B.Sc. III Semester Degree Examination (NEP), April 2023
Subject : MATHEMATICS (Paper – I)
Paper : DSC – I : Ordinary Differential Equations and Real Analysis – I

Time : 2 Hours

Max. Marks : 60

Instruction : *Answer all the Sections.*

SECTION – A

Answer **any five** of the following :

(5×2=10)

1. a) Solve $P^2 + P - 6 = 0$.

   b) Define orthogonal trajectory.

   c) Solve $[D^2 - 6D + 9] y = 0$.

   d) Define Cauchy's Euler equation.

   e) Define a sequence.

   f) State the comparison test.

   g) Test the convergence of the series

   $$1 + \frac{5}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \dots$$

SECTION – B

Answer **any four** of the following :

(4×5=20)

2. Find the general and singular solution of $y = px + 1 - p^2$.

3. Solve $[D^2 - 5D + 6] y = \sin 3x$.

4. Solve by variation of parameter $y_2 + y = \sec x$.

5. Show that the sequence $\left\{ \frac{n}{n^2 + 1} \right\}$ is bounded.

P.T.O.

6. Test the convergence of the series

$$\frac{1}{1.3.5} + \frac{2}{3.5.7} + \frac{3}{5.7.9} + \ldots\ldots$$

7. Find the nature of the series

$$x + \frac{1.2}{1.3} x^2 + \frac{1.2.3}{1.3.5} x^3 + \ldots\ldots \quad (x > 0).$$

SECTION – C

Answer **any three** of the following :   (3×10=30)

8. a) Solve $[x^2D^2 + xD - 4]y = x^2$.   6

   b) Verify the condition of the integrability and solve $(y + z) dx + dy + dz = 0$.   4

9. a) Solve $\dfrac{dx}{1} = \dfrac{dy}{-2} = \dfrac{dz}{3x^2 \sin(y + x)}$.   6

   b) Solve $y = 3x + \log P$.   4

10. a) Solve $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$.   6

    b) Find the orthogonal trajectory of $xy = a^2$.   4

11. a) Show that every monotonically increasing sequence which is bounded above converges to its least upper bound.   6

    b) Test the convergence of the series

    $$1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \ldots$$ 4

12. a) If a sequence $\{x_n\}$ converges to the limits $l > 0$ then show that $\exists\, m \in N$ such that $x_n > 0\ \forall\, n \geq m$.   6

    b) Find the nature of the series $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{2}{n}\right)^{n^2}$.   4

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Paper Code : MATDSC 13L

B.Sc. III Semester Degree Examination (NEP), March/April 2024
Subject : MATHEMATICS Paper – I
Paper : Ordinary Differential Equations and Real Analysis – I

Time : 2½ Hours

Max. Marks : 60

Instruction : Answer **all** Sections.

SECTION – A

I. 1) Answer **any five** of the following. (5×2=10)

a) Solve $p^2 + 2px - 3x^2 = 0$.

b) Solve $[D^2 - 7D + 12]y = 0$.

c) Define complementary function and particular integral.

d) Define convergent sequence with an example.

e) Show that sequence $\{x_n\}$ where $x_n = 3 + \dfrac{1}{n}$ is monotonic.

f) State the Raabe's test.

g) Find the nature of $\displaystyle\sum_{n=1}^{\infty}\left(1+\dfrac{2}{n}\right)^{n^2}$.

SECTION – B

II. Answer **any four** of the following. (4×5=20)

2) Solve, $y = 2px + y^2p^3$.

3) Find the general and singular solution of the equation $y = px + p^2$.

4) Verify the condition of integrability and solve $(y + z)dx + dy + dz = 0$.

5) Show that "Every convergent sequence has a unique limit".

6) Using Cauchy's criterion of convergence, show that the sequence $\{x_n\}$ where

$x_n = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots\ldots + \dfrac{1}{n}$ is not convergent.

7) Find the nature of the series,

$$\dfrac{1}{1\cdot2\cdot3} + \dfrac{3}{2\cdot3\cdot4} + \dfrac{5}{3\cdot4\cdot5}.$$

P.T.O.

SECTION – C

III. Answer **any three** of the following.   (3×10=30)

8) a) Solve $(2x^2y+y^2)dx + (2x^3 - xy)dy = 0$.   6

   b) Solve $\dfrac{dx}{z^2y} = \dfrac{dy}{z^2x} = \dfrac{dz}{xy^2}$.   4

9) a) Solve, $\dfrac{dx}{\cos(x+y)} = \dfrac{dy}{\sin(x+y)} = \dfrac{dz}{z}$.   6

   b) Solve, $y = p\sin p + \cos p$.   4

10) a) Solve by Clairaut's form

   $y^2 - 2pxy + p^2(x^2 - 1) = m^2$.   6

   b) Solve, $\dfrac{dx}{mz-ny} = \dfrac{dy}{nx-lz} = \dfrac{dz}{ly-mx}$.   4

11) a) Prove that "Every monotonically decreasing sequence which is
      bounded below converges to its greatest lower bound".   6

   b) Discuss the convergence of the series.   4

   $$\sum_{n=1}^{\infty}\left[\left(\frac{n+1}{n}\right)-\left(\frac{n+1}{n}\right)^{n+1}\right]^{-n}.$$

12) a) Test the series $1-\dfrac{1}{4}+\dfrac{1}{7}-\dfrac{1}{10}+\ldots$ for   6

   i) Convergence

   ii) Absolute convergence

   iii) Conditional convergence.

   b) Show that the sequence $\left\{\dfrac{n}{n^2+1}\right\}$ is bounded.   4