \cos^2 \theta \sin \theta d\theta}{4\pi \epsilon_0 r^2} \quad (64)$$



The total field E_4 due to the charges on the entire (full) surface of cavity is obtained by integrating (64) between limits 0 to π .

$$\int dE_4 = \frac{2\pi r^2 P}{4\pi \epsilon_0 r^2} \int_0^\pi \cos^2 \theta \sin \theta d\theta$$

$$E_4 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta \quad (65)$$

Put $x = \cos \theta$

$$dx = -\sin \theta d\theta$$

$$\therefore -dx = \sin \theta d\theta$$

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Integrating Eq.(65) without limits,

$$\therefore E_4 = \frac{P}{2\epsilon_0} \int x^2(dx) = \frac{P}{2\epsilon_0} \left[\frac{x^3}{3} \right]$$

Sub for x and put the limits

$$\begin{aligned} E_4 &= \frac{-P}{3\epsilon_0} \left[\frac{(\cos\theta)^3}{3} \right]_0^\pi \\ &= -\frac{P}{2\epsilon_0} \times \frac{1}{3} [(\cos\pi)^3 - (\cos 0)^3] \\ &= -\frac{P}{6\epsilon_0} [-1 - 1] \\ E_4 &= \frac{2P}{6\epsilon_0} = \frac{P}{3\epsilon_0} \\ \boxed{E_4 = \frac{P}{3\epsilon_0}} \end{aligned} \quad (66)$$

\therefore The internal field E_i is given by

$E_i =$ Intensity of the applied field + field due to polarisation

$$\boxed{E_i = E + \frac{P}{3\epsilon_0}} \quad (67)$$

Here E_i is called as **internal field or Lorentz field**.

Classius - Mosotti relation

Let us consider an elemental dielectric having cubic structure. Since there are **no ions and permanent dipoles** in these materials, the ionic polarizability α_i and orientation polarizability α_0 are zero that is

$$\alpha_i = \alpha_0 = 0 \quad (68)$$

Since only **electronic polarizability** occurs, the polarisation P is given by

$$P = N \alpha_e E_i \quad (69)$$

Sub for E_i

$$\begin{aligned} P &= N \alpha_e \left[E + \frac{P}{3\epsilon_0} \right] \\ P &= N \alpha_e E + N \alpha_e \frac{P}{3\epsilon_0} \\ P \left[1 - \frac{N \alpha_e}{3\epsilon_0} \right] &= N \alpha_e E \end{aligned} \quad (70)$$

$$P = \left(\frac{N \alpha_e E}{1 - \frac{N \alpha_e}{3\epsilon_0}} \right) \quad (71)$$

We know electric flux density in the material in the presence of external field is given by

$$D = P + \epsilon_0 E \quad (72)$$

$$P = D - \epsilon_0 E \quad (73)$$

Divide by E

$$\frac{P}{E} = \frac{D}{E} - \epsilon_0$$

But $\frac{P}{E} = \epsilon = \epsilon_0 \epsilon_r$ (74)

$$\frac{P}{E} = \epsilon - \epsilon_0$$

$$= \epsilon_0 \epsilon_r - \epsilon_0$$

$$\frac{P}{E} = \epsilon_0(\epsilon_r - 1)$$

$$P = E \epsilon_0(\epsilon_r - 1) \quad (75)$$

Comparing (75) and (71) are can write

$$E \epsilon_0(\epsilon_r - 1) = \frac{N \alpha_e E}{1 - \frac{N \alpha_e}{3\epsilon_0}}$$

$$1 - \frac{N \alpha_e}{3\epsilon_0} = \frac{N \alpha_e}{\epsilon_0(\epsilon_r - 1)}$$

$$1 = N \alpha_e \left[\frac{1}{3\epsilon_0} + \frac{1}{\epsilon_0(\epsilon_r - 1)} \right]$$

$$1 = \frac{N \alpha_e}{3\epsilon_0} \left[1 + \frac{3}{\epsilon_r - 1} \right]$$

$$\frac{N \alpha_e}{3\epsilon_0} = \frac{1}{1 + \frac{3}{\epsilon_r - 1}} = \frac{\epsilon_r - 1}{\epsilon_r - 1 + 3}$$

$$\frac{N \alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad (76)$$

Where N = number of molecules per unit volume. The equation (76) is known as **Classius-Mosotti equation**. This equation is used to calculate α_e (**Electronic polarizability**), knowing the value of ϵ_r .

5.3.9 Dielectric loss

Definition

When a dielectric is subjected to an a.c. voltage, the electric energy is absorbed by the material and is dissipated in the form of heat. This *dissipation of energy* is called as *dielectric loss*. Since this involves *heat generation* and *heat dissipation*, the dielectric loss plays an important role in high voltage applications.

(a) Expression for dielectric loss

When the period of applied voltage is *greater than relaxation process*, the polarisation is completed at any time within a cycle. Here the *charging current leads the voltage* as in capacitor. Hence electric energy is lost during charging. This is applicable for a perfect insulator like vacuum or purified gas.

When the period of the applied voltage is in the *same range as relaxation time of polarisation process*, the *resonance occurs*. Here the *current leads the voltage by* $(90 - \delta)$ where δ is called as the *loss angle* and $\tan \delta$ is taken as a measure of electrical loss due to resonance and is known as *loss tangent*. Here the charging current can be resolved in to two components, one is parallel and another is perpendicular (90°) to the applied voltage as in capacitor. This is shown in Fig.32.

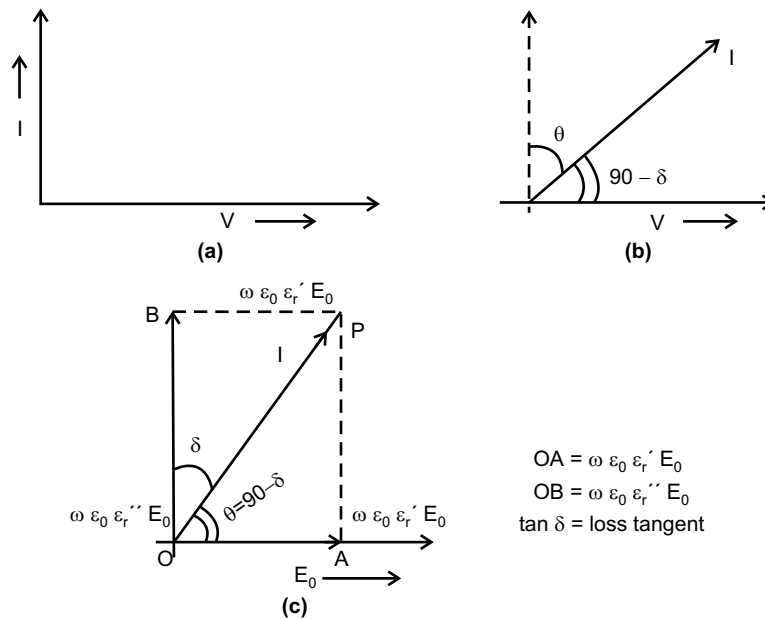


Fig. 32

The parallel component is the real part (OP is charged current) and results in $I^2 R$ loss (power loss).

Loss tangent can be expressed as

$$\boxed{\tan \delta = \frac{\epsilon_r''}{\epsilon_r'}} \quad (77)$$

Where ϵ_r' , ϵ_r'' are real and imaginary parts of relative permittivity for a dielectric in a capacitor having capacitance C and f is the frequency of the applied voltage V , then the **dielectric power loss** is given by

$$P = VI \cos \theta \quad (78)$$

$$\text{But } \boxed{V = I X_C} \quad (79)$$

where X_C is the capacitive reactance

$$\therefore X_C = \frac{1}{2\pi f C}$$

$$\boxed{I = \frac{V}{X_C}} \quad (80)$$

$$\text{Also } \boxed{\cos \theta = \cos(90 - \delta) = \sin \delta} \quad (81)$$

Where δ is the angle made by the charging current with the applied voltage sub in (78)

$$\boxed{P = \frac{V^2}{X_C} \sin \delta} \quad (82)$$

Since δ is very small, $\sin \delta = \tan \delta$. Hence Eq.(82) can be written as

$$P = \frac{V^2}{X_C} \tan \delta \quad (83)$$

Sub for X_C we get

$$\boxed{P = \frac{V^2 \tan \delta}{2\pi f C}} \quad (84)$$

Here $\tan \delta$ is called as the **power factor of the dielectric**. Hence power **loss depends on $\tan \delta$ only**. Since other factors like voltage (V) frequency of applied voltage (f) and capacitance are constants.

(b) Frequency regions of dielectric losses

1. The dielectric losses in **radio frequency** region are usually due to the **dipole rotation** or due to ions jumping from one equilibrium position to another.
2. The dielectric losses in the **lower frequencies** are mainly due to d.c. resistivity (P).

3. The dielectric losses in the *optical region* are associated with electrons and this loss is known as *optical absorption*. Hence the *electronic polarisation* produces power loss at optical region and at lower frequencies electronic polarisation produces no power loss.
4. The dielectric losses associated with ionic vibrations the frequencies fall in *Infrared region* and is known as *Infrared absorption*.

(c) Dependence of dielectric loss

Factor affecting dielectric loss. Increase in values of the following factors will increase the dielectric loss.

- (a) Temperature
- (b) Humidity
- (c) Applied voltage
- (d) Frequency of the applied voltage.

(d) Significance of dielectric loss

Dielectric loss plays a dominating role in high voltage applications. Dielectric loss is an engineering problem involving *heat generation and heat dissipation*. Hence for the usage of insulating materials, the proper understanding of balance between heat generation and heat dissipation is a must.

The dielectric materials having very large value of dielectric loss is said to be *lossy dielectric*.

5.4 Dielectric breakdown

5.4.1 Definition of dielectric breakdown

Definition of dielectric breakdown is applied to a dielectric and if it exceeds the critical value, the dielectric material loses its resistivity (insulating property) and permits a very large current to flow through it. This phenomenon is known as *dielectric breakdown*. The electric field strength at which the breakdown takes place is known as *dielectric strength* and the voltage corresponding to this is known as *breakdown voltage*. This breakdown produces a devastating effect like firing.

5.4.2 Definition of dielectric strength

The dielectric strength is defined as the *breakdown voltage per unit thickness* of the dielectric material. Unit is Vm^{-1} .

The failure (or) breakdown of a dielectric material is of great importance for electronic engineers. There are many reasons for this breakdown to occur.

Let us now discuss the different types of dielectric breakdown and their mechanism briefly.

5.4.3 Types of dielectric breakdown

The different types of dielectric breakdown are

- (a) Intrinsic breakdown
- (b) Thermal breakdown
- (c) Discharge breakdown
- (d) Electrochemical breakdown
- (e) Defect breakdown.

(a) Intrinsic breakdown

In a perfect dielectric, there are *no free electrons* and the *conductivity is almost zero*.

When the applied voltage is large, some of the electrons in the valence band cross over to the conduction band after crossing through the large forbidden energy gap, there by giving rise to large conduction currents. The liberation (or) movement of electrons from the valence band is known as *field emission*. This process of crossing of electrons from valence band to conduction band is a *chain reaction*. As a result, a large amount of current flows through the dielectric material causing dielectric breakdown (losing the resistive property). The breakdown produced in this manner is known as *intrinsic breakdown* (or) *avalanche breakdown*. This breakdown occurs may be due to the presence of imperfections, vacancies at the intensities or filled with ions and some extra atoms present at the lattice other than normal ones.

