Paper Code : 11128

B.Sc. I Semester Degree Examination (NEP), April 2022
Subject : MATHEMATICS
Paper : Algebra – I and Calculus – I
Paper : DSC 1

Time : 2 Hours   Max. Marks : 60

***Instruction** : Answer **all** the Sections.*

SECTION – A

Answer **any five** of the following : **(5×2=10)**

1. a) Define symmetric matrix.

   b) Verify the system
   $x + y - z = 1$
   $4x + 4y - z = 2$ and
   $6x + 6y + 2z = 3$ for consistency.

   c) Define pedal equation.

   d) Find the n^{th} derivative of $\log[3x + 4]$.

   e) Define Rolle's theorem.

   f) Evaluate $\lim\limits_{n\to\infty}\left[\dfrac{n^2}{3n^2+2n+1}\right]$.

   g) Find the n^{th} derivative of $y = \sin^{-1}\left[\dfrac{2x}{1+x^2}\right]$.

SECTION – B

Answer **any four** of the following : **(4×5=20)**

2. Find for what values of λ and μ the system
   $x + 2y + z = 3$
   $x + 2y + 2z = 6$ and
   $x + \lambda y + 3z = \mu$ has unique solution.

3. Show that square of radius of curvature at any point (r, θ) on the curve $r = a[1+ \cos\theta]$ is $\dfrac{8ar}{9}$.

P.T.O.

4. Find the envelope of $y = mx - a\sqrt{1+m^2}$ where 'm' is the parameter.

5. Find $\frac{d^n}{dx^n}\left[\sin^6 x\right]$.

6. If $\cos^{-1}y = \log[x^2 - 2x + 1]$ show that

$(x-1)^2 y_{n+2} + (2n+1)(x-1) y_{n+1} + (n^2+4) y_n = 0$.

7. Find 'm' and 'n' such that $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ mx+n, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$ is a continuous function.

SECTION – C

Answer **any three** of the following : **(3×10=30)**

8. a) Reduce $\begin{bmatrix} 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 6 & 4 & 6 \\ 7 & 8 & 5 & 10 \end{bmatrix}$ to the echelon form and hence find the rank. **6**

b) Find the real μ for which the system

$3x + y + 2z = \mu y$

$2x + 3y + z = \mu z$

$x + 2y + 3z = \mu x$ posses a non-trival solutions. **4**

9. a) Show that the following pairs of curves are orthogonal, $r^2 = a^2 \sin 2\theta$ and $r^2 = b^2 \cos 2\theta$. **6**

b) Find the radius of curvature for $y^2 = x^3 + 8$ at $(-2, 0)$. **4**

10. a) State and prove Lagranges mean value theorem. **6**

b) Show that $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$ using Maclaurins expansion. **4**

11. a) Find the n^{th} derivative of $\frac{6x}{x^3 - x^2 - 4x + 4}$. **5**

b) Evaluate $\lim_{x \to 0}\left\{\frac{1}{\tan x} - \frac{1}{x}\right\}$ by using L-Hospital's Rule. **5**

12. a) Trace the curve $y = a\cosh\left(\frac{x}{a}\right)$, $(a > 0)$ catenary. **6**

b) Find the n^{th} derivative of $y = \sin(ax + b)$. **4**

Paper Code : 11128

B.Sc. I Semester Degree Examination (NEP), April 2023
Subject : MATHEMATICS (Paper – I)
Paper : DSC – I : Algebra – I and Calculus – I

Time : 2 Hours | Max. Marks : 60

*Instruction : Answer **all** the Sections.*

SECTION – A

1. Answer **any five** of the following : **(5×2=10)**

a) Define skew symmetric matrix.

b) Verify the system $x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$ for consistency.

c) Find the angle between the radius vector and tangent to the curve $r = a(1 + \cos\theta)$.

d) Evaluate $\lim_{x \to 0} \frac{\tan 3x}{\tan 5x}$.

e) Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 4$ on $[1, 4]$.

f) Find the n^{th} derivative of e^{ax}.

g) Find the n^{th} derivative of $\sin(3x + 2)$.

SECTION – B

Answer **any four** of the following : **(5×4=20)**

2. Verify the Cayley-Hamilton theorem for $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.

3. Verify the system
$$\begin{aligned} x + y - z &= -4 \\ x - 2y + 3z &= 5 \\ 4x + 3y + 4z &= 7 \end{aligned}$$
for consistency and hence solve.

