Paper Code : MATHDSC 12L

B.Sc. II Semester Degree Examination (NEP), October 2022
Subject : MATHEMATICS
Paper : Algebra – II and Calculus – II (Paper – I)

Time : 2.30 Hours | Max. Marks : 60

***Instruction** : Answer **all** the Sections.*

SECTION – A

I. Answer **any five** of the following : **(5×2=10)**

1) Define Least upper bound of the set.
2) Find the Supremum and Infimum of $\left\{\frac{4n+3}{n}, \forall n \in N\right\}$.
3) Find the number of generators of a cyclic group of order 10.
4) Find all the right cosets of $H = \{1, -1\}$ in the multiplicative group $G = \{1, -1, i, -i\}$.
5) Find the total differential of $u = x^3 + y^3 + x^2y + xy^2$.
6) Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2)\, dydx$.
7) Evaluate $\int_0^1 \int_0^1 \int_0^1 xyz\; dxdydz$.

SECTION – B

II. Answer **any four** of the following : **(4×5=20)**

8) Show that the set of natural numbers has no limit points.
9) State and prove Lagranges theorem for the finite group.
10) Prove that a non-empty subset H of a group G is a subgroup of G if and only if $\forall a, b \in H \Rightarrow a. b^{-1} \in H$.
11) If $u = \log(\tan x + \tan y + \tan z)$. Then show that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$.
12) If $u = e^{\left(\frac{x^2y^2}{x^2+y^2}\right)}$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \log u$.
13) Evaluate $\int_0^1 \int_0^x \sqrt{x^2 + y^2}\, dydx$ by changing into polar Co-ordinates.

P.T.O.

Paper Code : MATHDSC 12L

SECTION – C

III. Answer **any three** of the following : **(3×10=30)**

14) a) Prove that the intersection of two closed sets is a closed set.

b) Define countable set and prove that the set of all integers z is Countable.

15) a) Prove that intersection of two sub groups is also a sub group.

b) If G is a Cyclic group generated by a, i.e., G = <a> then prove that 0(G) = 0(a).

16) a) If $x = r\cos\theta$ and $y = r\sin\theta$ then prove that $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$.

b) Verify Euler's theorem for the function $u = y^3 . \log(x/y)$.

17) a) If $u = \sqrt{xy}$ $v = \sqrt{yz}$, $w = \sqrt{zx}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1}{4}$.

b) Evaluate $\iint_R x^2y^2 \, dxdy$ where R is triangular region with vertices (0, 0), (2, 0) and (2, 3).

18) a) By changing the order of integration evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$.

b) Find the volume of the sphere $x^2 + y^2 + z^2 = 4$.

Paper Code : MATHDSC 12L

B.Sc. II Semester Degree Examination (NEP), Oct./Nov. 2023

Subject : MATHEMATICS (Paper – I)

Paper : Algebra – II and Calculus – II

Time : 2½ Hours Max. Marks : 60

Instruction : *Answer **all** Sections.*

SECTION – A

I. Answer **any five** of the following : **(5×2=10)**

1) a) Define upper bound.

b) Define semi group.

c) Define cyclic group.

d) Define partial differentiation.

e) Define line integral.

f) Define homogeneous function.

g) State generalized rule of integration by parts.

SECTION – B

II. Answer **any four** of the following : **(4×5=20)**

2) If $H = \{1, -1\}$ is a subgroup of G. Then $H^{-1} = H$.

3) The order of any integral of an element 'a' can not exceed the order of 'a' (i.e. $O(a^K) \leq O(K)$).

4) If $u = \cos^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{x}{y}\right)$ show that $xu_x + yu_y = 0$.

5) Evaluate $\int_1^2 \int_0^x \frac{dydx}{y^2 + x^2}$.

6) If $u = e^{3x^2 + xy}$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

7) Evaluate $\int_C (2y + x^2)dx + (3x - y)dy$ along the curve $x = 2t$, $y = t^2 + 3$ where $0 < t < 1$.

P.T.O.

SECTION – C

III. Answer **any three** of the following : **(3×10=30)**

8) a) Find all the cosets of the subgroup H(0, 3) in the group $(Z_6, +_6)$.

b) Find the order of the cyclic group G of order $(Z_{12}, +_{12})$ generated by 3.

9) a) If $z = e^{ax+by} \sin(ax - by)$, show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

b) If $u = x^3 + y^3 + z^3 - 3xyz$, show that $xu_x + yu_y + zu_z = 3u$.

10) a) Find the total differential of the following $u = xy^2 + yz^2 + zx^2$.

b) Find the total derivative of the function $u = xy^2 + x^2y$ where $x = 2at$, $y = at$.

11) a) Define Jacobian if $u = x + y$, $v = \frac{1}{x+y}$. Show that $\frac{\partial(u, v)}{\partial(x, y)} = 0$.

b) Evaluate $\int_C (xydx + yzdy + zxdz)$ where C is the curve given by $x = t$, $y = t^2$, $z = t^3$ and t is varying from – 1 to 1.

12) a) Evaluate $\int_0^2 \int_1^2 \int_1^2 \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right) dxdydz$.

b) Find the volume of the region above the xy-plane bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 1$.