Characteristics of this breakdown:

1. It can takes place even at *low temperature*.
2. It requires relatively *large electric field*.
3. It does not depends on the *electrode configuration* and the *shape of the material*.
4. This kind of breakdown mostly occurs in *thin samples*.

(b) Thermal breakdown

If an electric field is applied to a dielectric material, some amount of heat is produced. This *heat produced* in the material *should be dissipated*. In some cases, the amount of *heat generated* is *larger* than the heat *dissipated* in it, then this excess of heat results in local melting. Once the melting start, that particular region becomes more conductive and allows a very large current flow through it there by causing dielectric breakdown. This type of breakdown is known as *thermal breakdown*.

Characteristics of this breakdown:

1. It occurs at *high temperatures*.

2. The breakdown strength depends on the *size and shape* of the *dielectric material*.
3. It also depends on the *geometry* and thermal property of electrodes.
4. The breakdown time is of the *order of milliseconds* or much longer.
5. The *breakdown strength* is lower in a.c. fields than in D.C., since the power loss increases with the *frequency* (frequency of a.c).

(c) Discharge breakdown

This breakdown is classified in two types

1. External breakdown.
2. Internal breakdown.

The *external breakdown* caused by a glow (or) corona discharge and such a discharge occurs at the sharp edges of the electrodes. The large electric field due to this glow (discharge) causes a damage on the adjacent solid dielectric medium. The carbon formed due to this damage makes the dielectric more conducting and there by causing dielectric breakdown. Dust or moisture present on the surface of the dielectric may also produce this external discharge breakdown.

Internal breakdown occurs when the insulator contains occluded (hiding) gas bubbles. If these dielectric materials are subjected to *high voltage*, the *gaseous substances* are *dissociated* into ions and there by producing a large *ionisation current*. This large ionisation current may produce an electric breakdown known as discharge breakdown.

Characteristics:

When a large number of *occluded gas bubbles are present* in a dielectric materials, the breakdown takes place even at low voltages.

(d) Electrochemical breakdown

The chemical and electrochemical breakdown are closely related to thermal breakdown. If the temperature of the dielectric material increases, it will increase the mobility of the ions and hence the electrochemical reaction takes place. When the ionic *mobility increases*, large current will flow there by *decreasing the insulation* resistance and this will result in *dielectric breakdown*. Due the chemical reaction, the electrical and mechanical properties of the dielectric materials are changed and finally these will lead to breakdown. Insulating materials become oxides and thereby decreasing the dielectric properties. For example in the case of rubber in *air* the oxide forms on the surface and, produce cracks on the surface. Organic materials working at high temperatures have lost their stability towards chemical properties.

Characteristics of this breakdown

1. Electrochemical breakdown is determined by the leakage current, density of ions, temperature and permanent dipoles present in the materials.
2. To avoid the electrochemical breakdown, the impurities should not be mixed with the pure dielectric materials.
3. Electrochemical breakdown are accelerated by the increase of temperature. Hence to avoid electrochemical breakdown, the dielectric materials should not be operated at high temperature (or) heat dissipation facilities should be provided.

(e) Defect breakdown

Some dielectric materials may have defects *like cracks, and pores* etc. *Moisture and other impurities* can be get filed up in these places leading to the breakdown. This is known as *defect breakdown*.

Criteria for the selection of a dielectric materials

Some of the important factors considered for the selection dielectrics are

1. It should have *high value* of *dielectric constant*.
2. It should have a *low dissipation factor* (or) *power factor*.
3. It should have a *high dielectric strength*.
4. It should have a *high insulation resistance*.
5. It should have a *high thermal conductivity* and *low thermal expansion*.

5.4.4 Classification of dielectric (or) insulation materials

The insulating materials (dielectric) are classified into three categories namely

- (a) **Solid insulating materials:**
Eg: mica, porcelain, bakelite asbestos, pre ceramics, glass, rubber & cotton.
- (b) **Liquid insulating materials:**
Eg: varnish, transformer oil, cable oil, silicon fluids and fluoro organic fluids.
- (c) **Gaseous insulating materials:** Eg: Air, hydrogen, nitrogen, halogens and sulphur hexa fluorides (SF₆).

5.5 Formulae and Problems in Magnetism

5.5.1 Important formulae in magnetism

$$\begin{aligned}\mu_\phi &= 4\pi \times 10^{-7} H/m \\ B &= \frac{\phi}{A}\end{aligned}\tag{85}$$

5.54 *Applied Physics*

$$\phi = B A \quad (86)$$

B = Magnetic induction (or) flux density

ϕ = Magnetic flux

$$\mu = \frac{B}{H} \quad (87)$$

$$\mu = \mu_0 \mu_r \quad (88)$$

$$\chi = \frac{I}{H} \quad (89)$$

Dividing Eq.(87) by (89) we get

$$\boxed{\frac{\mu}{\chi} = \frac{B}{I}} \quad (90)$$

$$\mu_r = 1 + K \quad (91)$$

$$B = \mu_0(I + H) \quad (92)$$

I = Intensity of magnetisation

H = Magnetic field strength (or) Intensity of magnetic field

μ = absolute permeability

χ = magnetic susceptibility

μ_r = Relative permeability

μ_0 = Permeability of air (or) free space

Bohr magneton $\boxed{\mu_B = \frac{eh}{4\pi m}}$ (93)

h = plancks constant = 6.62×10^{-34} Js.

e = charge of electron = 1.6×10^{-19} C

m = mass of electron = 9.1×10^{-31} Kg.

5.5.2 Problems of magnetic materials

□ Solved Problem 1

The magnetic induction produced in a bar of iron is 8π W b/m². Its area of cross section is 4×10^{-4} m². Calculate the magnetic flux (ϕ).

► Solution

Given $B = 8\pi$ W b/m²; $A = 4 \times 10^{-4}$ m².

$$\text{Magnetic induction} \quad B = \frac{\phi}{A}$$

$$\begin{aligned} \therefore \text{Magnetic flux } \phi &= B A \\ \phi &= 8\pi \times 4 \times 10^{-4} \\ \phi &= 32\pi \times 10^{-4} \\ \phi &= 100.5 \times 10^{-4} = 1.005 \times 10^{-2} \text{ Wb.} \end{aligned}$$

□ Solved Problem 2

Intensity of magnetic field (or) field strength (H) 2000 AT/m produces a magnetic induction (B) of $4\pi \text{ W b/m}^2$ in a bar of iron. Calculate the relative permeability and susceptibility ($\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$).

► Solution

$$\text{Given } B = 4\pi \text{ W b/m}^2; H = 2000 \text{ AT/m}; \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\mu = \frac{B}{H} = \frac{4\pi}{2000} = 2\pi \times 10^{-3} \text{ H/m}$$

$$\text{But } \mu = \mu_0 \mu_r$$

$$\therefore \mu_r = \frac{\mu}{\mu_0} = \frac{2\pi \times 10^{-3}}{4\pi \times 10^{-7}} = \frac{1}{2} \times 10^4$$

$$\boxed{\mu_r = 0.5 \times 10^4 = 5000}$$

$$\text{But } \mu_r = 1 + \chi$$

$$\boxed{\chi = \mu_r - 1 = 5000 - 1 = 4999}$$

□ Solved Problem 3

A magnetic material having magnetisation 2185 A/m , produces a flux density of 0.004 Wb/m^2 . Calculate the magnetising force field strength (H) and relative permeability (μ_r) of the material. ($\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$).

► Solution

$$\text{Given } I = 2185 \text{ A/m}$$

$$B = 0.004 \text{ Wb/m}^2 = 4 \times 10^{-3} \text{ Wb/m}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m.}$$

Using the relation

$$\therefore B = \mu_0(H + I)$$

$$H + I = \frac{B}{\mu_0} = \frac{4 \times 10^{-3}}{4\pi \times 10^{-7}} = \frac{10^4}{\pi}$$

5.56 *Applied Physics*

$$H + I = \frac{10000}{\pi} = 3185$$

$$H = 3185 - I = 3185 - 2185$$

$$\boxed{H = 1000 \text{ AT/m}}$$

But
$$\mu_r = \frac{B}{B_0} = \frac{B}{\mu_0 H}$$

$$\begin{aligned} \mu_r &= \frac{4 \times 10^{-3}}{4\pi \times 10^{-7} \times 10^3} \\ &= \frac{10^{-3}}{\pi \times 10^{-4}} = \frac{10}{\pi} \end{aligned}$$

$$\boxed{\mu_r = 3.185}$$

□ **Solved Problem 4**

The susceptibility of a diamagnetic material is -5×10^{-5} and magnetic field strength is 10^5 AT/m . Calculate the intensity of magnetisation produced in it?

► **Solution**

Given $\chi = -5 \times 10^{-5}$; $H = 10^5 \text{ AT/m}$; $I = ?$

$$\therefore \chi = \frac{I}{H}$$

$$I = \chi H = -5 \times 10^{-5} \times 10^5$$

$$\boxed{I = -5 \text{ A/m}}$$

(-ve sign shows, the magnetisation produced in the material is in the opposite direction of the external field).

□ **Solved Problem 5**

The intensity of magnetisation produced in a diamagnetic material is -5 A/m for the applied field of strength 10^5 AT/m . Calculate the flux density (B). ($\mu_0 = 4\pi \times 10^{-7}$).

► **Solution**

Given $H = 10^5 \text{ AT/m}$; $I = -5 \text{ A/m}$; $B = ?$.

$$B = \mu_0(H + I)$$

$$B = 4\pi \times 10^{-7}(10^5 - 5)$$

$$B = 4\pi \times 10^{-7} \times 10^5 \quad (\text{approx})$$

$$B = 4\pi \times 10^{-2} \text{ Wb/m}^2$$

□ Solved Problem 6

The susceptibility of a paramagnetic material is 4×10^{-3} at 27° . Calculate of susceptibility at 127° .

► Solution

$$\chi = 4 \times 10^{-3}$$

$$T = 273 + 27 = 300 \text{ K}$$

By curie law

$$\therefore \chi = \frac{C}{T} \quad \text{where } C = \text{curie constant}$$

$$C = \chi T = 4 \times 10^{-3} \times 300$$

$$C = 1200 \times 10^{-3}$$

$$\boxed{C = 1.2}$$

$$T = 127 + 273 = 400$$

$$\chi_{127} = \frac{C}{T} = \frac{1.2}{400} = \frac{12}{4000}$$

$$\boxed{\chi_{127} = \frac{3}{10^3} = 3 \times 10^{-3}}$$

□ Solved Problem 7

Calculate the power loss due to hysteresis in a transformer core of volume 0.02 m^3 and 50 Hz frequency. Area of cross section of hysteresis loop (or) energy loss/ m^3 is 500 J/m^3 .

