B.Sc. IV Semester Degree Examination (NEP), October/November 2023
Subject : MATHEMATICS (Paper – I)
Paper : Partial Differential Equations and Integral Transforms

Time : 2½ Hours

Max. Marks : 60

Instruction : Answer all the Sections.

SECTION – A

Answer **any five** of the following.   (5×2=10)

1. Form the PDE by eliminating the arbitrary constants a and b from
   $z = (x + a)(y + b)$.

2. Solve $z = px + qy + (p^2 + q^2)$.

3. Define parabolic PDE.

4. Find the Laplace transform $e^{-4t} + 3e^{-2t}$.

5. Define convolution theorem.

6. If $f(x)$ is a periodic function of the period 2π in any interval of length 2π, then write the formulae for a_n.

7. Define sine half range Fourier series of $f(x)$ in $(0, L)$.

SECTION – B

Answer **any 4** of the following.   (4×5=20)

8. Form the PDE by eliminating the arbitrary functions $z = f(x + ay) + g(x - ay)$.

9. Solve $p\tan x + q\tan y = \tan z$.

10. Solve the homogeneous linear PDE with constant co-efficient of $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} = 0$.

11. If $L[f(t)] = F(s)$, then prove that $L[e^{at}f(t)] = F(s - a)$.

12. Find $L\left[\dfrac{e^{-at} - e^{-bt}}{t}\right]$.

13. Find the Fourier series of $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$.

P.T.O.

SECTION – C

(3×10=30)

Answer **any 3** of the following.

14. a) Find the complete integral of $p + q = \sin x + \sin y$.   4

    b) Solve $px + qy + pq = 0$ by Charpits method.   6

15. a) Solve the elliptic PDE $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = -1$.   4

    b) Reduce a PDE to its conical form and solving it.   6

$$\frac{\partial u}{\partial t} + 2x\left(\frac{\partial u}{\partial x}\right) = 0.$$

16. a) Find the Laplace transform of $t^2 . u(t - 2)$.   4

    b) Find the inverse Laplace transform of $\dfrac{1}{(s+2)(s+4)}$ using convolution theorem.   6

17. a) Find $L^{-1}\left[\dfrac{e^{-s}}{(s+1)(s-2)}\right]$.   4

    b) Obtain the Fourier series of $f(x) = x - x^2$ in $-1 < x < 1$.   6

18. a) Obtain the half range sine series of $f(x) = x^2$ in $0 < x < \pi$.   4

    b) Obtain the half range cosine series of $f(x) = e^x$ in $0 < x < \pi$.   6

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