Paper Code : MATHDSC 15L

B.Sc. V Semester Degree Examination (NEP), March/April 2024
Subject : MATHEMATICS Paper – I
Paper : Real Analysis – II and Complex Analysis

Time : 2½ Hours

Max. Marks : 60

Instruction : *Answer* ***all*** *the Sections.*

SECTION – A

Answer **any five** of the following. **(5×2=10)**

1. a) Define norm of the partition.

   b) Define Beta function.

   c) Examine the convergence of $\int_{0}^{\infty} \frac{1}{1+x^2}\,dx$.

   d) Write polar form of Cauchy-Riemann Equations (i.e. C-R Equation in polar-form).

   e) Define Cauchy's integral function.

   f) Define Bilinear transformation.

   g) Define $U(p, f)$ and $L(p, f)$.

SECTION – B

Answer **any four** of the following. **(4×5=20)**

2. If $f : [a, b] \rightarrow R$ is a bounded function and $p \in [a, b]$ then prove that $m(b-a) \leq L(p, f) \leq U(p, f) \leq M(b-a)$ where m, M are infimum and supremum of f on [a, b].

3. Show that $f(x) = x$ is R-integrable on [0, 1] also prove that $\int_{0}^{1} f(x)\,dx = \frac{1}{2}$.

4. State and prove the fundamental theorem of integral calculus.

5. Show that $f(z) = (\sin x \cosh y + i \cos x \sinh y)$ is analytic every where.

P.T.O.

Paper Code : MATHDSC 15L

6. Find the analytic function whose real part is $u = x^3 - 3xy^2$.

7. Find the Bilinear transformation which maps the points (2, 1, 0) into 1, 0, *i* respectively.

SECTION – C

Answer **any three** of the following. **(3×10=30)**

8. i) If $f : [a, b] \to R$ is monotonic on [a, b] then prove that *f* is also R-integrable on [a, b].

   ii) If $f : [a, b] \to R$ is a Riemann integrable then prove that $|f|$ is also Riemann integrable.

9. i) Using first mean value theorem show that $\frac{2\pi}{13} < \int_0^{2\pi} \frac{dx}{10 + 3\cos x} < \frac{2\pi}{7}$.

   ii) Define Gamma function and evaluate $\Gamma\left[-\frac{5}{2}\right]$.

10. i) Show that $\int_0^{\pi/2} \cos^5\theta \cdot \sin^2\theta \, d\theta = \frac{8}{105}$.

    ii) Evaluate $\int_{(0,1)}^{(2,5)} [(3x + y)\,dx + (2y - x)\,dy]$ along the curve $y = x^2 + 1$.

11. i) Evaluate $\int_C \frac{1}{z(z-1)} dz$ where C is a circle $|z| = 3$.

    ii) Evaluate $\int_C \frac{e^{2z}}{(z-2)^2} dz$ where C is a circle $|z| = 3$.

12. i) State and prove necessary condition for analytic function.

    ii) Find the analytic function $f(z) = u + iv$ given that $u + v = e^x[\cos y - \sin y]$.

Paper Code : MATHDSC 25L

B.Sc. V Semester Degree Examination (NEP), March/April 2024
Subject : MATHEMATICS Paper – II
Paper : Vector Calculus and Analytical Geometry

Time : 2½ Hours Max. Marks : 60

***Instruction** : Answer all Sections.*

SECTION – A

I. Answer **any five** of the following : **(5×2=10)**

1) Define the scalar triple product of three vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

2) Show that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)+\vec{b}\left(\vec{c}\times\vec{a}\right)+\vec{c}\left(\vec{a}\times\vec{b}\right)=\vec{0}$.

3) Define the surface integral.

4) Evaluate by Stokes theorem.
$\int_C\left(e^x dx+2ydy-dz\right)$ where 'C' is the curve $x^2 + y^2 = 4, z = 2$.

5) If 'r' represents the position vector of a point P then show that div r = 3.

6) Define the pair of planes.

7) Find the angle between the two planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$.

8) Define a conicoid equation of second degree in x, y and z.

SECTION – B

II. Answer **any four** of the following : **(5×4=20)**

1) Prove that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$.

2) Show that $\vec{A}(t)\times\frac{d\vec{A}}{dt}=0$, if and only if $\vec{A}(t)$ has a fixed direction.

3) Show that $\int_S(axi+byj+czk).ndS=\frac{4}{3}\pi(a+b+c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

P.T.O.

Paper Code : MATHDSC 25L

4) By using the Greens theorem evaluate $\int_C [(y - \sin x)dx + \cos x \, dy]$ where 'C' is triangle formed by the lines $y = 0$, $x = \frac{\pi}{2}$, $y = 2x$.

5) To find the equation of the plane which makes the intercept a, b and c on the co-ordinate axes.

6) To find the equation of straight line in symmetrical form.

7) Find the equation of sphere which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at (1, 2, −2) and passes through (0, 0, 0).

SECTION – C

III. Answer **any three** of the following : **(10×3=30)**

1) If $\vec{A}(t) = 2ti + t^2j - (t^2 + 1)k$, $\vec{B}(t) = (1 - t^2)i + j - t^2k$, $\vec{C}(t) = ti + t^2j + t^3k$ then find

i) $\frac{d}{dt}\left(\vec{A} \times \vec{B}\right)$

ii) $\frac{d}{dt}\left(\vec{A} . \left(\vec{B} \times \vec{C}\right)\right)$ at, $t = 1$.

2) Verify the Greens theorem for $\int_C [(3x^2 - 8y^2)dx + (4x - 6xy)dy]$ where 'C' is the boundary of the region between the curves $x = y$ and $y^2 = x$.

3) Prove that $\text{grad}(f.g) = f \times \text{curl } g + g \times \text{curl } f - (f.\nabla)g + (g.\nabla)f$.

4) a) Find the equation of planes bisecting the angle between the planes $x + 2y + 2z = 19$ and $4x - 3y + 12z + 3 = 0$.

b) To find the length of perpendicular from (−6, 2 , 6) on the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{5}$.

5) To find the length of perpendicular from a point to the plane.