► Solution

Given

$$\text{volume } V = 0.02 \text{ m}^3$$

$$\text{Frequency } f = 50 \text{ Hz}$$

$$\text{Area of loop} = 500 \text{ J/m}^3 \quad (\text{energy loss})$$

\therefore power loss due to hysteresis = workdone

$$\begin{aligned} \text{workdone} &= W_G = A V f & V &= \text{volume} \\ &= 500 \times 0.02 \times 50 \end{aligned}$$

$$\begin{aligned}
 &= 25 \times 10^3 \times 0.02 \\
 &= 50 \times 10^3 \times 2 \times 10^{-2} \\
 &= 50 \times 10^1 \\
 W &= P(\text{loss}) = 500 \text{ W}.
 \end{aligned}$$

□ Solved Problem 8

A magnetic flux of $32\pi \times 10^{-4} \text{ Wb}$ produces a magnetic induction (B) $8\pi \text{ Wb/m}^2$ in a bar of iron the volume of the magnet is $2 \times 10^{-5} \text{ m}^3$, calculate the length of bar magnet?

► Solution

Given $\phi = 32\pi \text{ Wb}; B = 8\pi \text{ Wb/m}^2; A = ?$

$$\phi = B.A$$

$$A = \frac{\phi}{B} = \frac{32\pi \times 10^{-4}}{8\pi} = 4 \times 10^{-4} \text{ m}^2$$

$$A = 4 \times 10^{-4} \text{ m}^2$$

$$V = 2 \times 10^{-5} \text{ m}^3$$

$$V = 2 \times 10^{-6} \text{ m}^3$$

$$V = A \times l$$

$$l = \frac{V}{A} = \frac{2 \times 10^{-5}}{4 \times 10^{-4}} = \frac{20 \times 10^{-6}}{4 \times 10^{-4}}$$

$$l = 5 \times 10^{-2} \text{ m}$$

$$= 5 \text{ cm}$$

□ Solved Problem 9

The retentivity (B_r) and coercivity (H_C) of a magnetic material are $6\pi \text{ Wb/m}^2$ and 1000 AT/m . Calculate the energy product of the material?

► Solution

$$B_r = 6\pi \text{ Wb/m}^2$$

$$H_C = 1000$$

$$= 10^3 \text{ AT/m}$$

$$E_P = B_r \times H_C$$

$$E_P = 6\pi \times 10^3$$

$$E_P = 6000\pi \text{ J}.$$

□ Solved Problem 10

A magnetic field of intensity 10^4 A/m is applied to a paramagnetic material having susceptibility 3.7×10^{-3} . Calculate the intensity of magnetization (I) & flux density (B) (A.U.2002)

► Solution

Use formulae

$$H = 10^4 \text{ A/m}$$

$$X_e = \frac{I}{H}$$

$$\boxed{I = X_e \times H}$$

$$B = \mu_0(H + I)$$

Ans.

$$I = 37 \text{ A/m}$$

$$B = 0.0126 \text{ Wb/m}^2$$

5.6 Formulae and Problems in dielectrics

5.6.1 Formulae in materials

Electric intensity E

1. F = Force in Newton.
 q = Charge in Coulomb.
 V = Applied voltage.
 d = Distance between plates.

$$E = \frac{F}{q} \quad \text{N/c}$$

$$E = \frac{V}{d} \quad \text{N/m} \quad (\text{or}) \quad \boxed{F = Eq}$$

2. $\psi = DA$
 ϕ = Electric flux in Coulomb.
 D = Electric flux density in C/m^2
 A = Average of cross section.

3. $D = \epsilon E$ similar to $B = \mu H$

5.60 *Applied Physics*

(a) $\epsilon = \frac{D}{E}$ $\epsilon =$ absolute permittivity of a medium

$\epsilon = \epsilon_0 \times \epsilon_r$ $\epsilon_0 =$ Permittivity of free space

(b) $D = \epsilon_0 \epsilon_r E$

$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

$\epsilon_r =$ Relative permittivity

4. Dipole moment (μ)

$$\mu = q \times d$$

unit of μ is $\text{C} \cdot \text{m}$

$q =$ Value of one of the charge of dipole

$d =$ Dist. between dipoles

5. Electric polarization

$$p = \frac{\mu}{V}$$

$V =$ Volume of dielectric in m^3

6. $P = \epsilon_0 (\epsilon_r - 1)E$

7. $\mu = \alpha E$

$$\alpha = \frac{\mu}{E}$$

$\alpha =$ polarizability

Unit of $\alpha = \text{F/m}^2$

Electric flux density in a dielectric material under external field is (E).

8. Where

$N =$ Number of units/ m^3

$$P = N\mu = N\alpha E$$

9. $D = \epsilon_0 E + P$

10. Electric susceptibility X_e is given by the relation

$$\epsilon_r = 1 + X_e$$

$$X_e = \epsilon_r - 1$$

11. Capacitance of a conductor with air as dielectric

$$C = \frac{\epsilon_0 A}{d}$$

A = Area of one of the plate

d = Dist between plates unit of C is farad(f) but practical units are μf and pf

12. Capacitance with a dielectric medium (slab) between the plates,

$$K = \epsilon_r$$

$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{\epsilon_0 K A}{d}$$

$$\therefore \boxed{\frac{C_1}{C} = \epsilon_r = K} = \text{Dielectric constant of the material,}$$

13. Electronic polarization

$$\mu_e = \alpha_e E$$

$$p = N \mu_e = N \alpha_e E$$

α_e = electronic polarizability

14. Lorentz force on nucleus (or) electron

$$F = Eq$$

$$q = Ze$$

$$\boxed{F = E(Ze) = zeE}$$

$$15. \alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

16. In Ionic polarization

$$\mu_i = \alpha_i E$$

α_i = Ionic polarization

$$P_i = N \mu_i$$

$$P_i = N \alpha_i E$$

$$17. \alpha_i = \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right) = \frac{e^2}{\omega_0^2} \left(\frac{M+m}{Mm} \right)$$

m = Mass of cation (Ionic nucleus)

M = Mass of negative (electron) or anion

e = Charge of electron = $1.6 \times 10^{-19} e$

ω_0 = angular frequency of molecule

$$\omega_0 = 2\pi f_0$$

f_0 = frequency of rotation of molecule

5.62 Applied Physics

18. Orientation polarization

$\alpha_0 =$ orientation Polarizability

$$P_0 = N\alpha_0 E$$

$$P_0 = \frac{N\mu^2 E}{KT}$$

Where $K =$ Boltzmann constant

$$\alpha_0 = \frac{\mu^2}{3kT}$$

Here α_0 depends on temperature

$$\alpha_0 \propto \frac{1}{T}$$

19. Space charge polarization

$$P_s = N\alpha_s E$$

P_s is very small where compared to other polarizations in dielectric.

20. Internal field in a dielectric (E_i)

$$E_i = E + \frac{P}{3\epsilon_0}$$

$E =$ applied electric field intensity

$P =$ Polarization vector

21. Classios-Mosotti equation (for the determination of α_e)

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\alpha_e = \frac{3\epsilon_0}{N} \frac{(\epsilon_r - 1)}{\epsilon_r + 2}$$

22. Energy stored in a capacitor

$$E = W = \frac{1}{2}CV^2 = \frac{Q^2}{2C} \text{ Joules.}$$

Problems

Dielectric materials

□ Solved Problem 11

Calculate the capacitance of a capacitor having square plates of area $4 \times 10^{-4} m^2$ and separated by a distance of $0.004 m$.

► Solution

Given $d = 0.004m = 4 \times 10^{-3}m$; $A = 4 \times 10^{-4}m^2$

$$C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85 \times 10^{-12} F/m$$

$$C = \frac{8.85 \times 10^{-12} \times 4 \times 10^{-4}}{4 \times 10^{-3}}$$

$$C = 8.85 \times 10^{-16} \times 10^3$$

$$C = 8.85 \times 10^{-13}$$

$$C = 0.885 \times 10^{-12} f$$

$$C = 0.885pf$$

□ Solved Problem 12

The capacitance of a capacitor with air as dielectric between the plates is $2\mu f$ when air is replaced by a dielectric slab, the value of capacitance becomes $10\mu f$. Calculate the dielectric constant of the material of the slab

► Solution

Given $C_1 = 10\mu f$; $C = 2\mu f$

$$\epsilon_r = K = \frac{C_1}{C}$$

$$\epsilon_r = \frac{10}{2} = 5$$

□ Solved Problem 13

The charge on an electric dipole is $2 \times 10^{-6}c$ and distance between the dipoles is 2mm. Calculate the dipole moment of the dipole

► Solution

$$q = 2 \times 10^{-6}C = 2\mu C$$

$$d = 2mm$$

$$= 2 \times 10^{-3}m$$

$$\mu = q \times d$$

$$= 2 \times 10^{-6} \times 2 \times 10^{-3}$$

$$\mu = 4 \times 10^{-9}c - m$$

□ Solved Problem 14

The capacitance of a parallel plate capacitor is $4\mu f$ is with a dielectric of permittivity $\epsilon_r = 50$. Find the energy stored in the capacitor and energy stored in polarizing the dielectric for an applied voltage of $500V$. (Anna University Model 2001 & 2002)

► Solution

Given

$$C = 4\mu f = 4 \times 10^{-6} f$$

$$\epsilon_r = 50$$

$$V = 500V$$

$$\begin{aligned} \therefore \text{Energy stored in the capacitor } E &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} \times 4 \times 10^{-6} \times (500)^2 \\ &= \frac{1}{2} \times 4 \times 10^{-6} \times 25 \times 10^4 \\ &= \frac{1}{2} \times 100 \times 10^{-2} = \frac{1}{2} \times 10^2 \times 10^{-2} \\ &= \frac{1}{2} J \end{aligned}$$

Energy stored in the capacitor $E_1 = 0.5J$

Energy of capacitor when dielectric is removed (air as dielectric)

$$C_0 = \frac{C}{\epsilon_r} \quad \boxed{\frac{C}{C_0} = \epsilon_r}$$

$$\epsilon_r = 50$$

$$C = 4\mu f$$

$$\epsilon_r = 50$$

$$C_0 = \frac{4 \times 10^{-6}}{50}$$

$$C_0 = \frac{8}{100} \times 10^{-6} = 0.08 \times 10^{-6} f$$

$$\boxed{C_0 = 8 \times 10^{-8} f}$$

Energy stored in the capacitor with vacuum (or) air as dielectric

$$E_2 = \frac{1}{2} C_0 V^2$$

$$E_2 = \frac{1}{2} \times 8 \times 10^{-8} \times (500)^2$$

$$E_2 = \frac{1}{2} \times 8 \times 10^{-8} \times 25 \times 10^4$$

$$E_2 = 4 \times 10^{-8} \times 25 \times 10^4$$

$$E_2 = 100 \times 10^{-4} = 1 \times 10^{-2} = 0.01 \text{ J}$$

Energy stored in dielectric in polarizing the dielectric

$$E = E_1 - E_2 = 0.50 - 0.01 = 0.49 \text{ J}$$

□ Solved Problem 15

In an ionic crystal, the polarization produced is $4.425 \times 10^{-8} \text{ m}^2$, when an electric field of intensity 10^3 V/m is applied to it. Calculate the relative permittivity of the crystal.

► Solution

Given $E = 10^3 \text{ V/m}$; $P = 4.425 \times 10^{-8} \text{ m}^2$; $\epsilon_r = ?$

$$P = \epsilon_0 (\epsilon_r - 1)E$$

$$\epsilon_r - 1 = \frac{P}{\epsilon_0 E}$$

$$= \frac{4.425 \times 10^{-8}}{8.85 \times 10^{-12} \times 10^3} = \frac{10^4}{2 \times 10^3} = \frac{10}{2}$$

$$\epsilon_r - 1 = 5$$

$$\boxed{\epsilon_r = 6}$$

□ Solved Problem 16

Calculate the electronic polarizability of Argon atom.

