Paper Code : MATHOEC 12L

II Semester Open Elective Degree Examination (NEP), October 2022
Subject : MATHEMATICS – II
Paper : OE – II

Time : 2½ Hours

Max. Marks : 60

Instruction : *Answer all Sections.*

SECTION – A

I. Answer **any five** of the following : **(5×2=10)**

1) Define abelian group.
2) State Fermats theorem.
3) If $x = r\cos\theta$, $y = r\sin\theta$, show that $\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$.
4) Define homogeneous function.
5) State Euler's theorem.
6) Evaluate $\int_0^1\int_0^2 (x+y)\,dy\,dx$.
7) Evaluate $\int_0^1\int_0^2\int_1^2 xyz\,dx\,dy\,dz$.

SECTION – B

II. Answer **any four** of the following : **(4×5=20)**

1) If G is a group, then show that the identity element of G is unique.
2) Prove that every cyclic group is abelian.
3) Verify $\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}$ if $u = x$.
4) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

P.T.O.

5) Evaluate $\int_C (3x - 2y)dx + (y + 2z)dy - x^2 dz$, where C is $x = t$, $y = 2t^2$, $z = 3t^3$ and $0 \le t \le 1$.

6) Evaluate $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \sin(x + y)\, dx\, dy$.

SECTION – C

III. Answer **any three** of the following : **(10×3=30)**

1) a) Show that the cube root of unity form a group under multiplication.

b) Find the order of the elements of the group $G = (z_4, +_4)$.

2) a) If H is a subgroup of a group G, then $H^{-1} = H$.

b) Find all the left cosets of $H = \{0, 4, 8\}$ in $(z_{12}, +_{12})$.

3) a) If $u = xy(x + y)$, where $x = at^2$, $y = 2at$ find $\frac{du}{dt}$.

b) Find the homogeneous, function of degree if $f(xy) = \frac{x^4 - y^4}{x + y}$.

4) a) Using partial differentiation find $\frac{dy}{dx}$ if $y^2 = 4ax$.

b) Test for minima given $f(x, y) = x^2 + 2xy^2 + y^2 + 2x + y$.

5) a) Evaluate $\int_0^{\pi} \frac{dx}{a + b\cos x}, |b| < a$, hence deduce that

$$\int_0^{\pi} \frac{\cos x\, dx}{(a + b\cos x)^2} = \frac{-\pi b}{(a^2 - b^2)^{3/2}}.$$

b) If $u = e^{3x^2 + xy}$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

Paper Code : MATHOEC 12L

B.A./B.Sc./B.B.A./B.Com./B.C.A. II Semester Degree Examination (NEP), October/November 2023
Subject : MATHEMATICS – II (Open Elective)

Time : 2½ Hours | Max. Marks : 60

Instruction : Answer all Sections.

SECTION – A

I. Answer **any five** of the following. **(2×5=10)**

1) Define group.

2) Define cyclic group.

3) If $u = f\left(\frac{y}{x}\right)$ then show that $\frac{x\partial u}{\partial x} + \frac{y\partial u}{\partial y} = 0$.

4) Find the total differential of $\phi = \sin^{-1}\left(\frac{y}{x}\right)$.

5) If $x = r\cos\theta$, $y = r\sin\theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

6) Evaluate $\int_0^1\int_0^2 (x+y)\, dydx$.

7) Write any two properties of line integral.

SECTION – B

II. Answer **any four** of the following. **(4×5=20)**

1) Show that every cyclic group is an abelian group.

2) Find the number of generator of cyclic group $(z_{18}, +_{18})$. Write all the generators.

3) If $u = \sin(3x + 2y^2)$ then find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

P.T.O.

4) If $u = x^2(y - z) + y^2(z - x) + z^2(x - y)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

5) Show that $\int_C [(x + y)dx + (x - y)dy] = 0$ where 'C' is the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$.

6) Evaluate $\int_0^1 \int_1^2 (x^2 + y^2)\, dydx$.

SECTION – C

III. Answer **any three** of the following. **(3×10=30)**

1) a) State and prove Lagrange's theorem.

b) Show that the multiplicative group of cube root of unity is cyclic.

2) a) State and prove the Fermat's theorem.

b) Find all the right cosets of the sub-group H = {0, 3} in the group $(z_6, +_6)$.

3) a) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ then verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

b) If $u = x^3 + y^3 + z^3 - 3xyz$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$.

4) a) Verify the Euler's theorem for the function $u = y^3 \log\left(\frac{x}{y}\right)$.

b) If $u = x + y + z$, $v = y + z$, $w = z$ then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 1$.

5) a) Evaluate $\int_C [(x^2 - y)dx + (y^2 + x)dy]$ where 'C' is the curve given by $x = t$, $y = t^2 + 1$, $0 \le t \le 1$.

b) Evaluate $\int_0^1 \int_0^1 (x^2 + xy)\, dydx$.