Paper Code : 95510

I Semester Degree Examination (NEP), April 2022
Subject : MATHEMATICS
Paper : Mathematics – I
Paper : OE1

Time : 2 Hours | Max. Marks : 60

***Instruction** : Answer **all** Sections.*

SECTION – A

Answer **any five** of the following. **(5×2=10)**

1. a) Define symmetric matrix. Give an example.

   b) Define a Rank of Matrix.

   c) Determine whether the system

   $$x + 2y + 3z = 0$$
   $$3x + y + 2z = 0$$
   $$2x + 3y + z = 0$$

   possesses a non-trivial solution.

   d) Discuss the continuity of f(x) at the point x = 0. $f(x) = \begin{cases} 2x - 1, & \text{if } x < 0 \\ 6 - 2x, & \text{if } x \geq 0 \end{cases}$

   e) Expand the series expansion of e^x is power of x.

   f) Evaluate $\lim_{x \to 0} (\sin x)(\log x)$

   g) Find the n^{th} derivative of log(ax + b) w.r.t. x.

SECTION – B

Answer **any four** of the following. **(4×5=20)**

2. Find the rank of $\begin{bmatrix} -8 & -1 & -3 & 4 \\ 0 & 3 & 2 & 2 \\ 8 & 1 & 3 & 6 \end{bmatrix}$ by reducing to normal form.

3. Solve the system

   $$x + 17y + 4z = 0$$
   $$2x - y - 3z = 0$$
   $$-3x + 5y - 4z = 0$$
   $$x + y + z = 0$$

4. If $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$, then prove that $\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$.

5. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ over [2, 3].

6. Find the n^{th} derivative of $y = e^{ax}\cos(bx + c)$.

7. Find the $D^n(\sin^3 x)$.

SECTION – C

Answer **any three** of the following **(3×10=30)**

8. a) Verify Cayley-Hamilton theorem for $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. Also find the inverse of the given matrix. **6**

   b) Find the rank of matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$ by reducing to echelon form. **4**

9. a) Check for consistency and solve

   $x + 2y + 3z = 14$

   $3x + y + 2z = 11$

   $2x + 3y + z = 11$. **6**

   b) Find the Eigen values of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **4**

10. a) State and prove Lagrange's mean value theorem. **6**

    b) Evaluate $\lim_{n \to \infty} n(\sqrt{n^2 + 6} - n)$. **4**

11. a) Evaluate $\lim_{x \to 0} \left(\frac{x - \tan x}{x^2 \tan x} \right)$ **5**

    b) If $\cos^{-1} y = \log(x^2 - 2x + 1)$, show that $(x - 1)^2 y_{n+2} + (2n + 1)(x - 1)y_{n+1} + (n^2 + 4)y_n = 0$. **5**

12. a) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$ $(a > 0)$ Astroid. **6**

    b) Find the n^{th} derivative of $\frac{x + 1}{(2x - 3)(3x + 1)}$. **4**

Paper Code : 95510

I Semester B.A./B.Com./B.B.A./B.Sc./B.C.A. Degree
Examination (NEP), April 2023
Subject : MATHEMATICS – I (Open Elective)

Time : 2 Hours Max. Marks : 60

Instruction : *Answer **all** the Sections.*

SECTION – A

I. Answer **any five** of the following. **(2×5=10)**

1) a) Define symmetric matrix.

b) State Cayley Hamilton theorem.

c) Define eigen values.

d) Define successive derivatives.

e) State Rolle's theorem.

f) Define greatest lower bound (glb) or infimum.

g) If $y = \tan^{-1}x$ prove that $(1 + x^2)y_2 + 2xy_1 = 0$.

SECTION – B

II. Answer **any four** of the following. **(5×4=20)**

2) If A and B are symmetric then show that AB is also symmetric.

3) Verify Cayley Hamilton theorem for $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.

4) Find the value of a such that the Rank of $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 4 & 4 & 1 & -3 \\ 2 & a & 2 & 2 \\ 9 & 9 & 3 & a \end{bmatrix}$ is 3.

5) Solve the system of equations

$x - y + 2z = 0$

$4x + y + 5z = 0$

$3x + 2y + z = 0$.