► Solution

Given $\epsilon_r = 1.0027$

$$N = 2.7 \times 10^{25} \text{ atom/m}^3$$

5.66 *Applied Physics*

$$P = \epsilon_r (\epsilon_r - 1)E$$

$$P = N\alpha_e E$$

$$\alpha_e = \frac{P}{NE} = \frac{\epsilon_0 (\epsilon_r - 1)E}{NE}$$

$$\boxed{\alpha_e = \frac{\epsilon_r (\epsilon_r - 1)}{N}}$$

$$\alpha_e = \frac{8.85 \times 10^{-12}(1.0027 - 1)}{2.7 \times 10^{25}}$$

$$\alpha_e = \frac{8.825 \times 10^{-12} \times (0.0024)}{2.7 \times 10^{25}} = \frac{8.85 \times 10^{-12} \times 2.7 \times 10^{-3}}{2.7 \times 10^{25}}$$

$$\alpha_e = 8.85 \times 10^{-15} \times 10^{-25}$$

$$\alpha_e = 8.85 \times 10^{-40} \text{ Fm}^2$$

□ **Solved Problem 17**

Find the total polarizability of CO_2 if its susceptibility is 0.985×10^{-3} Density of CO_2 is 1.977 kg/m^3 .

► **Solution**

Given

$$X_e = 0.985 \times 10^{-3}$$

$$d = 1.977 \text{ kg/m}^3$$

$$\text{Molecules weight of } \text{CO}_2 = 44 \text{ kg}$$

$$= 44 \times 10^{-3} \text{ kg}$$

$$\epsilon_r = 1 + X_e \text{ Similar to}$$

$$\epsilon_r = 1 + 0.985 \times 10^{-3}$$

$$\mu_r = 1 + X_m$$

Take

$$\boxed{N = 2.688 \times 10^{25}}$$

$$\alpha = ?$$

$$\alpha = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

$$\alpha = \frac{8.85 \times 10^{-12}(1 + 0.985 \times 10^{-3} - 1)}{2.688 \times 10^{25}}$$

$$\alpha = \frac{8.85 \times 10^{-12} \times 0.985 \times 10^{-3} \times 10^{-25}}{2.688}$$

$$\alpha = \frac{8.85 \times 0.985 \times 10^{-40}}{2.688}$$

$$\alpha = 3.24 \times 10^{-4} F/m^2$$

□ Solved Problem 18

The following data refers to a dielectric material. $\epsilon_r = 4.94$ $n^2 = 2.69$. When n is the index of refraction. Calculate the ratio between electronic and ionic polarizability of this material.

► Solution

Given $\epsilon_r = 4.94$ $n^2 = 2.69$

Using

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

Where $\alpha = \alpha_e + \alpha_i$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0}$$

$$\begin{aligned} \frac{4.94 - 1}{4.94 + 2} &= \frac{3.94}{6.94} \\ &= 0.5677 = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0} \end{aligned}$$

$$\frac{N(\alpha_e + \alpha_i)}{3\epsilon_0} = 0.5677$$

But $\epsilon_r = n^2$

At optical frequencies $\alpha_i = 0$

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_0}$$

$$\begin{aligned} \frac{2.69 - 1}{2.69 + 2} &= \frac{1.69}{4.69} = 0.3603f \\ &= \frac{N\alpha_e}{3\epsilon_0} \end{aligned}$$

$$\frac{N\alpha_e}{3\epsilon_0} = 0.36034$$

Divide

$$\frac{\alpha_e + \alpha_i}{\alpha_e} = \frac{0.5677}{0.36034} = 1.575$$

$$1 + \frac{\alpha_i}{\alpha_e} = 1.575$$

$$\frac{\alpha_i}{\alpha_e} = 0.575$$

$$\frac{\alpha_e}{\alpha_i} = \frac{1}{0.575} = 1.738$$

5.7 Modern Engineering Materials

Materials play an important role in the designing and fabrication of equipments and appliances used in engineering, medicine, science, industry and day to day life. Before designing and fabrication, a complete study of physical, chemical, electrical, magnetic etc properties of materials is a must. A thorough study of properties of materials and their relation to the microstructure, form the basic concepts of material science.

The development of material science help us to design the equipments and appliances to meet the requirements of modern engineering, medical and industrial needs. The materials used in 'Hi-Tec' are designed to give maximum performance (efficiency) and long life. Let us study some of the newly developed engineering materials, along with their production, properties and applications.

5.8 Shape Memory Alloys (SMA)

5.8.1 Definition of SMA

Shape memory alloys are special alloys which after being *deformed can remember and recover their original shape or size* when they are *subjected* to appropriate *thermal process*. Generally these materials can be physically deformed at some relatively low temperature and upon exposure to some high temperature, they will return to their original shape.

5.8.2 Types of SMA

- Materials that exhibit shape memory only upon heating are referred to have a *one way shape memory*.
- Some materials also undergo a change in shape on recooling. These materials are said to have *two way shape memory*. *Examples of SMA:* Nickel - Titanium ($Ni - Ti$) alloys and copper base alloys such as Cu Zn Al and Cu Al Ni.

5.8.3 Crystal structure of SMA

Shape memory alloys display *two distinct* crystal structures or *phases*. The temperature and stresses will determine the phase of SMA at that moment.

The crystal structure or phase of a SMA at *lower temperature* is called *martensite* and the crystal structure or phase of a SMA at *higher temperature* is called as *austenitic*.

When a SMA is at martensite form at low temperature, it can be deformed into any shape. When the alloy is heated it goes through a transformation from martensite to austenite. In the *austenite phase*, the memory metal *remembers the shape before it was deformed*.

Thus when a SMA alloy of given shape is cooled from a given temperature T_1 , to a lower temperature T_2 , it deforms and changes in shape. Again upon heating from T_2 to T_1 , the alloy regains its original size (or) shape. This *property* is known as *Thermoplasticity*. It is a *natural property of the alloy* in solid-state transformation and the transformation is known as *martensite transformation*. There is a microstructural change takes place when the *initial crystal structure* is transformed in to a *new crystal structure*. This mechanism *does not* involve the mixing of atoms by *diffusion* but there is a *coordinated shear displacement*, in the sections of the crystal.

5.8.4 General characteristics of SMA

Temperature - Transformation - Temperature Curve for SMA under constant load (stress) as it is cooled and heated.

T_h : Transformation hysteresis

M_s : martensite start

M_f : Martensite finish

A_s : Austenite start

A_f : Austenite finish

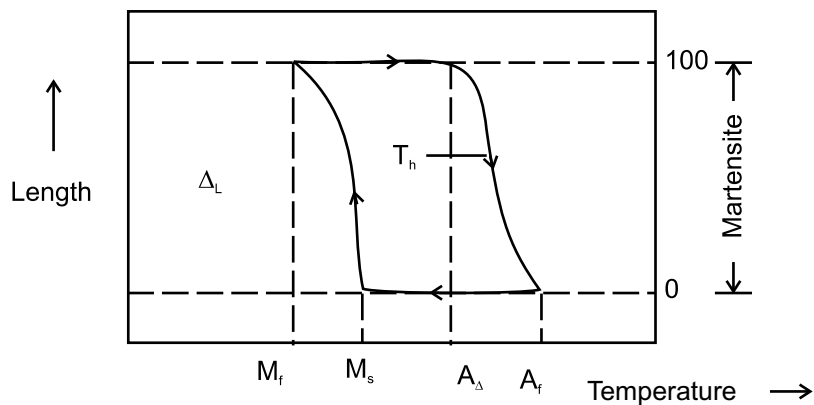


Fig.33

stress behaviour as seen in the part of the curve AB, on unloading though the material reverts back to austenite at lower stress as shown by the curve CD and shape recovers not by the application of heat but by the reduction on of stress. This effect makes the material to become extremely elastic and it known as *pseudo elasticity* (or) *super elasticity*, pseudo elasticity is non linear. Hence young's modulus is not able to define in this range since it shows both temperature and strain dependence.

Memory alloys shows greater stress rates of elasticity. For example *eyeglass frames* made up of SMA are martensite in phase. Bending the arms (application of stress) in half (at room temp) introduces a phase change at the bends to make it to austenite. Austenite is not stable at room temperature and the system seeks lower energy states, the austenite reverts back to martensite and because of this, the arm of eyeglass bend back.

5.8.6 Commercial alloys having shape memory effect

The most common shape memory alloy is Nitinol consisting of equal parts of Nickel and Titanium. The following table gives a list of SMA with their composition and transformation on temperature.

S.No	SMA Alloy	Composition Atomic (or) weight percentage	Transformation Temperature (celsius)
1	$N_i - T_i$	49/51 at% N_i	-50 to 110°
2	$M_n - C_u$	5/35 at % C_u	250 to 180°
3	$C_u - S_n$	Approx 15 at% S_n	-120 to 30°
4	$N_i - Al$	36/38 at% Ni	-180 to 100°
5	$Ag - cd$	44/49 at% cd	190 to -50°
6	$Au - cd$	46.5/50 at% cd	30 to 100°
7	$Cu - Al - Ni$	14/4.5 at% Al 3/4.5 at% Ni	-140 to 100°

5.8.7 Application of SMA

Shape memory alloys have a variety of applications. Some of their applications are listed below.

1. Industrial applications

(a) Control of increased heat in industries

Lines that carry high inflammable and toxic (unpleasant (or) pungent smell) fluids (liquids and gases) should have a greater amount of control so that the dangerous (unwanted) events can be prevented. For that the systems are programmed with SMA in such a way as to shut down the production of increased (unwanted) heat, there by preventing the the devastating effects. Industries like *petrochemicals, semiconductors, phar-*

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maceutical, and large oil and gas fields are using this kind of technique.

(b) **Fire Safety Valve**

This equipment uses *Cu Zn Al actuator* which is designed to shut off toxic (or) inflammable gas flow when fire occurs. Here SMA is used as a *force actuator* on a spring.

(c) **Valve Control**

A device has been developed using SMA in which a *valve controls the rate* of flow of heat carefully to any desired amount.

(d) **Croft Hydraulic coupling**

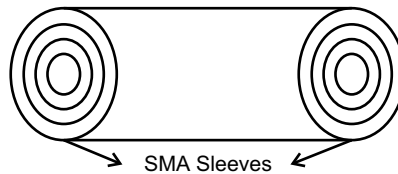


Fig. 35

Here fittings are manufactured as cylindrical sleeves with SMA, which are slightly smaller than metal tubing used to join their *diameters* get expanded while being *martensite* and on warming they transformed to *austenite* in which they *shrink in diameter* and strongly hold the ends. This joint is superior to welding.

(e) **Helicopter blades**

The performance of helicopter blades depends on the vibrations of SMA used in micro processing control tabs. These control tabs are used for the trailing ends of the blades and hence a pilot can fly with increased accuracy.

2. Commercial Applications

(a) **Eye Glass Frames**

Eye glass companies demonstrate the eye glass frames that can be bent back and forth and retain their shape. Here the frames are made up of SMA which shows *super elasticity*. In eye glass *Nickel - Titanium* alloy is used as SMA, which absorb (or) withstand large deformation without any damage.