4. Find the pedal equation for the curve $r^2\cos 2\theta = a^2$.

5. Discuss the continuity of $f(x)$ at $x = 2$

$$\text{if } f(x) = \begin{cases} x + 1, & x < 2 \\ 3, & x = 2 \\ 5 - x, & x > 2 \end{cases}.$$

P.T.O.

6. Expand sinx in Maclaurin's series.

7. If $y = \sin(m \sin^{-1}x)$ then show that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)y_n = 0$.

SECTION – C

Answer **any three** of the following : **(10×3=30)**

8. a) Find the Eigenvalues and Eigenvectors of $\begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$. **6**

b) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$ to the echelon form and hence find the rank. **4**

9. a) Find the radius of curvature at any point of the curve $r^n = a^n \cos n\theta$. **6**

b) Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$. **4**

10. a) Expand logsinx about a upto the term containing $(x - a)^3$ by using Taylor's expansion. **6**

b) Evaluate $\lim_{x \to 0}\left[\frac{1}{\tan x} - \frac{1}{x}\right]$. **4**

11. a) If $y = e^{\sin^{-1}x}$ then show that $(1 - x^2)y_2 - xy_1 = 0$ and hence prove that

$$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2y_n = 0.$$ **6**

b) Find the n^{th} derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$. **4**

12. a) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$ $(a > 0)$. **6**

b) Find $\frac{ds}{dt}$ for the curve $x = a[\cos t + t \sin t]$ and $y = a[\sin t - t \cos t]$. **4**

Paper Code : 11128

B.Sc. I Semester Degree Examination (NEP), March/April 2024
Subject : MATHEMATICS Paper – I
Paper : Algebra – I and Calculus – I

Time : 2½ Hours

Max. Marks : 60

*Instruction : Answer **all** the Sections.*

SECTION – A

Answer **any five** of the following. (5×2=10)

1. Define Symmetric matrix and given example.
2. Verify the system
   $x + y - z = 1$, $4x + 4y - z = 2$, $6x + 6y + 2z = 3$ for consistency and hence solve.
3. Find the angle between the radius and tangent to the curve $\frac{m}{r} = 2 + e\cos\theta$.
4. Evaluate $\lim_{x \to 0}\left(\frac{\sin px}{\tan qx}\right)$.
5. State the Rolle's theorem.
6. To find the n^{th} derivative of $\log(4 - 4x + x^2)$.
7. Define Envelope.

SECTION – B

Answer **any four** of the following. (4×5=20)

8. Find the value of λ for which the system. $x + y + z = 1$; $x + 2y + 4z = \lambda$; $x + 4y + 10z = \lambda^2$ is consistent and hence solve.
9. Show that the pairs of curves are orthogonal, $r = 9\sec^2\left(\theta/2\right)$ and $r = 3\operatorname{cosec}^2\left(\theta/2\right)$.
10. Find the envelope of the family $\frac{m^2}{x}\cos\theta - \frac{n^2}{y}\sin\theta = k^2$, where θ is parameter.

P.T.O.

11. Evaluate $\lim_{\theta \to 0} \left(\frac{e^{\sin\theta} - e^{\theta}}{\sin\theta - \theta} \right)$.

12. Find the n^{th} derivative of sinh3xsin3x.

13. If $y = \cos p\theta$ and $x = \sin\theta$ show that

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 - p^2) y_n = 0.$$

SECTION – C

Answer **any three** of the following.

14. a) Reduce $\begin{bmatrix} 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 6 & 4 & 6 \\ 7 & 8 & 5 & 10 \end{bmatrix}$ to the echelon form and hence find the rank. 6

b) Find Eigen values and Eigen vectors of $A = \begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$. 4

15. a) Find radius of curvature for the curves at the specified point $xy = a^2$ at (x, y). 6

b) Find all the asymptotes of the curves $y^2 = 4ax$. 4

16. a) State and prove Lagrange's mean value theorem. 6

b) Obtain the Taylor's expansion of tanx about $\pi/4$ upto the term containing $\left(x - \frac{\pi}{4} \right)^3$. 4

17. a) Find the n^{th} derivative of $\frac{6x}{x^3 - x^2 + 4x + 4}$. 6

b) Evaluate $\lim_{x \to 0} \left(\frac{1}{\tan x} - \frac{1}{x} \right)$ by using L-Hospital's rule. 4

18. a) Trace the curve cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. 6

b) Find the n^{th} derivative of $y = \sin(ax + b)$. 4