6) Find eigen values of $\begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$.

7) Find the n^{th} derivative of $\cosh 2x.\sin^2 2x$.

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SECTION – C

III. Answer **any three** of the following. **(10×3=30)**

8) a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$ to the echelon form and find the Rank of the given matrix.

b) Find the n^{th} derivative of e^{ax}.

9) a) Find λ and μ for which the system

$x + 2y + z = 3$

$2x + 6y + 8z = 10$

$x + 3y + \lambda z = \mu$.

1) Unique solutions 2) No solutions.

b) Find the n^{th} derivative of $\sin^2 2x$.

10) a) Verify Rolle's theorem for $f(x) = x^2 - 5x + 6$ over [2, 3].

b) Evaluate $\lim_{x \to 0} \left\{ \frac{x^2 + 2x}{3x} \right\}$.

11) a) Obtain the Taylor's expansion of the tan x about $\frac{\pi}{4}$ upto the term containing $\left(x - \frac{\pi}{4}\right)^3$.

b) Evaluate $\lim_{x \to 5} \left\{ \frac{x^3 - 125}{x - 5} \right\}$.

12) a) Find the position and nature of the double points, $a^4y^2 = x^4(2x^2 - 3a^2)$.

b) Discuss the continuity of f(x) at x = 0 if $f(x) = \begin{cases} 2x - 1, & x < 0 \\ 2x + 1, & x > 0 \end{cases}$

Paper Code : 95510

B.A./B.Sc./B.Com./B.B.A./B.C.A. I Semester Degree Examination (NEP), March/April 2024

Subject : MATHEMATICS (Open Elective)

Paper : Mathematics – I

Time : 2½ Hours

Max. Marks : 60

Instruction : *Answer **all** the Sections.*

SECTION – A

Answer **any five** of the following. (5×2=10)

1. a) If A, B are symmetric, show that (A + B) is also symmetric.
   b) Define rank of a matrix.
   c) Define eigen values and eigen vectors of a matrix.
   d) Evaluate $\lim_{x \to 0} (\operatorname{cosec} x - \cot x)$.
   e) Obtain the series expansion of e^x in powers of x.
   f) Find the n^{th} derivative of e^{mx}.
   g) Find the n^{th} derivative of $x^3 e^{ax}$.

SECTION – B

Answer **any four** of the following. (4×5=20)

2. Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}$ to the echelon form and hence find its rank.
3. Verify Cayley – Hamilton theorem for $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and find A^{-1}.
4. Verify Rolle's theorem for the function $f(x) = x^2 - 4x + 8$ over [1, 3].
5. Evaluate $\lim_{x \to 0} \left\{ \frac{x - \tan x}{x^2 \cdot \tan x} \right\}$.
6. If $y = \tan^{-1} x$, then prove that $(1 + x^2)\, y_{n+2} + 2(n + 1)\, x y_{n+1} + n(n + 1) y_n = 0$.
7. Trace the curve $y^2(a - x) = x^3$, $a > 0$.

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SECTION – C

Answer **any three** of the following. (3×10=30)

8. a) Check the consistency of $x - y - z = 3; -x - 10y + 3z = -5; 2x - y + 2z = 2$ and hence solve. 5

b) Find the inverse of $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ by using elementary row operations. 5

9. a) Find the eigen values and eigen vectors of $\begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}$. 5

b) State and prove Cauchy's Mean Value Theorem. 5

10. a) Show that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$. 4

b) i) Find $\lim_{\theta \to 0} (\cot\theta - \operatorname{cosec}\theta)$.

ii) Find $\lim_{x \to 1} \left\{ \frac{x}{\log x} - \frac{1}{\log x} \right\}$. 6

11. a) Find the n^{th} derivative of $\frac{1}{(x+2)(x-1)}$. 4

b) If $y = \left[\log\left(x + \sqrt{1+x^2}\right) \right]^2$, then show that

$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0$. 6

12. a) Verify Cauchy's Mean Value Theorem for $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ in [1, 3]. 5

b) Find the n^{th} derivative of $\sin^2 x \cdot \cos^3 x$. 5