Super elasticity is the capacity of some poly crystalline material to exhibit large deformation without fracture (broken) (or) necking. They can withstand elongation from 100% to > 1000% without breaking.

3. Medical Applications

(a) Blood clot filter

The main applications of SMA is the blood clot filter. Here $N_i - T_i$ wire is shaped to **anchor (forced) in to a vein** and catch the passing clots. This part is chilled, so it can be collapsed and inserted in to the vein. The body heat is sufficient to turn the part in to its functional shape.

(b) Artificial hip joints and bone plates for healing bone fractures

- i. $N_i - T_i$ shape memory alloys have been employed in artificial joints such as **artificial hip joints**.
- ii. Because of their **flexibility** they are used as, **bone plates** for narrow joint for healing bone fracture and for connecting broken bones.
- iii. They are also used in medical applications like **teeth braces, artificial muscles and orthopaedic devices**.

5.9 Nano Phase Materials

5.9.1 Definition and introduction

Nanophase materials are the nano structures materials having a characteristic **length**, less than **100nm**. Their **grain size** is the order of **1nm to 100nm**. They exhibit greatly altered physical chemical and mechanical properties when compared to their normal, large grained (macro size) counter parts with the same chemical composition.

Nano phase material is identified as one of the major field in modern material science by the research work done by Gleiter and coworkers in 1990. A cluster (group) of nano particle contains less than 10^4 molecules or atoms corresponding to a diameter of a few nanometers. Hence one can conclude the size of a nano particle is about 1nm. Since the size of nanoparticles lie between **molecular and bulk material they have hybrid properties**. They have **non linear optical** and **magnetic properties**.

5.9.2 Production of Nano Structured Materials

It is possible to produce nano, structured materials by using a variety synthetic methods. Depending upon the desired properties or applications, each method will have some advantages and disadvantages.

a) Vapour Condensation

Method of synthesis:

In this method clusters of atoms are synthesized through vapour condensation. This method involves the process of **evaporation** of a solid metal followed by

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rapid(fast) condensation in order to form nanosize clusters. Thus the resulting powder can be *used as filter* for composites materials or consolidated in to bulk material.

Application:

This method is used to produce ceramic or metal nano structured powders.

b) Chemical Synthesis

Method synthesis

By using a *variety of chemical approaches* such as *sol-gel or thermal decomposition*, a nanostructured materials are synthesized. These methods provide large quantities of nano sized materials at low cost.

Application

This method is used to produce both metals and ceramics of nano material size.

c) Mechanical deformation

Method:

This method involves *milling or shock deformation*. It is a common method used to produce nanostructured powders which turns produces nano structured materials. By mechanical milling, it is possible to produce nano structured powders of different systems.(other wise it become immisible)

Application

This method is applied to produce nano structured powders of a bulk material (or) different systems.

d) Thermal crystallization

Method:

By controlling the nucleation and growth rate during annealing (slow cooling) of an amorphous materials, we can produce bulk materials with an average grain size of 20nm without using condensation and sintering steps.

Application

This method is used to produce three dimensional nano materials (crystalline materials).

5.9.3 Ball milling

Principle

This techniques uses the principle of mechanical crushing.

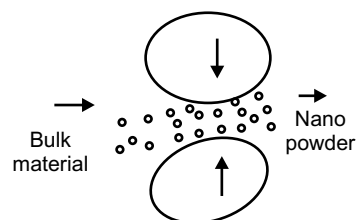
In ball milling, small hard balls are allowed to rotate inside a drum and then it is made to fall on a solid with high force to crush the solid into nano crystal.

Types of ball mill

- Attrition ball mill
- Planetary ball mill
- Vibrating ball mill
- Low energy tumbling ball mill
- High energy ball mill

Construction and working

Ball mills rotate around a horizontal axis, partially filled with the material to be ground plus the grinding medium.



Different types of materials are used as media like steel, ceramics, flint pebbles etc.,

When the ball is rotating, the material is forced to get crushed against the walls. The milling balls impart energy on collision and produce smaller size of nano particles from bulk material.

The grinding balls in the grinding jar are subjected to superimposed rotational movements, coriolis forces. The difference in speeds between the balls and grinding jars produces an interaction between frictional and impact forces which releases high dynamic energies. The interplay between these forces produces the high and very effective degree of size reduction.

5.9.4 Properties of nanostructured particles, physical properties

(a) Variation of physical properties with size

In this section let us discuss how the geometrical arrangement of atoms and hence their stability changes with size.

Discussion

When the bulk material is reduced to nanoparticle size, it creates more surface sizes (area). This will in turn change, the surface pressure and leads to a change in the inter particle spacing. This effect is

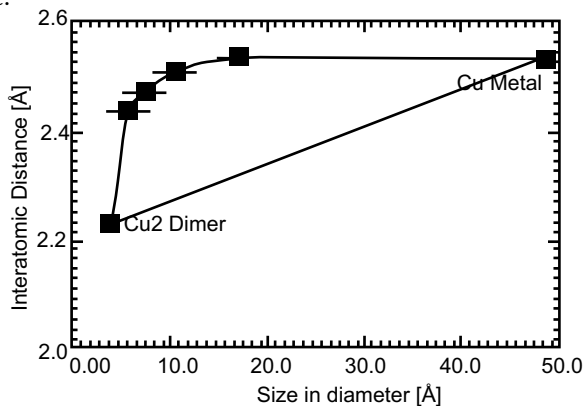


Fig. 36

shown in the graph drawn between size of particle and inter atomic distance for Cu_n . (36)

It is quite interesting to note that as the *inter particle spacing decreases as the size decreases*. It is apical of metal clusters (groups). This is due to the competition (interaction) between the *long range electronic forces and short range cove cove repulsive forces*. The change in inter particle spacing and the *large surface area to the volume ratio* of the particles have a *combined effect* on the properties of materials. The variations in the surface free energy (use to form clusters) changes the chemical potential. This will in turn alter the thermodynamic properties like meting point. This effect is shown in Fig.37 drawn between inter atomic distance and meting point for Au_n . Here the meting point decreases as the size decreases. The rate of decrease, increases substantially at very small sizes.

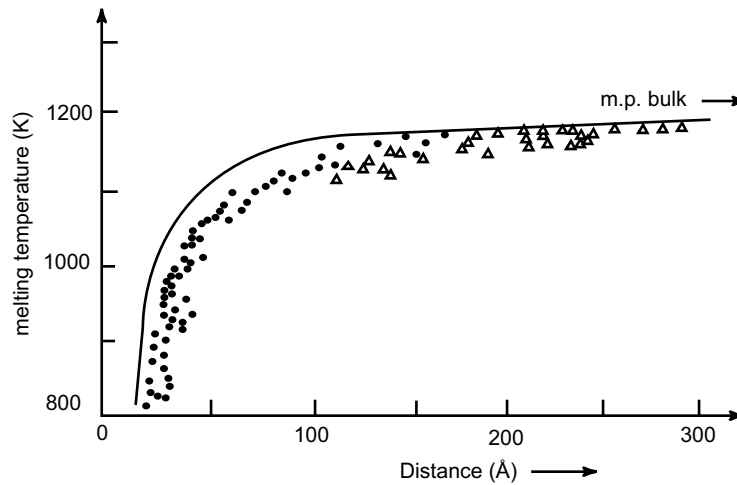


Fig.37

5.9.5 Electron affinities and chemical properties

The variations in the inter particle spacing and geometry (arrangement) also result in the variation of electronic properties with size.

a) Ionization potential

When the size is reduced from the bulk, the electronic bands in the metals become narrower and the delocalise a electronic states are transformed in to more localized molecular bonds. This will affect the ionisation potential (Energy required to remove an electron in the outer most orbit of an atom in the vapour state). It has been noticed in the experiment that the ionization potentials at small size of the particles are higher than at the bulk material work function. The ionization potential shows a marked fluctuations with the size of the particles.

b) Reactivity (catalytic property)

The large surface to volume ratio of the particles, variations in geometry and the electronic structure have a strong effect on catalytic properties. For example the reactivity of small clusters (collection) has been found to vary greatly in magnitudes when the cluster is changed by only a few atoms Fig.38 shows the activity of clusters of Fe_n with hydrogen.

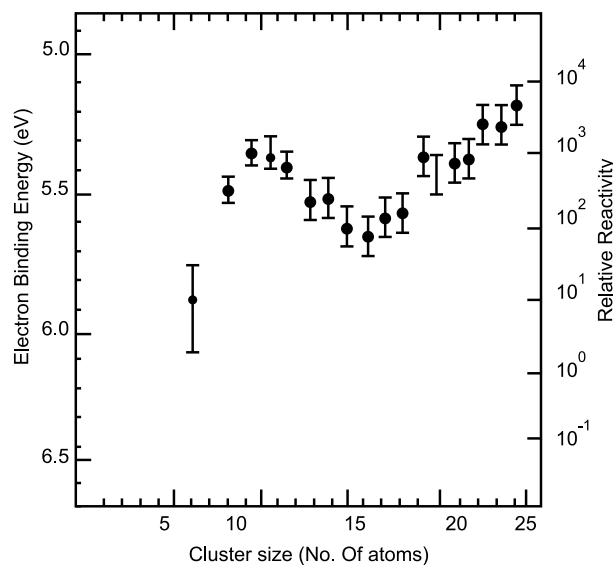


Fig.38

c) Hydrogen storage (absorption)

Another important possibility is the absorption (storage) of hydrogen in metals. It is well known that the metals do not absorb hydrogen. If they do so (if the metal absorbs), the hydrogen is absorbed dissociatively by the metal on the surface in the ratio 1 : 1. This limit is notably increased in particles of smaller size. It has been found that small positively charged clusters of Ni , Pd , and Pt generated (produced) in molecular beam and containing 2 to 60 atoms, can absorb eight hydrogen atoms per metal atom. (**H: metal: 8:1**). Also the number of *absorbed H atoms decreases as the cluster size increases*. This clearly shows that the *smaller particles* are very useful in *hydrogen storage devices*. This principle is used in the industry in the process of hydrogenation of oil and fats using powdered Nickel as catalyst instead of using bulk Nickel metal (to increase surface area to volume).

5.9.6 Magnetism in nanoparticles

Nanoparticles of magnetic and nonmagnetic solids exhibit an entirely new class of magnetic properties, which are marked by quantum that are far from complete understanding.

a) Domains

A bulk ferromagnetic material domain contains several domains. Each domain contains several thousand atomic spins. Generally the spins are aligned within the domain but different domains point in different directions. The following table will briefly explain the difference in magnetic behaviour of a bulk material and clusters of the same metal.

S.No	Meatal	Bulk	Cluster
1	<i>Na – K</i>	Paramagnetic	Ferro magnetic
2	<i>Fe, Co, Ni</i>	Ferro magnetic	Super paramagnetic
3	<i>Gd, Tb</i>	Ferro magnetic	Rotators/super paramagnetic
4	Cr	Antiferro magnetic	Frustrated paramagnetic
5	Rh	Para magnetic	Ferro magnetic

b) Geomagnetic resistance, coordination number & magnetic moment (GMR)

Ferromagnetic and anti ferromagnetic *multilayers* have been found to exhibit *giant magneto resistance* (GMR). Small particles differ from the bulk, since large fraction of the atom resides on the surface itself. The atom residing on the surface of smaller particles will have lower *coordination number* than to the interior atoms.

Fig.39 shows the dependance of magnetic moment on the coordination number. It is clear from the graph as the *coordination number decreases, magnetic moment increases towards the atomic size*. In other words the *smaller particles* are more magnetic *than the bulk material*.

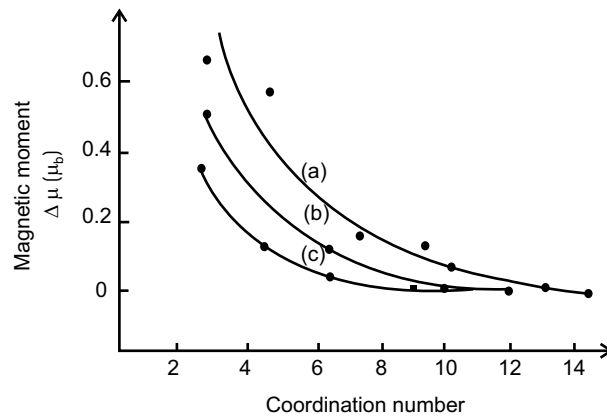


Fig. 39

5.9.7 Mechanical properties

(a) Dependance of hardness and strength

Hardness and strength of conventional grain size are the function of *grain size*.

For ductile poly crystalline materials, the grain size depends on *flow stress* which is given by empirical *Hall-petch* equation.

$$\sigma_0 = \sigma_i + K d^{-\frac{1}{2}}$$

where σ_0 = yield stress

σ_i = friction stress which opposes the dislocation motion

K = constant

d = grain diameter

A few observations on an particles prepared by gas condensation method are useful to explain their mechanical properties similarly *hardness* is given by the equation.

$$H_o = H_i + K d^{-\frac{1}{2}}$$

d = grain diameter

H_o = Hardness after stress

H_i = Initial hardness

The experimental results of hardness measurements shows a different behaviour namely positive slopes zero slope and negative slope depending on the grain size, when it is less than 20 nm.

5.9.8 Conclusion

Thus the hardness, strength and deformation behaviour of nano-crystalline materials is *unique* and *not yet well understood*.

Thus the nano structured particles have the following properties

1. High *magnetic resistance*
2. *Lower melting point*
3. High solid state *phase transition temp*
4. Lower *debye temp*
5. High self *diffusion coefficient*
6. High *catalytic activity*
7. Low *ferromagnetic temp*
8. High *hardness & strength*.

We have also seen the physical, electronic, magnetic and chemical properties of nano materials depends sensitively on size and the small particles display behaviours which are quite different from those of the individual atoms of the bulk material. These new behaviours are not only important from the fundamental point of view but also *lead to new applications*. In fact, we are capable of *controlling*

the properties of materials by *changing the size, composition and dimension* that leads to atomically designed materials with additional properties.

They are listed below

1. They have 30 to 50% *lower elastic* module than for conventional grain size.
2. They possess *super plastic* behaviour at low homologous temperature.
3. They possess 2 to 7 times *higher hardness and strength* values for nano crystalline pure metals ($\sim 10\text{nm}$ grain size) than those of larger grain size ($7(\mu\text{m})$).
4. There is a *decreasing hardness* with *decreasing grain size*.

Applications

In nano phase materials, there are two types of applications

- a) Structural applications
- b) Functional applications.

a) Structural applications

These applications are mainly based on the mechanical properties of nano phase materials. They are used to produce *plastic ceramics* and *extremely hard materials*.

b) Functional applications

These applications are mainly based on the *transformation of external signals* like the filtering of incident light and the *change of electrical resistance* in different gas concentrations.

c) Other applications

1. They are used to produce *soft magnetic materials* and *tiny permanent magnets* with high energy product ($B_r \times H_C$). Hence they are used in *high density magnetic recording, information storage devices*.
2. They are used as magnetic nano-composite *refrigerants* and *magneto resistance spin values applications*.
3. Quantum wells quantum dots and quantum wires having quantum confinement are mainly produced from fermi conductor nano materials are used in *computer storage (memory) devices*.
4. Since these materials have large ratio between surface area and volume, they are used to improve the mechanical behaviour of materials in *potential structural applications*.

5.10 Metallic Glasses

Definition

The engineering materials which share the properties of both metals and glasses are called *metallic glasses* simply the metallic glasses are *glasses having metallic properties*.

Eg: Pd₇₇, Cu₆ Si₁₇.

Explanation

A glass is a solid material obtained from *a liquid which does not crystalline on cooling*. They are *amorphous solid* in which atoms are packed in *random fashion*. The glass is a word generally associated with *transparent silicate glasses* containing silica (SiO₂) and oxides of some metals like Al, K, Na, Pb and Mg. These glasses are non metallic act as electrical insulators and does not exhibit ferromagnetism. But metals are malleable, ductile act as good conductors and posses crystalline structure.

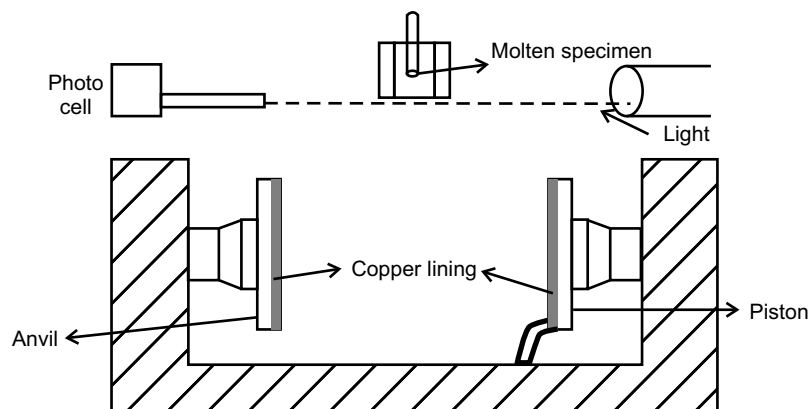


Fig.40 Piston-Anvil liquid quenching device

But metallic glasses exhibit metallic properties even though they are amorphous in nature. Oxides of glasses are amorphous, in nature and it is common feature but a metallic material (metallic glasses) to have amorphous *nature is quite new*.

Metallic glasses are usually prepared by *cooling a metallic liquid* (mentioned already) which has a disordered structure *so quickly* so that there is no enough time left for the formation of crystalline structure. The rate of cooling is of the order of 2×10^6 °C per second.

During solidification there is no change in interatomic spacing. Thus a metallic glass may be considered as a **solid with frozen liquid structure**. The temperature at which the transition takes place from liquid to solid is known as *glass transition temperature* (T_g). On heating the metallic glasses show a reversible glass-liquid

transition at glass transition temperature T_g . The metallic glasses developed forty years ago required faster cooling of the order of million degrees of Celsius per second. But the new alloys recently developed that form metallic glasses require lower cooling rates at the order of 1 to 100°C per second. This slower cooling facilitates to fabricate large parts of a machinery using metallic alloys in bulk ingot.

5.10.1 Preparation of metallic glasses

(a) Small scale production

The original method used for preparing metallic glasses was the *gun technique* (or) *splat cooling*. Here a small liquid globule is propelled (pumped) by means of a shock tube and spraying it to form a thin foil (coating) on copper substrate (Cu base). Here the substrate is curved so that the *centrifugal force promotes* a good thermal contact between liquid layer (sprayed) and the substance. In this method heat is extracted (absorbed) from the melt from only one side. This limitation is eliminated in an improved method in which the liquid globules are *squeezed* between a *fast-moving* piston and a fix *anvil* that are lined with copper. This copper coating removes the heat from melt (liquid sprayed) quickly. An advantage of this *piston-anvil method* is that it provided a good parallelism between two faces of foil (cooled film) which is about 25mm in diameter and 40 μ m in thickness.

(b) Large scale (or) mass production

The laboratory methods discussed above are not suitable for large scale (or) mass production of metallic glasses. For practical applications we require metallic glasses in the form of thin sheets. Hence large effort was devoted to various methods so as to freeze the metallic melt rapidly in the form of a continuous ribbon. The principle of this method is to feed (pump) a continuous jet of liquid alloy on the outer rim of a rapidly rotating cylinder. Hence the molten alloy solidifies rapidly in to a thin ribbon (50 μ m thick) that is ejected out tangentially (to the rotating cylinder) at the rate as high as 6600 ft/min (or) 2200 m/min. The width of ribbon obtained is limited to few millimeters.

This method mainly involves the following steps.

- (a) A stream of melt is injected through a nozzle (small hole) into a rotating drum containing water, instead of pumping on to a rotating cylinder. Wires with circular cross sections can be formed.
- (b) A very rapid cooling at the rate of 10⁶ K/sec called *splat cooling* is obtained in this method.

5.10.2 Properties of metallic glasses

1. Composition

The metallic glasses are metal alloys which have *no long range atomic order*.

2. Metallic properties

They are all strong, *ductile, malleable brittle* and opaque.

3. Mechanical properties

They have *high rupture strength*, high toughness and better *corrosion resistance*.

4. Electrical and Heat properties

The *electrical resistivity* of metallic glasses is *high* and it does not vary with temperature. Their temperature coefficient can *even be zero or negative*. It has *negative temperature coefficient*, that is the electrical resistivity decreases as the temperature increases.

5. Magnetic properties

They have high *magnetic permeability* (μ is high) and hence exhibit *ferromagnetism*. They have *low magnetic losses*, saturation magnetisation with *low coercivity* and *zero magnetostriction*. (They will not elongate or contract under a.c.field producing varying magnetic flux).

5.10.3 Applications of metallic glasses

Due to their *unusual physical and chemical properties*, metallic glasses play an important role as *an engineering material*. Some of the applications of metallic glasses are listed below.

1. Electrical and electronic applications

- (a) Because of their *insensitivity to temperature*, they are used in electronic circuits.
- (b) Since their *resistivity is high*, they are widely used as *resistance elements* in electric circuits.

2. Thermal applications

The metallic glass $\text{Pd}_{80}\text{Si}_{20}$ in which Pd is replaced by chromium (Cr) up to 7% are used as *thermometers for measuring very low temperature*. Such a thermometer is called as *cryo thermometers* (cryo means low).

3. Safety applications

Since metallic glass can withstand high doses (amount) of radiations without a change in electrical properties, they are used in *fission and fusion reactor and environmental devices*.

4. Magnetic applications

- (a) Since metallic glasses possess *high vicker's hardness* (8 GPa) (vicker hardness is a standard for measuring the micro hardness of a material) and *good corrosion resistance*, they are used in the manufacture of *magnetic tape recording heads*.

- (b) The high value of *magnetic permeability* makes the metallic glasses ribbons and wires ideally suitable for generating harmonics when they are placed in suitable AC and DC and as antitheft tags for protecting *store merchandize* (articles for sale).
- (c) Because of their low magnetic loss, they are used in motors which *reduces the core loss* as much as 90% when compared to conventional crystalline magnets.
- (d) Super conducting metallic glasses like Zr_{75} and Rh_{25} are used in producing *high magnetic fields* and for magnetic *levitation effect* (magnetic suspension used in fast train track).
- (e) Metallic glasses are used in magnetic devices like *magnetic sensors* (or) *transducers, security systems* and *power transformer cores* and *scientific applications*.
- (f) Since metallic glasses may be regarded *as liquids with frozen structure*, they are ideally suited to produce materials for *low temperature transport* and *critical behaviour studies* and they are most suited for the *study of electrons in non crystalline metals*.

5.10.4 Metallic glasses as transformer core material

Since metallic glasses exhibit excellent ferromagnetic properties *with low magnetic loss, high permeability, magnetic saturation, low coercivity* and *zero magnetostriction effect with extreme hardness* they are ideally suited to produce *large sheets used as transformer cores*.

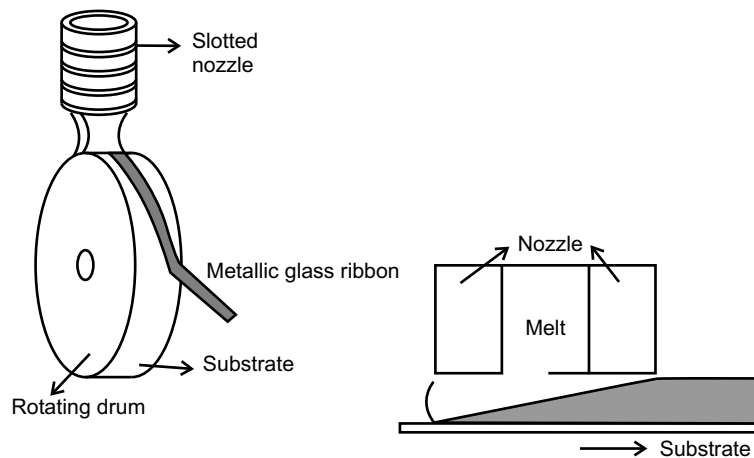


Fig.41 Planer flow casting (for the manufacture of ribbons of metallic glasses)

Core sheets (or) wide ribbons up to 15 cms can be produced by using a planar flow casting method in which a *narrow slitted nozzle for feeding molten alloy* is situated very close to rotating cylinder surfaces.

These large sheets of metallic glasses are widely used in power distribution transformers which convert high voltage electricity to power lines of 240 V used for domestic purposes (Step down transformer). Another *advantage in using* metallic glasses as core in transformer, is that the *size* of transformer becomes *so small* and *efficient also* in performance when compared to the conventional transformers which are very large in size.

5.11 Ceramics

5.11.1 Definition

Ceramics is usually refers to an *art of pottery*. That is, products such as pots, pans, bricks and tea cups made out of clay. But all these clay products come under *traditional ceramics*.

Another class of ceramics is known as *modern advanced ceramics*, which composed of materials containing *metallic and non metallic elements*, chemically bonded together *primarily ionic bond* or *covalent bonds*. They can be *crystalline (or) non crystalline* (or) mixtures of both.

5.11.2 Properties

1. **Mechanical property:**

Most of the ceramics have *high hardness*.

2. **Thermal property:**

They have *high temperature strength*.

3. **Chemical property:**

They possess a good *chemical resistance*.

4. **Electrical property:**

Generally they have *low electrical* and *thermal conductivities*, which makes them useful for *electrical and thermal insulations*.

5.11.3 Explanation

The exhibition of the above properties can be explained from the electronic behaviour of constituent atoms of the bulk material. The constituent of ceramics are metallic and non metallic atoms. The metallic elements release the outer most electrons (valence electrons) and non metallic atoms which receive these electrons. As a result of this transfer electrons are immobilized (not moving) and this situation indicates *the absence of electrons in the conduction band*. Hence a good *ceramic material* behaves as a good *insulator both thermally and electrically*. Thus the insulating character of ceramics is explained.

For heat chemical and mechanical resistance characters, the following explanation can be given. The metallic positive ions (after losing electrons) and negative ions

(after gaining electrons) develop a strong electrostatic attractive force between them. Each cation (positive ion) is surrounded in all sides by anions (negative charge). Hence a ***considerable energy is required to separate*** these two ions. That is why the ***ceramic materials have mechanical hardness***, thermal resistance (refractory) and chemically inertness (chemical resistance).

5.11.4 Classification of ceramics

Generally ceramic materials can be classified in two classes, namely

- (a) Traditional ceramics
- (b) Advanced ceramics (Technical ceramics and Engineering ceramics).

(a) Traditional ceramics

These ceramics includes clay made materials like brick, tile, sanitary ware, dinner ware, clay pipe and electrical porcelain, commonly used glass, cement abrasives and refractory materials also come under traditional ceramics. Usually the traditional ceramics are made from the following basic components clay, silica, flint and felspar. Also ***traditional ceramics*** are made from the ***materials (or) minerals*** obtained in their ***natural state***.

(b) Advanced ceramics

Advanced ceramics which is also known as technical ceramics or engineering ceramics are ***based on pure (or) nearly pure ceramic*** components or their ***combination***. The raw materials used for advanced ceramics must be processed carefully so that a controlled product can be produced. Also ceramics are made by using various composition mixing and processing techniques. Therefore the starting materials used for the manufacture of ceramics are ***not naturally occurring (got) materials***, but materials that have already undergone ***some chemical transformation and refinement***.

Examples: Alumina (Aluminium oxide Al_2O_3) Zirconia (ZrO_2), Silicon carbide (SiC) Silicon nitride (Si_3N_4), barium titanate (BaTiO_3) and high temperature ***super conductors***.

5.11.5 Classification based on function (or) application

Based on the primary function or application, ceramics are also subdivided into two categories namely ***structural ceramics and electronic ceramics***.

(a) Examples of Structural ceramics:

Engine components, cutting tools, valves, bearings, fibres, heat exchangers, whiskers and biological implants.

(b) Examples of Electronic ceramics:

Optical and magnetic materials.

- (c) The *electronic-magnetic-optic functions* include electronic substrates, electronic packages, capacitors, magnets, transducers, wave guides, displays and sensors.

5.11.6 Fabrication of advanced ceramics

A variety of process are used to fabricate advanced ceramics. The process chosen for a particular product is based on the material, shape, complexity, property requirements and cost. Generally ceramics fabrication processes can be divided into four categories.

1. Powder process
2. Vapour process
3. Chemical process
4. Melt process.

Out of these processes let us now discuss only powder process method.

Manufacturing of advanced ceramics by powder process method

Traditional clay-made ceramics, refractories and majority of advanced ceramics are fabricated by powder process method. The method involves several steps which are listed as below

1. Preparation of the starting powders.
2. Forming the desired shape (green forming).
3. Removal of water and organics.
4. Heating with (or) without the application of pressure to density the powder.
5. Finishing.

(1) Powder preparation

The starting powders for traditional ceramics are mainly clay and other natural minerals which are first processed to achieve purity and particle size of desired. Size hence the methods involves *mining, crushing* and *milling* or process similar to those operations used in other industries.

Manufacturing process of ceramics

The methods used to manufacture modern ceramics have many similarities with the manufactures of traditional ceramics. The mass production of traditional ceramics involves the process like *milling, and blending* the naturally occurring (available) materials into a *fine clay powder plastic mass*. Then this mass is shaped by using techniques like *injunction moulding, extrusion moulding or slip casting*.

Injunction moulding is a process in which the clay is forced in to a closed mould. But in **extrusion moulding** clay is forced through a die having required cross section. **Slip casting** is a process in which **clay-water slurry** is poured in to a process mould which **absorbs excess of water**. Then the shaped product is dried in air and then taken to a **kiln**. Here it goes a through a process called **sintering**. This process occur at a temperature **lower than the melting point** of clay mixture. Then the product is goes through process like finishing, glazing etc. Glazing is done by a wax like material which is melted on the surface to give a uniform coating on the surface of the product. It gives a **glassy appearance on the surface of the product**.

The advanced ceramic manufacture involves very similar steps. **But water is not used in these process**. Instead of water, **organic polymer binders** are added to convert the finely **grounded powder into a malleable mass**. Bur during **sintering** these organic chemicals are eliminated by vapourisation (Or) chemical combination (or) both. The application of pressure before sintering (or) during sintering gives a denser product (densifying process). This will remove the voids (or) pores in the powdered mass. This also helps to eliminate cracks and reduce the shrinkage.

5.11.7 Properties of advanced ceramics

1. **Hardness:**

Advanced ceramics are **hard, stiff and inert**.

2. **Conduction:**

They are **poor conductors** of heat and electricity.

3. **Temperature strength:**

They can **withstand very high temperature**.

4. **Melting point:**

They possess **high melting point** due to their strong chemical bonds (ionic-covalent bonds).

5. **Wear resistance:**

They have **high wear** resistance.

6. **Stability:**

They are **chemically** very stable.(**inert**)

7. **Corrosion resistance:**

They have **very high corrosion** resistance.

8. **Fracture toughness:**

They have **very low fracture toughness** (brittle fracture) of about 1. Some ceramics have fracture toughness vary from 3 to 12. They are 50 times more susceptible brittle fracture than metals. But **fracture toughness can be increased** by using materials with no voids and chemical impurities for the fabrication of ceramics.

5.11.8 Applications of ceramics

The selection of materials for engineering applications is based on performance requirements, cost and other factors.

The general advantages of advanced ceramics over metals and polymers are,

- (a) High *temperature strength*
- (b) *Chemical stability*
- (c) *Wear resistance* and
- (d) *Corrosion resistance*.

Let us now discuss the various applications of ceramics in different fields.

(a) Structural applications

Advanced ceramics like silicon nitride, zirconia and alumina are used for manufacturing *cutting tools* and *as mechanical seal* which can encounter the heat and corrosion in the environment. Their fracture toughness can also be increased by reinforcing ceramic matrix with ceramic fibre.

(b) Electronic applications

Based on the specific composition and crystal structure ceramic exhibit *either insulator* (or) *fermi conductor* (or) *conductor behaviour* and very often they can change from one to another. They exhibit some useful *magnetic and electronic properties*.

Because of their above mentioned special characteristics, they are used as substrates for *electronic circuits, electronic packages, capacitors magnetic components electronic sensors piezoelectric components* and *integrated optic components*.

(c) Semiconductor applications

Since some ceramics exhibit semiconducting properties, they are used to fabricate semiconductor devices, one of the example is *thermistor* (or) a temperature sensitive resistor used for controlling and measuring temperature. Because of their transition property from non conducting to conducting, they are used as *varistor* which is a *variable resistance* device. *Zinc oxide* is an example of advanced ceramic material shows this switching over property from insulation barrier to metallic conduction, when applied voltage is increased. Switching voltage can also be controlled by varying the composition.

(d) Piezoelectric effect and piezoelectric ceramic applications

If a *mechanical pressure* applied on a quartz crystal is changed, a *varying voltage* which is proportional to the applied pressure is produced across the crystal in perpendicular direction. This is known as *piezoelectric effect* and the crystal is known as *piezoelectric crystal*.

Ceramic materials which exhibit piezoelectric effect are called piezoelectric ceramics. Lead zirconate titanate (PZT) is an excellent example of piezoelectric ceramics. Piezoelectric ceramics are used in *phonograph pickups, microphones, gas lighters* etc. When a PZT crystal is subjected to a mechanical pressure, it produces a very high voltage. This high voltage produces a spark due to breakdown of air and this spark is used to ignite the gas in gas stove and gas ovens.

Piezoelectric ceramics are used in *quartz watches, ultrasonic scanners in medicines and SONAR* (Sound Navigation and Ranging) used in submarines.

(e) Magnetic applications of ceramics

Ferrites, a class of magnetic ceramics used in *magnetic cores memories*, power transformer, T.V and Radio sets (as ferrite antenna) *recording* and *reading heads* for magnetic tape records and *bubble memory cores*.

(f) Ferroelectric ceramic applications

These ceramics have a unusual ability of converting *electrical signals* into *mechanical energy* such as sound.

They can also change *the sound, pressure* or *motion into electric signals*. Thus they function as *transducers*. (A device used to convert one energy into another). *Barium titanate* (BaTiO_3) is a good example of ferroelectric ceramics.

The *non conducting property* of ferroelectric ceramics is used in the manufacture of *capacitors* to store electric charge in different electric and electronic circuits, BaTiO_3 has a high value of dielectric. Its *dielectric constant* vary from *1200 to 1500*.

(g) Superconducting ceramics

When a super conductor is cooled below a certain temperature called *critical temperature* T_C , it *transforms from normal conductor to a superconductor*. Above T_C , it again reverts back in to a normal conductor. High value T_C superconducting ceramics like $\text{Ba}_2\text{Cu}_3\text{O}_7$ ($T_C = 90^\circ\text{K}$) and $\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ ($T_C = 125^\circ\text{K}$) are used in *power transmission, electromagnets and transformer windings, coil windings* in electrical machines and super *conducting ring in computers as storage element*. The main advantage of using super conducting ceramics is that they consume very low *power (0.5 W)*.

(h) Applications in automation

Advanced ceramic components are used in automotive and other engines. The automotive engines includes *turbo charger rotors, valves, valve guides, fuel injector nozzles, exhaust-port inserts and advanced diesel engine components*.

(i) Application of ceramics as insulator in space shuttle orbiter

Since advanced ceramics have high thermal strength, they are used in *thermal protection system* for the space shuttle orbiter and the space shuttle can be used for at least 100 missions. This *thermal insulation tile material* attached to the body of space shuttle orbiter is known as *high temperature reusable surface insulation* (HRSI). They are also used as *aerospace hardwares*.

(j) Medical applications

A number of advanced ceramics are used as *implants in the human body* especially for hip joint and dental implants and they are called as *biomedical materials*.(ceramics)

(k) Telecommunication application

Silica (SiO_2) is used to make *optical fibres* cables used for telecommunication purposes. Super plastic materials (ceramics) are used in advanced *nano phase* materials.

(l) Wear resistance and corrosive resistance applications

Since advanced ceramics have high value of *wear resistance* and *corrosive resistance*, they are used as *extrusion dies* and *pump components*.

5.12 Bio-Materials

Bio-materials are the materials which can be implanted in the human body under required medical conditions so that it heals the defect or wound in the body without creating any side effect. i.e. Bio-materials are completely accepted by human body system.

Bio-materials are nowadays used in the following categories:

1. Metallic bio-materials
2. Ceramic bio-materials
3. Polymeric bio-materials

1. Metallic bio-materials

Used as implants in human body. Implants are used in bone replacement, strengthening of bones and to connect broken bones.

Example: Stainless steel (ASTMF - 138 and 139) - Implant devices, bone screws.

Proto sul - 10 (Co-Ni-Cr-Mo) for Hip joint.

2. Ceramic bio-materials

Ceramic bio materials are used to prepare artificial bones. They are also used in dental and orthopaedic purposes.

Example: Apatite ceramics: synthetic bone.

Synthetic hydroxyapatite: dental and orthopaedic raw materials.

3. Polymeric bio-materials

Polymeric bio-materials find vast usage in many areas of medicine.

Example:

- PTFE - Polytetra fluoro ethylene: To prepare artificial valves
- PMMA - Polymethyl methacrylate: To prepare contact lenses and bone cement.
- PVC - Poly Vinyl chloride: To prepare artificial blood vessels and heart components.
- Polyesters - Used in preparing artificial lungs, livers and blood vessels
- Nylons - To prepare artificial joints.

Short Answer Questions

1. Define magnetic flux and magnetic induction and flux density. Mention their units?
2. Define magnetic permeability and susceptibility. Mention their units?
3. Define magnetic relative permeability, Intensity of magnetisation and magnetic field intensity (H). Give the relation between relative permeability and susceptibility.
4. What is Bohr magneton?

Ans: When an atom is placed in an external magnetic field, the orbital magnetic moment of electrons is quantized (have certain definite minimum value). A quantum (measure) of magnetic moment of an atomic system (Atomic magnetism) is known as Bohr magneton represented by μ_B .

$$\left(\mu_B = \frac{eh}{4\pi m} \right)$$

e = charge of electron

m = mass of electron

h = planck's constant

5. What is curie constant and weiss curie law?
6. What are domains?

7. What is Heissenberg theory on ferromagnetism?
8. What is hysteresis and hysteresis loss?
9. Define retentivity and coercivity of a magnetic material?
10. Define energy product of a magnetic material? Give its importance.
11. What are hard and soft magnetic materials? Give examples. Mention their uses.
12. What are ferrites? Mention their applications?
13. What are dielectrics? Mention some applications of dielectrics.
14. Define electric field intensity (E), electric flux (ψ) and Electric flux density (or) electric displacement vector (D). Mention their units?
15. Define electric permittivity and relative permittivity (Dielectric constant)?
16. Define dipole and dipolemoment.
17. Define electric polarisation (or) polarisation vector?
18. Define polarzability (α).
19. Name the four types of polarisation process and mention their frequency range of the applied field.
20. What is electronic and Ionic polarisation? Mention their ranges of frequency of applied field.
21. What is orientation and space charge polarisation. Mention their frequency ranges of the applied field.
22. Explain the terms dielectric loss and loss tangent?
23. Explain the significant of dielectric loss and list the factors affecting the dielectric loss?
24. Mention same characteristics for the selection of a dielectric material?
25. Explain different types of insulating (dielectric) materials and give examples?
26. Name different types of dielectric breakdown mechanism.
27. Give some examples of active dielectrics and mention their applications?

Ans: piezo electrics and pyroelectrics are two classes of active dielectrics.

(a) **Piezo electrics:**

Barium titanate (BaTiO_3) potassium dihydrogen phosphate (KDP) and Lithiumniobate (LiNbO_3).

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(b) **Pyroelectrics:**

Barium titanate (BaTiO_3) Lithium Niobate (LiNbO_3)

Uses: used for making, pressure transducers, ultrasonic transducers & microphones.

triglycine sulphate (TGS).

Uses: For making high sensitive infrared detectors.

28. Explain intrinsic breakdown mechanism and thermal breakdown mechanism in dielectrics.
29. Explain discharge breakdown mechanism and electro chemical breakdown mechanism in dielectrics.
30. Explain defect breakdown mechanism.
31. Explain the term internal field (or) local field.
32. What are bio materials? Mention their applications in medicine?

A **bio material** can be defined as a **synthetic material** used to **replace the part** of a **living system** or to function in intimate contact with the living tissue.

A **bio material** can be used for any period of time as a part of **system** which **treats, augments or replaces** any tissue organ or function of the body. **The bio material** should possess **compatibility** (matching characteristics) with human cells and tissues and **satisfy many** aspects with the **required properties** to act as implant materials.

Classification: Metallic biomaterials, ceramic biomaterials, polymeric biomaterials.

Eg:

- (a) **Bio plates** are implants which are used to aid in the fixation of **long bone fractures**.
 - (b) **Sutural**, screws used to **assist healing**.
 - (c) **Probes** and catheters used to **aid diagnosis**.
 - (d) **Cardiac pacemaker** used to improve **function of heart**.
 - (e) **Ceramic bio materials** used in **dentistry** and to replace **heart valves**.
 - (f) **PVC used** in dialysis devices polypropylene (PP) used in **disposable syringes**.
33. What are metallic glasses? Give some examples.
 34. Mention some unique properties of metallic glasses?
 35. List out some applications of metallic glasses.

36. What are the advantages of using metallic glasses as transformer core? (AU)
37. What are the nano particles?
38. Explain thermal crystallisation and mechanical deformation process to produce nano particles.
39. How does the properties of nano particles vary with size?
40. Explain the magnetic properties of nano particles.
41. Mention some special (unique) properties of nanophase materials.
42. Give some applications of nanophase materials.
43. What are shape memory alloys? Give examples. (AU)
44. What is one way and two way shape memory? Explain. (AU)
45. Mention two types of crystal structure exhibited by SMA? (martensite & austenite)
46. Mention some general characteristics of SMA.
47. Explain the thermo mechanical behaviour of SMA.
48. List out some applications of SMA. (AU)
49. What are ceramics? How they are classified. Give the general properties of ceramics.
50. Give some examples of advanced ceramics?
51. Mention the methods involved in the manufacture of advanced ceramics.
52. What is Green forming and Hot pressing?
53. Mention some applications of advanced (in electronic & structural fields) ceramics.
54. What are piezoelectric ceramics mention their applications. (AU)
55. What are the ferroelectric ceramics mention their applications. (AU)
56. What is the advantage of using ceramics for the fabrication of capacitors?
57. What are the superconducting ceramics? (AU)
58. List out the uses of semiconducting and superconducting ceramics.
59. What are ferrites? Mention some of their uses? (AU)
60. What is ceramic tile insulation (space shuttle orbiter).

Review Questions

1. Compare dia, para and ferromagnetism.
2. What is ferromagnetism? Explain weiss theory (or) Domain theory of ferromagnetism and explain how it explains the ferromagnetic properties.
3. Explain the term 'Hysteresis' and how hysteresis curve can be drawn for a ferromagnetic material. Explain how the hysteresis curve can be explained by domain theory.
4. What are the antiferromagnetic materials and ferrimagnetic materials. Discuss their properties and applications. Give examples.
5. Describe Heissenberg's theory of ferro magnetism (or) Heissenberg's criteria on internal field?
6. Explain energy product of a magnetic material and its application in electronic field?
7. Describe dielectric property in terms of band theory of solids and mention some applications of dielectrics.
8. Mention different types of polarisation mechanisms involved in a dielectric material? Derive an expression on the internal field (or) Lorentz field (or) local field produced in a dielectric under the external electric field. Hence derive claussius - Mosotti equation used for the calculation electronic polarisation (α_e) for a cubic structure. (AU)
9. Explain briefly the different kinds of dielectric breakdown? Explain the significance of dielectric breakdown.
10. Describe briefly electronic polarisation and ionic polarisation. Derive expressions for electronic and ionic polarizabilities.
11. Discuss space charge polarisation and orientation polarization. Derive expression for orientation and space charge polarizabilities.
12. Discuss briefly the effect of frequency of the applied field and temperature on polarisation and dielectric constant.
13. Explain the term "Dielectric loss". Derive an expression for the power factor of dielectric and loss tangent. Explain the significance of dielectric loss. (AU)
14. What are shape memory alloys? Explain their characteristics. List their applications.
15. What are nano phase materials? How their physical properties vary with geometry. Mentional their applications. (AU-2003)
16. What are metallic glasses? How are they prepared? Discuss some of their applications (or) Explain their use as transformer core material. (AU-2003)
17. What are ceramics? What are characteristics? How they are manufactured? Explain their mechanical, electrical and magnetic applications.
18. Discuss ferroelectric ceramics, piezoelectric ceramics. Superconducting ceramics and their applications.